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Selection of appropriate variables in the prediction of the coefficient of performance of absorption heat transformers

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ABSTRACT

This research consists of a proposal of three methods that analyze the contribution of each operational variable in the prediction of the coefficient of performance of three different absorption systems aimed at reducing the independent variables for a faster and more efficient prediction. Methods considered in this study are correlation analysis, principal component analysis combined with correlation analysis, and Garson's method. The experimental information includes (i) an absorption heat transformer with duplex components, (ii) an absorption single-state heat transformer, and (iii) a double-absorption heat transformer. For each case, the three methods were applied, the results were discussed in order to find the coincidences and discrepancies.

Keywords: Lithium Bromide solution; Correlation coefficient; Covariance

1. Introduction

In the field of the engineering processes, the relationships between operational variables and performance of the system are frequent topic and analysis under discussion. Variables such as temperature, pressure, solution concentration, pH or mass flow are commonly assumed as independent variables in empirical models to find the predictor. The purpose of this research is to propose three methods which analyze the contribution of each operational variable with the objective of reducing the number of variables for a faster and more efficient prediction of the coefficient of performance for the experimental heat transformer under study. A comparative study considering three methods is done. These methods are (i) correlation analysis, (ii) principal component analysis (PCA) with correlation analysis, and (iii) sensibility analysis on artificial neural network (ANN) model based on the Garson's method. For this purpose, these methods were applied to the experimental database of three absorption heat transformers aimed at finding the most optimal (significant) operational variables in the coefficient of performance prediction of the systems.

A correlation matrix analysis was presented as an option to know the degree of correlation between the independent variables with the prediction. Kachigan [1] describes the correlation matrix as many correlation coefficients arranged in a systematic and orderly fashion. In this way, Ramírez-Hernández et al. [2] presented the modeling of an absorption heat transformer with duplex components based on a correlation matrix to select the best independent variables to predict the coefficient of performance.

About the PCA, Karytsas and Choropanitis [3] proposed the PCA to categorize the main diffusion barriers of the ground source heat pump system routed in the domestic sector. The objectives of a PCA were (a) to extract the most important information from the data, (b) to compress the

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size of the data keeping only the essential information, (c) to simplify the description of the dataset, and (d) to analyze the structure of the observations and the variables. Popescu et al. [4] used the multivariable statistical technique commonly known as PCA for the extraction and interpretation of the systematic variance in spruce wood samples submitted to several thermal and hydrothermal conditions. Baklouti et al. [5] presented the Iterated Robust Kernel Fuzzy PCA as an improvement of the PCA assuming a more accurate multi-objective function to minimize the errors, optimize the robustness to outliers and improve the memory efficiency. The PCA is presented by Lefkir et al. [6] to reduce the dimensionality of the historical database to be used in the wastewater treatment modeling; the excess removal of organic pollution and the excess nitrates product in the performance of the purification process were identified with significant importance with the PCA method.

The ANNs have been used for modeling of thermal, chemical process and heat exchanging equipment. The analysis proposed by Garson [7] weighs the relevance of every one of the operational variables considering them as input neurons in the architecture of the ANN model. El-Hamzaoui et al. [8] trained an ANN model to predict the chemical oxygen demand (COD) removal during the degradation of alazine and gesaprim commercial herbicides under various experimental conditions, the Garson's method was applied to ANN model showing that the reaction time and herbicide concentration were the input variables with more significant importance in the prediction of COD. Díaz-Gómez et al. [9] developed an ANN model to predict the global solar radiation based on meteorological environmental data about Cuernavaca city, Mexico. The authors found that at the time, the atmospheric pressure and the temperature were variables that affect the global solar radiation as predicted using the Garson's method.

The objective of this research study is the absorption heat transformer. A heat driven heat transformer, also known as a reversed absorption heat pump or temperature amplifier, is described by Siqueiros and Holland [10]. The unique ability of heat transformer is to produce a high-temperature stream depending on heat supply in the generator Q_{GE} and evaporator Q_{EV}, as a consequence the heat is rejected in the condenser. The coefficient of performance is the relationship between the heat obtained in the absorber Q_{AB} divided by the total amount of heat supplied as follows:

$$\text{COP} = \frac{Q_{AB}}{Q_{GE} + Q_{EV}} \tag{1}$$

The novelty of this research is to present the correlation analysis, PCA with correlation analysis, and Garson's method to find the significant operational variables which affect the prediction of the coefficient of performance in three experimental absorption heat transformers. The results and discussion will help us to trace the optimal variables on which we must focus our attention on each of the systems studied from a new perspective. In addition, to focusing effort on the variables which must be measured for special significance. It is important to notice that there is no previous research where the PCA method has been used in experimental data of the absorption heat transformer.

2. Methods to select variables used in this research

In this section, to select variables, three methods are presented: (1) correlation matrix, (2) PCA, and (3) sensitivity analysis based on Garson's method.

Denote by $\mathbb{R}^n$ is the Euclidean space n is the dimensional, Cov(X,X) is the matrix covariance of the random variable X, Var(X) is the variance of the random variable X, $v_1 \cdot v_2$ is the inner product of vectors v_1 and v_2, X^T denotes the transpose of the vector X, and A^{-1} inverse of the matrix A.

2.1. Method 1: correlation matrix

Pearson correlation coefficient, also known as r or R is a measure of lineal dependence between random variables. Let x_i, x_j be random variables ($i,j = 1,2,\ldots,n$), r was defined in Kachigan [1] as:

$$r\left(x_i, x_j\right) = \frac{\text{cov}\left(x_i, x_j\right)}{\sqrt{\text{Var}\left(x_i\right)\text{Var}\left(x_i\right)}} \tag{2}$$

It has values in the interval (−1,1), that is, $-1 \le r(x_i,x_j) \le 1$ with $r(x_i,x_j) = 0$ only if $\text{Cov}(x_i,x_j) = 0$. Table 1 shows the interpretation of Pearson correlation coefficient according to the calculated value.

When the correlations that exist among several variables need to be studied, for instance, $x_1,x_2,\ldots,x_n$, the correlation coefficient is calculated between each pair of variables (x_i,x_j), $i,j = 1,2,\ldots,n$. Since there are many coefficients, it is convenient to arrange the coefficients in a systematic and orderly fashion. This is done in the form of the correlation matrix, Kachigan [1] for more details.

Remark 1: To select those variables that are highly correlated with each other, the correlation analysis is used to reduce the number of variables, that is, if in principle there are n original variables, $x_1,x_2,\ldots,x_n$, the correlation analysis allows us to have a smaller number of variables (x_5,x_8,x_4, three for example).

2.2. Method 2: principal components analysis combined with correlation analysis

This subsection briefly describes the PCA, as well as, the definitions of the linear algebra concepts used in the PCA. Our main sources are Jolliffe [11] and Grossman [12]. Let $X = (X_1,X_2,\ldots,X_n)^T$ be a random variable n is the dimensional and let $A = \text{Cov}(X,X)$.

Definition 1: The first principal component is defined as a lineal combination of the variables $X_1,X_2,\ldots,X_n$, that is $Z_1 = v_{11}X_1 + v_{12}X_2 + \ldots + v_{1n}X_n$, or in matrix form $Z_1 = v_1^T X$, $v^1 = (v_{11},v_{12},\ldots,v_{1n})^T \in \mathbb{R}^n$ satisfying that $\text{Var}\left(Z_1\right) = \max_{v\in\mathbb{R}^n} \{\text{Var}(v^T X) : v \cdot v = 1\}$.

Remark 2: To obtain the first principal component observe that:

(a)

$$\begin{aligned} \text{Var}\left(Z_1\right) &= \text{Var}\left(v_1^T X\right) \\ &= \text{cov}\left(v_1^T X, v_1^T X\right) \\ &= v_1 \,\text{cov}\left(X, X\right) v_1^T \left(\text{by properties of the covariance matrix}\right) \\ &= v_1^T \text{A} \text{v}_1 \end{aligned} \tag{3}$$

Thus, to find the first principal component by Eq. (1), the following optimization problem has to be solved:

$$\begin{cases} \max_{v\in\mathbb{R}^n}\left\{v^T \mathrm{A}v\right\}, \\ v\times v=1. \end{cases} \tag{4}$$

(b) The optimization Eq. (4) can be solved using the Lagrange multipliers technique as follows:

$$L(v)=v^T\mathrm{A}v-\lambda_1\left(v\times v-1\right)\times\frac{\partial L}{\partial v}2vA-2\lambda_1 v=0. \tag{5}$$

Eq. (5) implies that the vector $v \in \mathbb{R}^n$ which maximizes Eq. (4) satisfies $\mathrm{A}v = \lambda_1 v$, so, v is an eigenvector of the covariance matrix A and the scale factor λ_1 is the eigenvalue corresponding to that eigenvector.

(c) On the other hand, note by Eq. (3) and (b) that $\mathrm{Var}(Z_1) = v_1^T\mathrm{A}v_1 = \lambda_1 v_1\cdot v_1=\lambda_1$.
(d) Finally, by parts (b) and (c) the first principal component, Z_1, is developed using the highest eigenvalue of the covariance matrix and the eigenvector associated to this eigenvalue.

Definition 2: The second principal component is defined as a lineal combination of the variables $X_1,X_2,\ldots,X_n$ that is $Z_2 := v_{21}X_1 + v_{22}X_2 + \ldots + v_{2n}X_n$, or in matrix form $Z_2 = v_2^TX$, $v_2 = (v_{21},v_{22},\ldots,v_{2n})^T \in \mathbb{R}^n$ satisfying that $\mathrm{Var}(Z_2) = \max_{v\in\mathbb{R}^n} \{\mathrm{Var}(v^TX) : v\cdot v = 1 \text{ and } v\cdot v_1 = 0\}$, where v_1 is the vector that maximize the variance of the first principal component, Z_1.

Proceeding similarly to the development of the first principal component (Remark 2), it is obtained that to find the second principal component, the following optimization problem has to be solved.

$$\begin{cases} \max_{\in\mathbb{R}^n} v^T\mathrm{A}v \\ v\times v=1, \\ v\times v_1=0. \end{cases} \tag{6}$$

Using the Lagrange multipliers technique to solve the optimization Eq. (4) it gets $L(v) = v^T\mathrm{A}v - \lambda(v\cdot v - 1) - \mu(v\cdot v_1)$.

$$\frac{\partial L}{\partial v}=2\mathrm{A}v-2\lambda v-\mu v_1=0, \tag{7}$$

multiplying the equality Eq. (7) by v_1^T to get $2\mathrm{A}v^Tv_1^T - \mu v_1v_1^T = 0$ implying that $\mu = 0$ since by Eqs. (4) and (6) $v^Tv_1^T = v_1\cdot v = 0$ and $v_1v_1^T = v_1\cdot v_1 = 1$. So, substituting $\mu = 0$ in Eq. (7) it is concluded that the vector which maximizes the variance of the second principal component, Z_2, satisfies $\mathrm{A}v = \lambda v$, that is, v is again, an eigenvector of the covariance matrix A and the scale factor λ is the eigenvalue corresponding to that eigenvector. Moreover, proceeding similarly as in the case of the first principal component, it can obtain $\mathrm{Var}(Z_2) = \lambda$.

Remark 3: Notice that to find the first and second principal components, the optimization problems which had to be solved were Eqs. (4) and (6). The vectors that maximize these problems are the eigenvectors of the covariance matrix associated with the eigenvalues of this matrix. Similar arguments to the ones presented above, allows us to conclude that the principal components are linear of the eigenvectors of the covariance matrix. That is if $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$ are the eigenvalues of the covariance matrix ordered from higher to lower and $v_1,v_2,\ldots,v_n$ are eigenvectors associated with the eigenvalues, then the principal components are $Z_1 = v_1^TX$, $Z_2 = v_2^TX,\ldots,Z_n = v_n^TX$. Moreover, $\mathrm{Var}(Z_1) = \lambda_1$, $\mathrm{Var}(Z_2) = \lambda_2$, $\mathrm{Var}(Z_3) = \lambda_3,\ldots,\mathrm{Var}(Z_n) = \lambda_n$.

Variability percent: Let $X = (X_1,X_2,\ldots,X_n)^T$ be the original variables and $Z_1,Z_2,\ldots,Z_n$ is the principal component. It is known that $\mathrm{Var}(X_i) - \mathrm{Cov}(X_i,X_i)$.

So,

$$\sum_{i=1}^{n}\mathrm{Var}\left(X_i\right)=\mathrm{Trace}\left(A\right) \quad \text{with} \quad A=\mathrm{cov}\left(X,X\right) \tag{8}$$

Now, let $C = (v_1,v_2,\ldots,v_n)$ be the matrix of the eigenvectors associated to the eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$. Then, by Theorem 2 in Grossman [12], $D := C^{-1}AC = C^TAC$ since C is orthogonal, $(C^{-1} = C^T)$. Thus, using the properties of trace operator it gets $\mathrm{Trace}(D) = \mathrm{Trace}(C^TAC) = \mathrm{Trace}(AC^TC)$. Therefore,

$$\mathrm{Trace}\left(D\right)=\mathrm{Trace}\left(AC^TC\right)=\mathrm{Trace}\left(A\right) \\ \text{since } C \text{ isorthogonal}, C^TC=I. \tag{9}$$

On the other hand, by definition of the operator trace it gets, $\mathrm{Trace}(D) = \sum_{i=1}^{n}\lambda_i$, by Remark 3 $\mathrm{Var}(Z_i) = \lambda_i$, this implies that:

$$\mathrm{Trace}\left(D\right)=\sum_{i=1}^{n}\lambda_i=\sum_{i=1}^{n}\mathrm{Var}\left(Z_i\right) \tag{10}$$

So, by Eqs. (8)–(10), it is concluded that the variability of the original variables $X = (X_1,X_2,\ldots,X_n)^T$ is equal to the variability of the components $Z_1,Z_2,\ldots,Z_n$. Consequently, the variability percent explained by the first principal component and the variability percent explained by r principal components are given by $\frac{\lambda_1}{\sum_{i=1}^{i=n}\lambda_i}\times 100$, and $\frac{\sum_{i=1}^{i=r}\lambda_i}{\sum_{i=1}^{i=n}\lambda_i}\times 100$, respectively.

In practice, having N original variables in the beginning, $X_1,X_2,\ldots,X_n$ the PCA allows us to have a smaller number of variables (components, Z_1, Z_2, Z_3 for instance) that collects a large percentage of the total variability. This way, the PCA reduces the data dimension. The correlation coefficient between each principal component and the original variables are calculated. Those with the highest correlation coefficient from the first principal component will be the optimal variables.

Table 1
Values Pearson correlation coefficient

$r(x_i,x_j) = 1$	Indicates a total positive linear regression
$r(x_i,x_j) = 0$	Suggests nonlinear regression
$r(x_i,x_j) = -1$	Corresponds a total negative linear regression

2.3. Method 3: sensitivity analysis based on Garson's method [10]

An ANN is one of the most effective methods for the model physical phenomenon and engineering data analysis. Commonly, a series of impulses feed interconnected neurons in the input and hidden layers to predict a target. The magnitude of the impulse is related to the importance and variability of the input. The method can be analyzed from this perspective:

$$y = \text{ANN}\left(x_1, x_2, x_3, \ldots, x_N\right), \tag{11}$$

where y, x and N are the dependent variables, independent variables and the number of input operational variables, respectively. ANN averages the mathematical relationship usually referred to as ANN architecture. Mathematical functions such as logarithmic sigmoid, hyperbolic tangent sigmoid and radial are commonly applied in the hidden layer of an ANN architecture and a linear function is suggested in the output layer by several authors. The ANN training is the iterative procedure to obtain appropriate weights and bias to estimate the independent variable, along with it, an optimization algorithm is needed. Several works describe the following steps for ANN training:

- First, gather an experimental database based on the operational variable range of the available equipment. Not registered operational variables must be ruled out.
- Second, define the percentage of data for training and validation, also calculate all necessary parameters: architecture, number of hidden layers, optimization algorithm, activation function, number of iterations and, the most important factor, the number of hidden neurons per layer if possible. Each element of the hidden layers is connected to each input neuron through the weight matrix.
- Finally, calculate the difference between the target and network output, which is the error and it should be minimized.

The weight and bias matrices are the results of the training process in the ANN. The mathematical expression of the ANN model considering a tangent sigmoid transfer function and a linear function in the hidden and output layers is given by:

$$y_k = \sum_{s=1}^{s}\left[W_{0l,s}\left(\frac{2}{1+e^{\left(-2\sum_{n=1}^{N}\left(Wi_{s,n}p_r\right)+b1_s\right)}}\right)\right] + b2_1 \tag{12}$$

where S is the number of neurons in the hidden layer, W_i is the weights in the input-hidden layer, $b1_s$ is the j-th of bias in the hidden layer, W_0 are the weights in the hidden output layer, N is the input-neuron number, l is the output-neuron number and $b2_l$ is the k-th value of bias in the output layer.

As an example, for the specific case where: N = 4, S = 3, for the hidden layer and hyperbolic tangent sigmoid respectively, and in the output layer the linear function is selected, l = 1 and using Eq. (12) the following model is obtained:

$$y = 2\left[\frac{W_{0(1,1)}}{1+e^{\phi_1}} + \frac{W_{0(2,1)}}{1+e^{\phi_2}} + \frac{W_{0(3,1)}}{1+e^{\phi_3}}\right] - \left(W_{0(1,1)} + W_{0(2,1)} + W_{0(3,1)}\right) + b_2 \tag{13}$$

where $\phi_1 = -2(W_{i(1,1)}x_1 + W_{i(1,2)}x_2 + W_{i(1,3)}x_3 + W_{i(1,4)}x_4 + b1_1)$, $\phi_2 = -2(W_{i(2,1)}x_1 + W_{i(2,2)}x_2 + W_{i(2,3)}x_3 + W_{i(2,4)}x_4 + b1_2)$, $\phi_3 = -2(W_{i(3,1)}x_1 + W_{i(3,2)}x_2 + W_{i(3,3)}x_3 + W_{i(3,4)}x_4 + b1_3)$ or in matrix form $\phi = -2W_iX + b_1$, with

$$\phi = \begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\end{bmatrix}, \quad W_i = \begin{bmatrix} W_{i(1,1)} & W_{i(1,2)} & W_{i(1,3)} & W_{i(1,4)}\\ W_{i(2,1)} & W_{i(2,2)} & W_{i(2,3)} & W_{i(2,4)}\\ W_{i(3,1)} & W_{i(3,2)} & W_{i(3,3)} & W_{i(3,4)}\end{bmatrix},$$

$$b1 = \begin{bmatrix} b1_{(1,1)}\\ b1_{(1,2)}\\ b1_{(1,3)}\end{bmatrix}, \quad X = \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$$

This model should consider that the independent variables x_1, x_2, x_3 and x_4 have normalized values ranging from –1 to 1 and the dependent y variables from 0 to 1.

The validation process is the procedure in which the experimental results are paired with the ANN approach. A statistical tool such as a determination coefficient higher than 0.98 and ordinary linear regression analysis was reported to confirm the goodness of fit. As a result of the training and validation steps, an ANN model is presented.

Sensibility analysis: Garson [7] proposed a method to estimate the degree of relevance I_{x_j} of each input operational variable x_j in an ANN model based on the weights matrix, W_i and W_0.

Garson's method uses the absolute value of each weighting coefficient (W_i, W_0), ruling out their negative characteristic and keeping only their magnitude, then the bias values $b1$ and $b2$ are not considered. Following the method described by Garson [7], the I_{x_j} is obtained as the product of the absolute value of the input layer per its respective weight coefficient in the hidden layer as following.

$$I_{x_j} = \sum_{i=1}^{N}\left[\left(\frac{\left|W_{i_{(i,j)}}\right|}{\sum_{s=1}^{N}\left|W_{i_{(i,s)}}\right|}\right)\times W_{0_{(i,j)}}\right], \frac{I_{x_j}}{I_{\text{Den}}}\times 100, \quad \text{where } I_{\text{Den}} = \sum_{j=1}^{n} I_{x_j} \tag{14}$$

The methods described above are given in Fig. 1.

3. Applications

In this section, three systems are studied (1) absorption heat transformer with duplex function compounds by Morales et al. [13], (2) a single-state absorption heat transformer by Hernández et al. [14], and (3) a double-absorption heat transformer by Rivera et al. [15]. The methods previously described are applied to select the minor number of variables that predict the best coefficient of performance for

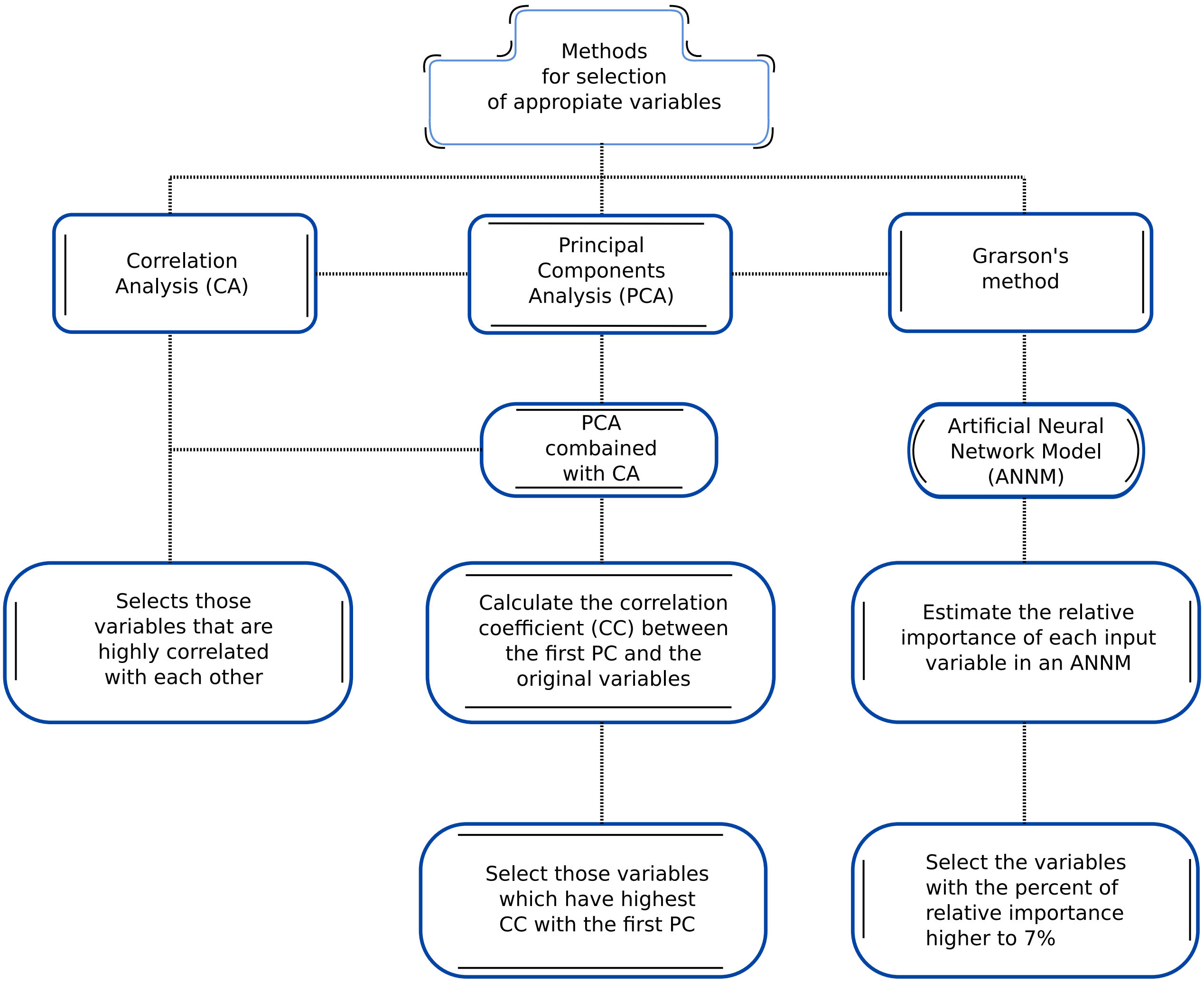

Fig. 1. Methods to select variable flow diagram.

these systems. To emphasize the discussion, the calculations related to the methods were synthesized in Tables A1–A13.

3.1. *Absorption heat transformer with duplex function compounds*

Fig. 2 shows a schematic diagram of an experimental absorption heat transformer with the purpose of water purification. Previous works by Ramírez-Hernández et al. [2], Morales et al. [13] and Martínez-Martínez et al. [16] reported theoretical and experimental studies of this equipment to predict the coefficient of performance. The system consists of two duplex units: generator–condenser and absorber–evaporator, these were built to reduce the heat and fluid-transport losses. Each equipment consists of concentric helical coils fed by a distributor to increase the heat transfer. Temperature, pressure, and mass flow sensors were installed in the shell of both duplex equipment to save all experimental data possible. The concentration of the bromide–lithium solution was registered as the indirect variable with the help of the refractive index. Table 2 shows a summary of the experimental database of absorption heat transformer with duplex components.

3.1.1. *Method 1: correlation matrix*

The correlation matrix of the operational variables from the studied system is obtained and those variables with the highest correlation coefficients are selected. As can be seen in Tables A1 and A2, the variables that are highly correlated with each other are x_1 with x_2, x_3, x_5, x_8; x_2 with x_4 and x_5; x_4 and x_5; x_5 and x_8; x_6 and x_7; x_{12} and x_{13}. So, the set of optimal variables for this system could be $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_{12}, x_{13}\}$.

3.1.2. *Method 2: PCA combined with correlation analysis*

PCA is used to select a set of variables. To do this, first, the covariance matrix is calculated; second, the eigenvalues from this matrix and its eigenvectors associated are obtained; and, third, the principal components are defined. Tables A3 and A4 show the covariance of each pair (x_i,x_j), $i,j = 1,2,\ldots,16$. The covariance describes the way two random variables vary together. Since the variance is a special case of covariance, $\mathrm{Cov}(x_i,x_i) = \mathrm{Var}(x_i)$, the diagonal of the covariance matrix contains the variances of the variables x_i.

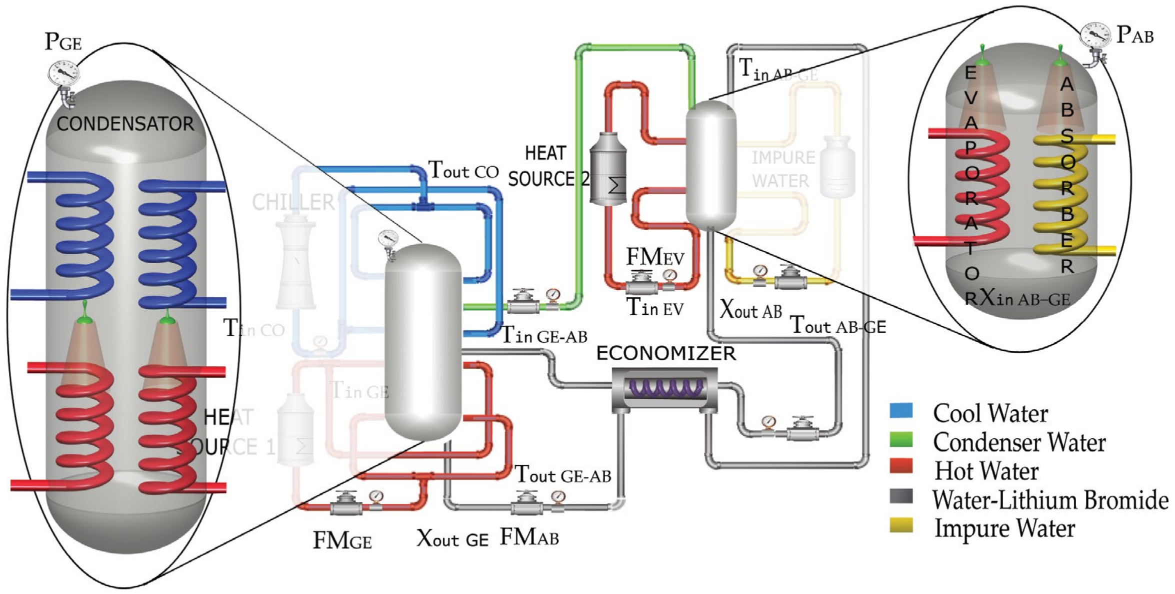

Fig. 2. Schematic diagram of absorption heat transformer with duplex compounds by Ramírez-Hernández et al. [2], the thermal load design is 2 kW, the approximate dimensions of system are 2.3 m × 2 m × 2 m.

Table 2
Heat transformer with duplex compounds. Relationship between mathematical and operational variables

Mathematical label	Temperatures (°C)	Operation range
x_1	T_{inGE}	65.61–85.68
x_2	$T_{inGE-AB}$	56.25–78.09
x_3	$T_{outAB-GE}$	77.98–97.05
x_4	$T_{inAB-GE}$	50.70–78.82
x_5	$T_{outGE-AB}$	57.33–81.38
x_6	T_{inCO}	13.38–20.20
x_7	T_{outCO}	16.02–29.26
x_8	T_{inVE}	66.61–86.03
	Concentrations (%)	
x_9	$X_{inAB-GE}$	51.67–53
x_{10}	X_{outAB}	47.68–57.06
x_{11}	X_{outGE}	51.55–58.14
	Pressure (in Hg)	
x_{12}	P_{AB}	29.73–82.39
x_{13}	P_{GE}	4.68–95.99
	Mass flow (kg/s)	
x_{14}	FM_{GE}	0.0721–0.1541
x_{15}	FM_{EV}	0.0727–0.1445
x_{16}	FM_{AB}	0.0049–0.0199
Y	COP	0.1–0.36

Now, the eigenvalues of the covariance matrix are calculated in order from the highest to the lowest (Table A5). The eigenvectors associated with the eigenvalues given in Table A5 are presented in Tables A6 and A7. These eigenvectors satisfy the condition $v_1 \cdot v_1 = 1$ and $v_1 \cdot v_2 = 0$, that is, have a norm of 1 and are orthogonal to each other.

According to the theory presented in section 1, the principal component are linear combinations from the original variables $x_1,\dots,x_{16}$ where the coefficients of each linear combination are the inputs of the eigenvectors, Definitions 1, 2 and Remark 3. For instance, the first and second principal components are $Z_1 = 0.3902x_1 + 0.3851x_2 + 0.3177x_3 + 0.4702x_4 + 0.4865x_5 + 0.0243x_6 + 0.0385x_7 + 0.3387x_8 - 0.0109x_9 - 0.0038x_{10} + 0.0113x_{11} - 0.1466x_{12} - 0.0476x_{13} - 0.0002x_{14} - 0.0001x_{15} - 0.00x_{16}$, $Z_2 = 0.2982x_1 + 0.2095x_2 + 0.135x_3 + 0.359x_4 + 0.0764x_5 + 0.3250x_6 + 0.6451x_7 + 0.3528x_8 + 0.0394x_9 - 0.2002x_{10} - 0.225x_{11} - 0.1516x_{12} - 00498x_{13} + 0.0007x_{14} - 0.00x_{15} - 0.0003x_{16}$.

Table A8 shows the information on the principal components. The first row corresponds to the first principal component, the second row corresponds to the second principal component and so on. The determination of the principal components number to be retained is, in part, arbitrary and remains under the judgment of the researcher. One criterion is to retain principal components that collect a large percentage of the total variability. In our case study, the principal components retained are Z_1, Z_2, Z_3 and Z_4 since the percentage of the total variability retained by these 4 components is 96.05%, see Table A8. In this way, the PCA reduces the dimension of the data since the N original variables, $x_1, x_2, \dots, x_n$ are reduced to a smaller number of variables (components), Z_1, Z_2, Z_3, Z_4.

Tables A9 and A10 show the correlation coefficient between each principal component and the original variables. The selected variables will be those that have the highest correlation with the first principal component. As can be seen in Tables A9 and A10 the variables set selected with the PCA are $\{x_1, x_2, x_3, x_4, x_5, x_8, x_{12}\}$.

3.1.3. Method 3: a sensitivity analysis based on Garson's method

Morales et al. [13] presented a training and validation of an ANN to model the coefficient of performance for absorption heat transformer with duplex components. The ANN model consists of 16 operational variables in the input layer of architecture, 7 neurons in the hidden layer to predict the coefficient of performance as the output layer. The weights and bias obtained by the authors are shown in Table A12. According to the procedure described in the previous section Sensitivity analysis based on Garson's method, Garson [7], the relative importance of each variable was calculated with Eq. (14) and shown in Fig. 3. The most important variables selected with the sensibility analysis considering percent of relative importance mayor to 7% are $\{x_1, x_4, x_9, x_{11}, x_{12}, x_{13}\}$.

3.2. Comparison of the methods and discussion

The variables selected by the three methods are presented in Tables 3 and 4. In order, the coefficient of performance prediction of the absorption heat transformer with two-duplex components, the authors in Ramírez-Hernández et al. [2] developed four polynomial models. The best polynomial model included inlet temperature in the generator, absorber–generator, and evaporator; output temperature in the absorber–generator and pressure in the generator, that is, $\{x_1, x_4, x_5, x_8, x_{13}\}$. The polynomial model showed an excellent correlation between experimental and simulated values of the coefficient of performance with a coefficient of determination $R^2 \geq 0.9910$. As can be seen in Table 3.

- PCA predicts four of the five variables used in the best polynomial model developed in Ramírez-Hernández et al. [2], but the other three polynomial models combine exactly the seven variables selected by the CPA;
- with respect to the correlation matrix, the selected variables set contained every variable used in the polynomial models developed in Ramírez-Hernández et al. [2]. This is because the authors used the correlation matrix variable selection;
- the selected variables set by sensibility analysis contained three variables used by the authors in Ramírez-Hernández et al. [2].

On the other hand, in the work Martínez-Martínez et al. [16] the authors developed ANN models in order the coefficient of performance prediction of the absorption heat transformer with two-duplex components. The best ANN model uses the variables $\{x_1, x_2, x_4, x_5, x_8, x_{12}\}$. Observing again Table 9, we can notice that:

- the PCA predicts all variables used in the best ANN model developed in Martínez-Martínez et al. [16];
- the correlation matrix also contains all the variables used in the best ANN model developed in Martínez-Martínez et al. [16], but this set is bigger than the set obtained with PCA;
- sensibility analysis only predicts three of the variables used in the best ANN model.

Finally, we can conclude that PCA is the best option to select variables since it also uses the correlation matrix method.

According to Table 4, the variables selected in principal components emphasized the absorption–desorption process using the temperature at the inlet and outlet of components. The temperature at the inlet of the evaporator, which shows the level of waste heat incorporated into the system, and

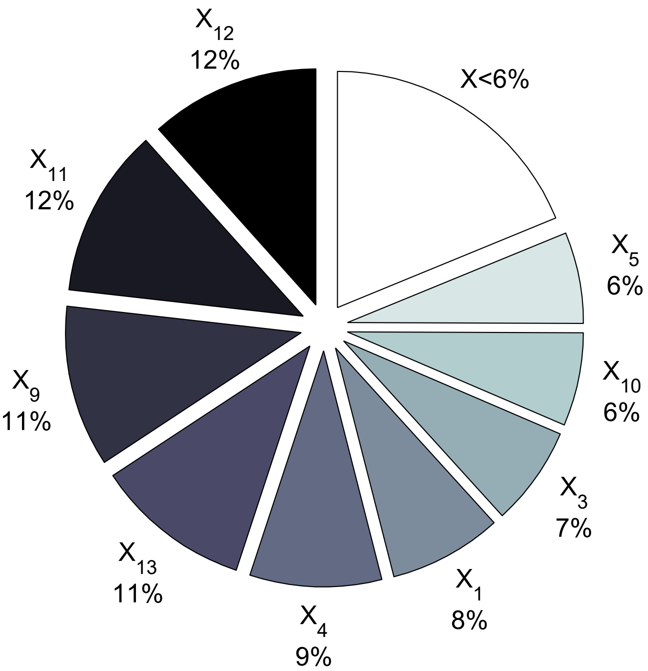

Fig. 3. Heat transformer with duplex compounds. Relative importance of input variables.

Table 3
Selected variables by three methods

Methods	Results
Principal components	$x_1, x_2, x_3, x_4, x_5, x_8, x_{12}$
Correlation matrix	$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_{12}, x_{13}$
Sensibility analysis	$x_1, x_4, x_9, x_{11}, x_{12}, x_{13}$

Table 4
Selected variables by the three methods for the absorption heat transformer with duplex function compounds

Methods	Results
Principal components	$T_{inGE}, T_{inGE-AB}, T_{outAB-GE}, T_{inAB-GE}, T_{outGE-AB}, T_{inEV}, P_{AB}$
Correlation matrix	$T_{inGE}, T_{inGE-AB}, T_{outAB-GE}, T_{inAB-GE}, T_{outGE-AB}, T_{inCO}, T_{outCO}, T_{inEV}, P_{AB}, P_{GE}$
Sensibility analysis	$T_{inGE}, T_{inAB-GE}, X_{inGE-AB}, X_{outGE}, P_{AB}, P_{GE}$

the pressure in the absorber–evaporator was signaled with significant importance. Referencing the variables selected by the correlation matrix, the condenser and generator contributed to the prediction of the coefficient of performance. Finally, the sensibility analysis changes the perspective, it includes the concentration of the lithium–bromide solution in the base of principal components. It is worth remembering that the concentration is a function of pressure and temperature as $X = f(P,T)$, therefore the coefficient of performance can be predicted as the help of indirect variables.

If we use the coincidences between the results of the methods, it obtains $\{T_{\text{inAB–GE}}, P_{\text{AB}}\}$. This could mean that the absorption carried out in the absorber–evaporator contributes significantly to the prediction of the coefficient of performance and curiously at a point where the risk of crystallization is considerable.

3.3. Single-state absorption heat transformer

Fig. 4 shows the schematic diagram for a single-state absorption heat transformer. Hernández et al. [14] and Escobedo-Trujillo et al. [17] presented the experimental and theoretical development of absorption heat transformer from coefficient of performance modeling, control, and statistical analysis. The entire system consists of an absorber, an evaporator, a generator, and a condenser. The system was designed for the water purification process taking advantage of that useful heat quantity in the absorber [18]. The experimental data provides from the portable design of the water purification process integrated into a single-state absorption heat transformer with energy recycling by [19]. Bromide–lithium solution flows between the absorber and generator; while water was used as a working fluid. Heat load was supplied in the generator and evaporator from the auxiliary service system. Transitory and steady states were taken into account for each initial concentration of the Bromide–lithium solution. After 2 h from start-up, data was collected for 4 h. Experiments were carried out at eight different initial conditions with at least two replicates. Then, a database of 6,786 samples was obtained. Table 5 shows a summary of the experimental database obtained, the position of each measurement in the system according to Fig. 4 and the mathematical variables used in this analysis.

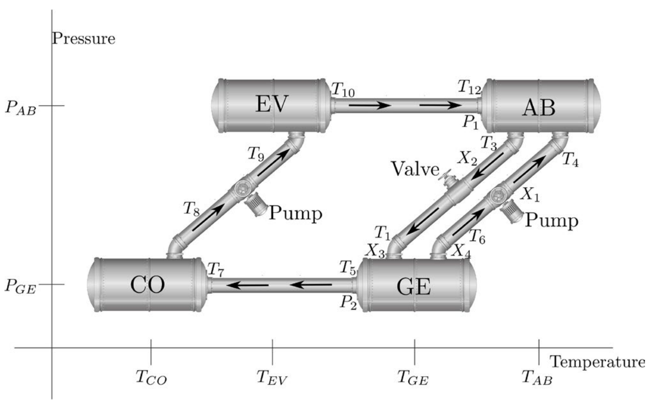

Fig. 4. Schematic diagram of single-state absorption heat transformer the experimental COP has range from 0.2 to 0.39.

3.3.1. Method 1: correlation matrix

The variables that are highly correlated with each other are x_1 with x_3, x_5; x_3 with x_1, x_5, x_6; x_4 with the variables x_6, x_{15}, x_{12}, x_{13}; x_6 with x_3, x_4; x_7 is highly correlated with x_8; x_{10} and x_2; x_{11} with the variables x_4, x_{12}, x_{13}, x_{14}, x_{16}; x_{12} with x_4, x_{11}, x_{13}, x_{14}, x_{16}; x_{13} with x_4, x_{11}, x_{14}, x_{16}; x_{14} with x_{11}, x_{12}, x_{13}, x_{16}; x_{16} with the variables x_{11}, x_{12}, x_{13}, x_{14}. Coefficient of performance is highly correlated with x_4, x_6, x_{12}, x_{13}, x_{14}. So, the set of selected variables for this system using the correlation matrix ($r \geq 0.4$) is $\{x_1, x_2, x_3, x_4, x_5, x_6, x_{11}, x_{12}, x_{13}, x_{14}, x_{16}\}$.

3.3.2. Method 2: principal components analysis combined with correlation analysis

According to the theory presented in section 1, the principal components are linear combinations from the original variables $x_1,\ldots,x_{16}$ where the coefficients of each linear combination are the inputs of the eigenvectors, see Definitions 1, 2 and Remark 3. For instance, the first and second principal components are: $Z_1 = 0.0555x_1 + 0.0378x_2 + 0.0723x_3 + 0.0260x_4 + 0.0555x_5 + 0.0211x_6 + 0.0206x_7 - 0.0240x_8 - 0.9896x_9 + 0.0378x_{10} + 0.0312x_{11} + 0.0328x_{12} + 0.0328x_{13} + 0.0291x_{14} - 0.0175x_{15} + 0.0041x_{16}$, $Z_2 = -0.1160x_1 + 0.5300x_2 - 0.2896x_3 - 0.1899x_4 - 0.1160x_5 - 0.0644x_6 + 0.4395x_7 + 0.0532x_8 + 0.0035x_9 + 0.5300x_{10} - 0.1235x_{11} - 0.1339x_{12} -0.1339x_{13} - 0.0646x_{14} - 0.1895x_{15} + 0.0079x_{16} + 0.0079x_{16}$.

Table A12 shows the information on the principal components of the system studied in this section. As in the previous application, the first row corresponds to the first principal component, the second row corresponds to the

Table 5
Range of experimental operating conditions used to obtain the coefficient of performance values

Temperatures (°C)	Operation range	Instrumentation label (Fig. 4)	Mathematical label
$T_{\text{inGE–AB}}$	76.29–91.53	T_1	x_1
$T_{\text{inEV–AB}}$	74.56–89.93	T_2	x_2
$T_{\text{outAB–GE}}$	84.31–98.27	T_3	x_3
$T_{\text{inAB–GE}}$	74.99–92.58	T_4	x_4
$T_{\text{outGE–CO}}$	76.29–91.53	T_5	x_5
$T_{\text{outGE–AB}}$	77.03–83.89	T_6	x_6
T_{inCO}	40.37–65.03	T_7	x_7
T_{outCO}	26.77–33.79	T_8	x_8
T_{inEV}	28.52–85.33	T_9	x_9
$T_{\text{outEV–AB}}$	74.56–89.93	T_{10}	x_{10}
Concentrations (%)			
X_{inAB}	51.66–55.36	X_1	x_{11}
X_{outAB}	50.75–54.36	X_2	x_{12}
X_{inGE}	50.75–54.36	X_3	x_{13}
X_{outGE}	53.16–56.07	X_4	x_{14}
Pressure (in Hg absolute)			
P_{AB}	7.00–11.50	P_1	x_{15}
P_{GE}	19.00–21.10	P_2	x_{16}

second principal component and so on. The variability percentage caught by the first three principal components is 81.33%. Depending on the research it is possible to work only with these three components or add another one to increase the variability percentage caught, for instance, with six principal components, the variability percentage caught 97.32%. In this application, only the first principal component was treated. Those variables with the highest correlation coefficient with the first principal component will be the selected variables. In this case the variables are $\{x_1, x_3, x_5, x_6, x_{11}, x_{12}, x_{13}, x_{14}\}$.

3.3.3. Method 3: a sensitivity analysis based on Garson's method

Hernández et al. [14] trained and validated an ANN model to predict the coefficient of performance in a single-state absorption heat transformer. This model involves 16 operational variables in the input layer and 3 neurons in the hidden layer to calculate coefficient of performance. Table A13 shows the weights and bias obtained by Hernández et al. [14]. Now, the sensibility analysis described in section 2.3 was applied. The variables $\{x_1, x_3, x_{11}, x_{12}, x_{13}, x_{14}\}$ are the most important variables selected with the sensibility analysis, considering the percentage of relative importance higher than 7%.

4. Comparison of the methods and discussion

The results of the three methods applied in experimental absorption heat transformer data were summarized in Table 6. Notice that the set generated by the correlation matrix contains the set obtained with the PCA. Moreover, the set generated by the CPA contains the set obtained by the sensibility analysis. The variables in the intersection of the methods are $\{T_{inGE-AB}, T_{outGE-CO}, X_{inAB}, X_{outAB}, X_{outAB}\}$.

As can be seen, the principal components method results presented in Table 7 reveal that the temperature and concentration of lithium bromide solution in the inlet and outlet of the absorber and generator are the most significant variables to be observed and recorded with the purpose of estimate the coefficient of performance. The correlation matrix results added the information about vapor temperature from the evaporator to the absorber, this variable is crucial for the exothermic reaction carried out in the absorber. The pressure in the generator is considered in this analysis, it reminds us that this variable is strongly related to condenser pressure. Finally, the sensibility analysis results emphasize the change in the lithium bromide concentration solution between the absorber and generator, in the flame of heat transformers, it can be explained as the function of coefficient of performance with the flow ratio presented by Siqueiros and Holland [10].

Table 6
Selected variables by the three methods

Methods	Results
Principal components	$x_1, x_3, x_5, x_6, x_{11}, x_{12}, x_{13}, x_{14}$
Correlation matrix	$x_1, x_2, x_3, x_4, x_5, x_6, x_{11}, x_{12}, x_{13}, x_{14}, x_{16}$
Sensibility analysis	$x_1, x_3, x_{13}, x_{11}, x_{12}, x_{14}$

4.1. Double-absorption heat transformer operating with H_2O/LiBr

Rivera et al. [15] present the experimental information of a double-absorption heat transformer, it consists of a generator, a condenser, an evaporator, an absorber, an absorber-evaporator, and an economizer. The authors emphasize that the system was effectively stable and repeatable. The entire system was built with stainless steel 316 with an approximate design power of 1 kW. Experimental information, instruments, and uncertainty analysis were presented and discussed by Rivera et al. [15]. The maximum coefficient of performance was 0.37, the experimental study reveals that the system is 30% more efficient than a single-state absorption heat transformer. The database consists of 51 tests ran in steady-state, the experimental information used in this research is presented in Table 8.

4.1.1. Method 1: correlation matrix

The correlation matrix of the measured variables in the double-absorption heat transformer given the variables that have a high correlation coefficient $r > 0.61$ with each other are $\{x_2, x_3, x_4, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{16}, x_{17}, x_{18}, x_{22}, x_{25}\}$.

4.1.2. Method 2: PCA combined with correlation analysis

The first principal component caught 56.3% of the variability of the information of the original variables, whereas, the second component caught 25.4%. Thus, taking the correlation coefficient between first and second principal components and the original variables, it obtains the following set of variables $\{x_1, x_2, x_5, x_9, x_{15}, x_{17}, x_{18}, x_{19}, x_{20}, x_{22}, x_{25}\}$.

4.1.3. Method 3: a sensitivity analysis based on Garson's method

In previous sections, Garson's method was used on the ANNs model developed by other authors. In this case, the ANN was trained and validated according to the procedure described in Hernández et al. [20]. Fig. 5 shows the

Table 7
Selected variables by the three methods for the single-state absorption heat transformer

Methods	Results
Principal components	$T_{inGE-AB}, T_{outAB-GE}, T_{outGE-CO}, T_{inGE-AB}, X_{inAB}, X_{outAB}, X_{inGE}, X_{outGE}$
Correlation matrix	$T_{inGE-AB}, T_{inEV-AB,} T_{outAB-GE,} T_{inAB-GE,} T_{outGE-CO}, T_{inGE-AB}, X_{inAB}, X_{outAB}, X_{inGE}, X_{outGE}, P_{GE}$
Sensibility analysis	$T_{inGE-AB}, T_{outAB-GE}, X_{inGE}, X_{inAB}, X_{outAB}, X_{outGE}$

Table 8
Experimental database of double-absorption heat transformer

Mathematical label	Temperatures (°C)	Operation range
x_1	T_E	34.90–66.19
x_2	T_C	29.47–46.39
x_3	T_G	66.91–98.35
x_4	$T_{G,\text{in}}$	67.24–96.63
x_5	$T_{E,\text{out}}$	23.89–97.61
x_6	T_A	93.93–128.08
x_7	$T_{A,\text{in}}$	83.07–113.42
x_8	$T_{\text{AB,out}}$	85.12–115.53
x_9	T_{AE}	64.71–84.30
x_{10}	$T_{\text{AE,in}}$	76.10–99.40
x_{11}	$T_{\text{AE,out}}$	61.70–79.00
x_{12}	$T_{\text{oil,in}}$	61.90–117.20
x_{13}	$T_{\text{oil,out}}$	81.40–118.20
x_{14}	$T_{\text{WA,in}}$	25.70–30.00
x_{15}	$T_{\text{WA,out}}$	29.70–36.50
	Concentrations (%)	
x_{16}	X_G	52.40–60.70
x_{17}	X_{AB}	49.10–60.30
x_{18}	X_A	51.90–59.60
	Heat flow (W)	
x_{19}	$Q_{E,\text{EX}}$	142.80–351.40
x_{20}	$Q_{G,\text{EX}}$	699.60–1045.20
x_{21}	$Q_{\text{AE,EX}}$	102.50–285.10
x_{22}	$Q_{A,\text{EX}}$	1.40–262.40
x_{23}	EFE	0.30–0.52
x_{24}	GTL	39.60–80.60
x_{25}	ΔT	0.64–30.44
Y	COP_{EX}	0.10–0.36

coefficient of performance predicted by the ANN model with one neuron in the hidden layer against the experimental coefficient of performance. A good fit is observed and proof of them is the mean value of 0.9835 and intercept to 0.00025916 in the linear regression. Then, we propose the calculation of standardized residuals as the difference between the experimental and predicted COP, the histogram of these standard residuals was plotted in Fig. 6, as can be seen, the image shows practically a normal distribution except for it does not do a perfect symmetry, the mean of standardized residuals is 0.00013905 and the variance is 0.000010321.

Garson's method was applied to the ANN trained and validated in this section. The most important selected variables with the sensibility analysis considering the percentage of relative importance greater than 7% are $\{x_3, x_4, x_6, x_{12}, x_{13}, x_{22}\}$. The results of Garson's method are illustrated in Fig. 7.

4.2. Comparison of the methods and discussion

As can be seen, multiple operational variables have a linear relationship with the COP according to the Correlation matrix method, it is not clear, what main variables are to consider. According to the principal components, method results show in Table 9, the principal operational variables are the temperatures around of evaporator, generator, and absorber–evaporator. These results indicate that the process in the absorber–evaporator is strongly influenced by the evaporator and generator performance, and subsequently in the coefficient of performance of the entire system. The results of Garson's method confirm the importance of the temperatures in the evaporator and generator and adds the oil temperatures as an important aspect to consider.

5. Conclusions

Three mathematical methods were satisfactorily applied in experimental data of three heat transformers, to give a

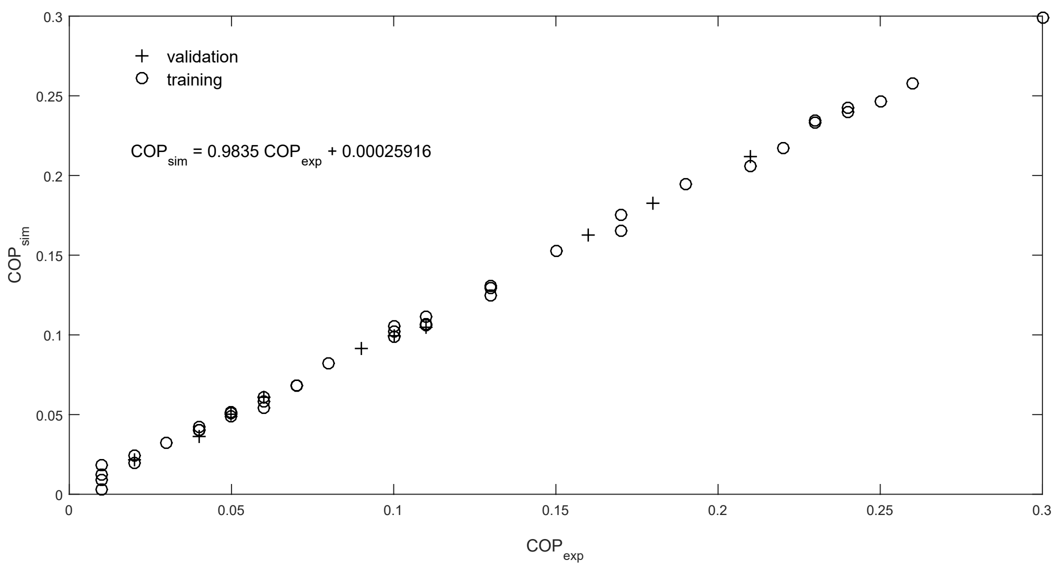

Fig. 5. COP_{exp} against COP predicted by artificial neural network with one neuron in the hidden layer considering experimental information by Rivera et al. [15].

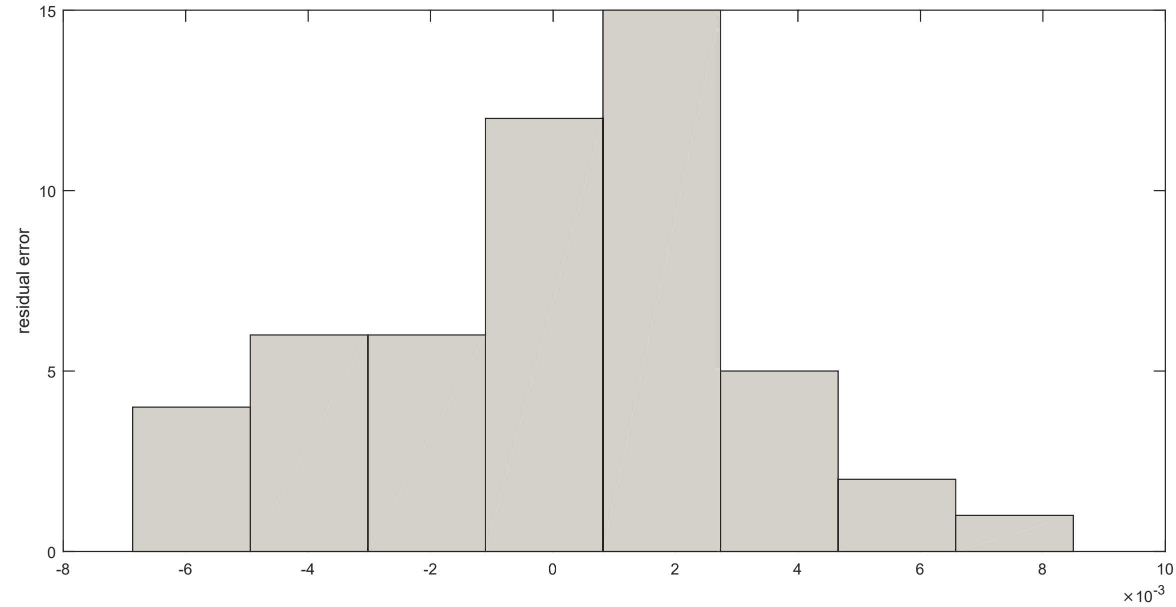

Fig. 6. Histogram of the standardized residuals.

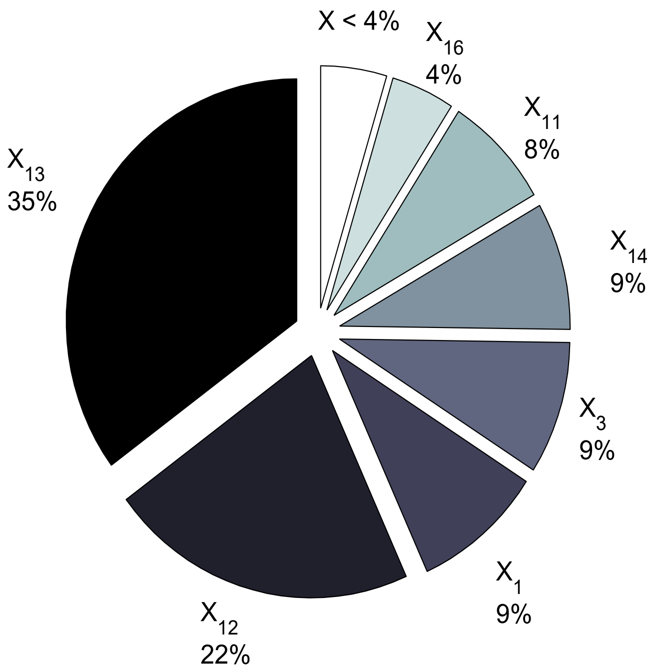

Fig. 7. A double effect heat transformer by Rivera et al. [15]. Relative importance of input variables.

new focus in the search of the main operating variables that influence the prediction of the coefficient of performance. The main contributions of this work are:

- This work proposes to combine the CPA method with the correlation analysis to select variables in this kind of energy system.
- The correlation matrix and principal components methods can be used in experimental data directly without the need for a physical or empirical model. Whereas, for Garson's method, it is necessary to train an ANN.
- For the absorption heat transformer with duplex function components, the methods enlisted in this work presented several perspectives to select the operational variables necessary to predict the COP. The three methods agree on the following variables: $T_{inAB}-_{GE}$ and P_{AB}. Special attention is provided by emphasizing the point where the risk of crystallization is significant.
- For the single-state absorption heat transformer, it is interesting to notice that the variables related to absorption–desorption are emphasized for the three methods.
- For the double-absorption heat transformer, the principal component method highlights in a clear way the relationship between certain variables and the COP. The influence of the evaporator and generator on the absorber–evaporator could be the research way for a detailed study in this kind of equipment.

Table 9
Selected variables by the three methods

Methods	Results
Principal components	$x_1, x_2, x_5, x_9, x_{15}, x_{17}, x_{18}, x_{19}, x_{20}, x_{22}, x_{25}$
Correlation matrix	$x_2, x_3, x_4, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{16}, x_{17}, x_{18}, x_{22}, x_{25}$
Sensibility analysis	$x_3, x_4, x_6, x_{12}, x_{13}, x_{22}$

This current research concludes mainly the following, linear algebra theory (eigenvalues and its eigenvectors) is correctly applied in the CPA. So, when it is combined with the correlation matrix gives a good method to select variables. In the analyzed applications, the variables selected by this method coincides with the variables used by other authors to predict the COP of the systems studied. On the other hand, the correlation matrix analysis is easy to implement. The researchers, according to the need, can use any of the three methods presented in this work.

The methods presented can be used for other types of heat transformers. Specifically, these methods have potential applications to locate the optimal variables that must be measured to predict the coefficient of performance of this kind of equipment with the aim of save the cost of instrumentation. In the future, the authors will analyze the effects of optimal operation variables selected with any of these methods on control or optimization problems of heat transformers.

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Appendix A

This appendix shows the correlation and covariance matrix, as well as, eigenvalues, eigenvectors, and the principal components for the absorption heat transformer with duplex function components Morales et al. [13]. Similar tables have been obtained for the other systems, only one shows the most important variables.

Table A1
Heat transformer with duplex components. Correlation matrix

Correlation matrix									
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x_1	1.0000	0.8485	0.7255	0.7972	0.9175	0.3502	0.1423	0.8914	–0.1524
x_2	0.8485	1.0000	0.5916	0.9547	0.9372	0.0688	–0.1978	0.7120	–0.4073
x_3	0.7255	0.5916	1.0000	0.5580	0.6713	–0.1136	–0.1732	0.7235	0.1693
x_4	0.7972	0.9547	0.5580	1.0000	0.9528	0.0476	–0.2885	0.7007	–0.4041
x_5	0.9175	0.9372	0.6713	0.9528	1.0000	0.1568	–0.1573	0.8273	–0.2561
x_6	0.3502	0.0688	–0.1136	0.0476	0.1568	1.0000	0.8273	0.3132	–0.0157
x_7	0.1423	–0.1978	–0.1732	–0.2885	–0.1573	0.8273	1.0000	0.0752	0.1006
x_8	0.8914	0.7120	0.7235	0.7007	0.8273	0.3132	0.0752	1.0000	–0.0150
x_8	–0.1524	–0.4073	0.1693	–0.4041	–0.2561	–0.0157	0.1006	–0.0150	1.0000
x_{10}	–0.0996	0.1386	0.0789	0.0453	–0.0131	–0.5191	–0.3590	–0.2868	0.1379
x_{11}	–0.0346	–0.1100	0.2951	–0.2078	–0.1322	–0.3832	–0.1000	–0.1864	0.5723
x_{12}	–0.8058	–0.6406	–0.7191	–0.6213	–0.7644	–0.2273	–0.0278	–0.9268	–0.0827
x_{13}	–0.4399	–0.3422	–0.1743	–0.3265	–0.4311	–0.2790	–0.0665	–0.3986	0.0899
x_{14}	0.0286	0.0087	0.4281	0.0137	0.0056	–0.4418	–0.2811	0.0998	0.3806
x_{15}	–0.0602	–0.1165	–0.0418	–0.0730	–0.0750	–0.0898	–0.0357	–0.0028	0.5296
x_{16}	–0.1271	0.1258	–0.2990	0.0373	–0.0537	–0.2317	–0.2562	–0.2449	–0.3567
COP	0.3213	0.2229	0.4307	0.2642	0.3965	–0.0876	–0.2035	0.3123	0.3021

Table A2
Heat transformer with duplex components. Correlation matrix (Continuation)

Correlation matrix								
	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	COP
x_1	–0.0996	–0.0346	–0.8058	–0.4399	0.0286	0.1832	–0.1271	0.3213
x_2	0.1386	–0.1100	–0.6406	–0.3422	0.0087	–0.1165	0.1258	0.2229
x_3	0.0789	0.2951	–0.7191	–0.1743	0.4281	–0.0418	–0.2990	0.4307
x_4	0.0453	–0.2078	–0.6213	–0.3265	0.0137	–0.0730	0.0373	0.2642
x_5	–0.0131	–0.1322	–0.7644	–0.4311	0.0056	–0.0750	–0.0537	0.3965
x_6	–0.5191	–0.3832	–0.2273	–0.2790	–0.4418	–0.0898	–0.2317	–0.0876
x_7	–0.3590	–0.1000	–0.278	–0.0665	–0.2811	–0.0357	–0.2562	–0.2035
x_8	–0.2868	–0.1864	–0.9268	–0.3986	0.0998	–0.0028	–0.2449	0.3123
x_9	0.1379	0.5723	–0.0827	0.0899	0.3806	0.5296	–0.3567	0.3021
x_{10}	1.0000	0.7287	0.2223	0.1429	0.3824	0.2448	0.1966	0.1293
x_{11}	0.7287	1.0000	0.1459	0.1638	0.5939	0.3640	–0.1719	0.1988
x_{12}	0.2223	0.1459	1.0000	0.3743	–0.0616	–0.0160	0.1197	–0.3875
x_{13}	0.1429	0.1638	0.3743	1.0000	0.1832	–0.0389	–0.1642	–0.6491
x_{14}	0.3824	0.5939	–0.0616	0.1832	1.0000	0.2970	–0.3489	0.0752
x_{15}	0.2448	0.3640	–0.0160	–0.0389	0.2970	1.0000	0.0924	0.0898
x_{16}	0.1966	–0.1719	0.1197	–0.1642	–0.3489	0.0924	1.0000	–0.0880
COP	0.1293	0.1988	–0.3875	–0.6491	0.0752	0.0898	–0.0880	1.0000

Table A3
Heat transformer with duplex components. Covariance matrix

Covariance matrix									
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x_1	25.5364	21.4352	18.8648	24.7572	27.9347	–0.0011	2.5182	21.2447	–0.4517
x_2	21.4352	24.9920	15.2190	29.3316	28.2296	0.6999	–3.4631	16.7888	–1.1942
x_3	18.8648	15.2190	26.4774	17.6451	20.8108	–1.1899	–3.1220	17.5576	0.5109
x_4	24.7572	29.3316	17.6451	37.7685	35.2776	0.5957	–6.2110	20.3103	–1.4563
x_5	27.9347	28.2296	20.8108	35.2776	36.2995	1.9226	–3.3187	23.5092	–0.9050
x_6	3.6011	0.6999	–1.1899	0.5957	1.9226	4.1411	5.8972	3.0064	–0.0187
x_7	2.5182	–3.4631	–3.1220	–6.2110	–3.3187	5.8972	12.2693	1.2418	0.2067
x_8	21.2447	16.7888	17.5576	20.3103	23.5092	3.0064	1.2418	22.2442	–0.0415
x_9	–0.4517	–1.1942	0.5109	–1.4563	–0.9050	–0.0187	0.2067	–0.0415	0.3439
x_{10}	–0.9191	1.2656	0.7414	0.5083	–0.1446	–1.9289	–2.2962	–2.4698	0.1477
x_{11}	–0.2519	–0.7916	2.1869	–1.8388	–1.1468	–1.1229	–0.5043	–1.2659	0.4833
x_{12}	–9.0022	–7.0804	–8.1799	–8.4419	–10.1821	–1.0226	–0.2157	–9.6640	–0.1072
x_{13}	–3.2446	–2.4970	–1.3092	–2.9285	–3.7916	–0.8287	–0.3401	–2.7445	0.0769
x_{14}	0.0035	0.0011	0.0535	0.0020	0.0008	–0.0219	–0.0239	0.0114	0.0054
x_{15}	–0.0052	–0.0100	–0.0037	–0.0077	–0.0077	–0.0031	–0.0021	–0.0002	0.0053
x_{16}	–0.0022	0.0021	–0.0052	0.0008	–0.0011	–0.0016	–0.0030	–0.0039	–0.0007

Table A4
Heat transformer with duplex components. Covariance matrix (Continuation)

Covariance matrix							
	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}
x_1	–0.9191	–0.2519	–9.0022	–3.2446	0.0035	–0.0052	–0.0022
x_2	1.2656	–0.7916	–7.0804	–2.4970	0.0011	–0.0100	0.0021
x_3	0.7414	2.1869	–8.1799	–1.3092	0.0535	–0.0037	–0.0052
x_4	0.5083	–1.8388	–8.4419	–2.9285	0.0020	–0.0077	0.0008
x_5	–0.1446	–1.1468	–10.1821	–3.7916	0.0008	–0.0077	–0.0011
x_6	–1.9289	–1.1229	–1.0226	–0.8287	–0.0219	–0.0031	–0.0016
x_7	–2.2962	–0.5043	–0.2157	–0.3401	–0.0239	–0.0021	–0.0030
x_8	–2.4698	–1.2659	–9.6640	–2.7445	0.0114	–0.0002	–0.0039
x_9	0.1477	0.4833	–0.1072	0.0769	0.0054	0.0053	–0.0007
x_{10}	3.3345	1.9161	0.8974	0.3810	0.0170	0.0077	0.0012
x_{11}	1.9161	2.0735	0.4645	0.3443	0.0208	0.0090	–0.0008
x_{12}	0.8974	0.4645	4.8876	1.2079	–0.0033	–0.0006	0.0009
x_{13}	0.3810	0.3443	1.2079	2.1307	0.0065	–0.0010	–0.0008
x_{14}	0.0170	0.0208	–0.0033	0.0065	0.0006	0.0001	–0.0000
x_{15}	0.0077	0.0090	–0.0006	–0.0010	0.0001	0.0003	0.0000
x_{16}	0.0012	–0.0008	0.0009	–0.0008	–0.000	0.000	0.0000

Table A5
Heat transformer with duplex components. Eigenvalues of the covariance matrix

λ	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8
Eigenvalues	148.9423	22.2273	16.9830	6.3483	2.5216	1.8846	1.2228	0.9231
λ	λ_9	λ_{10}	λ_{11}	λ_{12}	λ_{13}	λ_{14}	λ_{15}	λ_{16}
Eigenvalues	0.5915	0.4251	0.2094	0.1821	0.0384	0.0002	0.0001	0.0000

Table A6
Heat transformer with duplex components. Eigenvectors

λ	Eigenvectors								
λ16	–0.0015	–0.0010	0.0002	0.0004	0.0011	0.0004	0.0006	0.0013	–0.0006
λ15	0.0063	–0.0039	–0.0026	0.0098	–0.0103	–0.0056	0.0011	0.0012	0.0273
λ14	–0.0033	–0.0023	0.0016	0.0059	–0.0051	–0.0019	0.0010	0.0062	–0.0079
λ13	0.1965	–0.0074	0.0058	0.0297	–0.0929	–0.1618	0.0276	–0.0569	0.8114
λ12	0.4565	–0.3217	–0.0011	0.3363	–0.4600	0.1348	–0.1486	–0.1421	–0.2887
λ11	0.0772	0.1307	–0.0823	0.2403	–0.3333	–0.0005	–0.0311	–0.1465	0.3800
λ10	0.0420	0.2495	0.0532	–0.2794	0.0086	0.7763	–0.4360	–0.0915	0.1427
λ9	0.0222	0.0746	–0.0223	–0.2798	0.3476	–0.1934	0.0757	–0.4552	–0.0498
λ8	0.6222	–0.0500	–0.1913	–0.2474	0.0599	–0.2622	–0.2270	–0.0327	–0.1186
λ7	0.0721	0.6012	0.1213	–0.2320	–0.3941	–0.1609	–0.0160	–0.0377	–0.2400
λ6	–0.1298	0.3691	–0.1668	–0.0155	–0.3335	–0.1526	0.1006	0.4077	–0.0143
λ5	0.2063	0.0176	–0.3928	–0.3485	0.1209	–0.1325	–0.2117	0.2699	0.1039
λ4	0.2251	0.2819	0.1626	–0.0259	–0.0139	0.0377	0.4134	–0.4980	0.0041
λ3	0.0348	0.1984	–0.7816	0.2900	0.1420	0.2692	0.2916	–0.1035	–0.0767
λ2	0.2982	–0.2095	0.1351	–0.3595	–0.0764	0.3250	0.6451	0.3528	0.0394
λ1	0.3902	0.3851	0.3177	0.4702	0.4865	0.0243	–0.0385	0.3387	–0.0109

Table A7
Heat transformer with duplex components. Eigenvectors (Continuation)

λ	Eigenvectors						
λ16	–0.0003	0.0020	0.0013	0.0004	0.0171	–0.0734	0.9971
λ15	0.0020	0.0012	–0.0012	–0.0029	–0.1473	–0.9861	–0.0701
λ14	–0.0011	0.0141	0.0030	–0.0006	–0.9888	0.1456	0.0276
λ13	0.1415	–0.4792	0.1126	–0.0032	–0.0091	0.0266	0.0040
λ12	0.2829	–0.2118	–0.3064	–0.0277	0.0009	0.0037	0.0020
λ11	–0.4578	0.5552	–0.3323	–0.0775	0.0083	0.0154	0.0011
λ10	0.0326	–0.0358	–0.0191	0.1754	–0.0073	–0.0042	0.0004
λ9	–0.0796	–0.2209	–0.6749	0.1753	–0.0118	–0.0057	0.0015
λ8	–0.3292	0.0859	0.3432	0.3743	–0.0010	–0.0024	0.0001
λ7	–0.2021	–0.3080	0.0584	–0.4304	–0.0033	–0.0054	0.0015
λ6	0.2589	0.0097	–0.2128	0.6287	0.0030	0.0011	0.0001
λ5	0.4390	0.2944	–0.2105	–0.4440	0.0008	0.0039	0.0006
λ4	0.4707	0.3654	0.2502	0.0673	0.0013	0.0002	0.0001
λ3	–0.1112	–0.2092	0.0968	–0.0683	–0.0037	–0.0004	0.0002
λ2	–0.2002	–0.0225	–0.1516	–0.0498	–0.0007	–0.0000	–0.0003
λ1	–0.0030	–0.0113	–0.1466	–0.0476	0.0002	–0.0001	–0.0000

Table A8
Heat transformer with duplex components. Variance, standard deviation, variance of principal components

(λ) - Eigenvalues	% Variability percent	% accumulated	Standard deviation	Variance
148.9423	73.5519	73.5519	12.2042	148.9423
22.2273	10.9764	84.5283	4.7146	22.2273
16.9830	8.3867	92.9150	4.1210	16.9830
6.3483	3.1350	96.0500	2.5196	6.3483
2.5216	1.2452	97.2952	1.5879	2.5216
1.8846	0.9307	98.2259	1.3728	1.8846
1.2228	0.6039	98.8298	1.1058	1.2228
0.9231	0.4559	99.2856	0.9608	0.9231
0.5915	0.2921	99.5777	0.7691	0.5915
0.4251	0.2099	99.7876	0.6520	0.4251
0.2094	0.1034	99.8910	0.4576	0.2094
0.1821	0.0899	99.9809	0.4267	0.1821
0.0384	0.0189	99.9999	0.1959	0.0384
0.0002	0.000090876	100.0000	0.0136	0.00018402
0.0001	0.000049081	100.0000	0.0100	0.000099389
0.0000	0.00000090734	100	0.0014	0.0000018374

Table A9
Heat transformer with duplex components. Correlation coefficient: principal components vs. original variables

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
Z_{16}	–0.0000	–0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	–0.0000
Z_{15}	0.0000	–0.0000	–0.0000	0.0000	–0.0000	–0.0000	0.0000	0.0000	0.0005
Z_{14}	–0.0000	–0.0000	0.0000	0.0000	–0.0000	–0.0000	0.0000	0.0000	–0.0002
Z_{13}	0.0076	–0.0003	0.0002	0.0009	–0.0030	–0.0156	0.0015	–0.0024	0.2710
Z_{12}	0.0385	–0.0275	–0.0001	0.0234	–0.0326	0.0283	–0.0181	–0.0129	–0.2100
Z_{11}	0.0070	0.0120	–0.0073	0.0179	–0.0253	–0.0001	–0.0041	–0.0142	0.2965
Z_{10}	0.0054	0.0325	0.0067	–0.0296	0.0009	0.2487	–0.0812	–0.0127	0.1587
Z_9	0.0034	0.0115	–0.0033	–0.0350	0.0444	–0.0731	0.0166	–0.0742	–0.0653
Z_8	0.1183	–0.0096	–0.0357	–0.0387	0.0095	–0.1238	–0.0623	–0.0067	–0.1944
Z_7	0.0158	0.1330	0.0261	–0.0417	–0.0723	–0.0875	–0.0051	–0.0088	–0.4525
Z_6	–0.0353	0.1014	–0.0445	–0.0035	–0.0760	–0.1029	0.0394	0.1187	–0.0335
Z_5	0.0648	0.0056	–0.1212	–0.0900	0.0319	–0.1034	–0.0959	0.0909	0.2814
Z_4	0.1122	0.1421	0.0796	–0.0106	–0.0058	0.0466	0.2974	–0.2661	0.0178
Z_3	0.0284	0.1636	–0.6259	0.1945	0.0971	0.5452	0.3431	–0.0904	–0.5391
Z_2	0.2782	–0.1976	0.1238	–0.2758	–0.0598	0.7529	0.8683	0.3527	0.3165
Z_1	0.9423	0.9401	0.7535	0.9336	0.9855	0.1454	–0.1340	0.8764	–0.2262

Table A10
Heat transformer with duplex components. Correlation coefficient: principal components vs. original variables (Continuation)

	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	COP
Z_{16}	–0.0000	0.0000	0.0000	0.0000	0.0010	–0.0058	0.3984	0.0181
Z_{15}	0.0000	0.0000	–0.0000	–0.0000	–0.0604	–0.5737	–0.2059	0.0951
Z_{14}	–0.0000	0.0001	0.0000	–0.0000	–0.5519	0.1153	0.1103	0.0488
Z_{13}	0.0152	–0.0652	0.0100	–0.0004	–0.0734	0.3036	0.2281	0.1123
Z_{12}	0.0661	–0.0628	–0.0591	–0.0081	0.0163	0.0929	0.2482	–0.1314
Z_{11}	–0.1147	0.1764	–0.0688	–0.0243	0.1562	0.4104	0.1507	–0.0762
Z_{10}	0.0116	–0.0162	–0.0056	0.0784	–0.1956	–0.1584	0.0856	–0.0897
Z_9	–0.0335	–0.1180	–0.2348	0.0923	–0.3726	–0.2576	0.3319	0.1006
Z_8	–0.1732	0.0573	0.1491	0.2464	–0.0398	–0.1362	0.0404	–0.3023
Z_7	–0.1224	–0.2365	0.0292	–0.3261	–0.1513	–0.3509	0.4755	–0.0888
Z_6	0.1947	0.0093	–0.1321	0.5913	0.1678	0.0846	0.0538	–0.5734
Z_5	0.3817	0.3246	–0.1512	–0.4830	0.0542	0.3627	0.2634	0.3694
Z_4	0.6494	0.6394	0.2852	0.1162	0.1393	0.0353	0.0379	–0.0523
Z_3	–0.2510	–0.5986	0.1804	–0.1929	–0.6216	–0.1018	0.2733	–0.2840
Z_2	–0.5169	–0.0738	–0.3233	–0.1607	–0.1285	–0.0059	–0.3884	–0.0178
Z_1	–0.0200	–0.0960	–0.8091	–0.3978	0.0817	0.0696	–0.0769	0.3588

Table A11
Heat transformer with duplex components. Weights and biases for the ANN model proposed by Morales et al. [13]

W_i							
K	s_1	s_2	s_3	s_4	s_5	s_6	s_7
1	–1.9099	1.2920	0.5616	–1.5757	–0.5532	1.5157	0.4352
2	–0.0288	–0.3611	0.3854	–0.3885	–0.5210	–0.5062	0.9491
3	0.2129	2.2394	–1.1379	1.0966	–0.8497	–0.7690	1.6061
4	1.5230	1.0080	–1.4527	0.7660	0.9598	1.4658	1.9828
5	–1.3621	–0.4408	1.4821	0.3322	–1.7076	–0.6261	–0.5569
6	–0.0520	0.0135	–0.0236	0.0493	–1.0766	2.0584	–0.1266
7	–0.0949	0.1629	0.4055	–0.0468	0.1504	–1.4808	–0.3215
8	0.3049	–0.2157	0.4969	2.2333	–0.8665	–0.2973	0.7216
9	1.0391	0.6563	–1.7324	2.8432	1.2625	–1.1286	–1.5479
10	0.9664	–1.1222	1.0486	0.0371	–0.3499	–0.9110	–2.0244
11	–3.0315	–2.4746	–0.9918	–1.6792	–1.1325	–0.2667	0.8165
12	3.1316	1.6210	–1.0206	1.7662	–0.0674	–1.8688	0.8866
13	0.8523	–2.5339	–2.7502	–1.4656	1.3361	0.0175	–1.0743
14	0.0316	–1.4242	–0.0977	–0.2727	–1.7286	–0.9406	–0.6384
15	–0.5229	2.3381	–0.0735	0.2790	–0.0069	–0.3185	–0.8393
16	0.7108	1.4673	–0.6754	0.0599	1.9246	0.5640	0.0839
$W_0(s,l)(l = 1)$	0.1899	0.0610	0.1628	0.1498	0.0019	0.0011	0.1161
$b(1,s)$	0.8544	0.0116	4.2146	2.7413	2.1583	2.9889	2.5522
$b(2,1)$	0.0965						

Appendix B

Table A12
Absorption heat transformer. Variance, standard deviation, variance of principal components

(λ) - Eigenvalues	% Variability percent	% accumulated	Standard deviation	Variance
60.7767	55.5186	55.5186	7.7959	60.7767
15.2189	13.9022	69.4207	3.9011	15.2189
13.0428	11.9144	81.3351	3.6115	13.0428
9.9033	9.0465	90.3816	3.1469	9.9033
5.2302	4.7777	95.1593	2.2870	5.2302
2.3746	2.1691	97.3284	1.5410	2.3746
1.4656	1.3388	98.6672	1.2106	1.4656
0.6603	0.6032	99.2704	0.8126	0.6603
0.4933	0.4507	99.7210	0.7024	0.4933
0.2188	0.1998	99.9208	0.4677	0.2188
0.0442	0.0404	99.9613	0.2103	0.0442
0.0366	0.0335	99.9947	0.1914	0.0366
0.0058	0.0053	100.0000	0.0760	0.0058
0.0000	0.00000000000000071310	100.0000	0.0000000000000036802	1.3544e-29
–0.0000	0.00000000000000047462	100.0000	0.0000000000000029549	8.7312e-30
–0.0000	–0.00000000000000093569	100	0.0000000000000017129	2.9341e-30

Table A13
Absorption heat transformer. Weights and biases for the ANN model proposed by Hernández et al. [14]

K	s_1	s_2	s_3
1	1.0471	20.0710	3.7487
2	1.6234	6.4620	0.1130
3	8.5410	46.4600	3.5127
4	1.7987	6.3265	0.5450
5	7.1516	26.4226	0.6408
6	1.1960	16.5470	0.3451
7	1.0900	14.6380	0.0254
8	2.5118	37.4539	0.0000
9	0.0030	0.3227	0.0032
10	2.3420	4.4175	0.1114
11	242.3930	216.8700	0.4314
12	159.4000	135.7784	7.1419
13	172.5000	115.0829	12.6125
14	20.8370	10.2770	3.2912
15	19.4760	17.5255	0.1576
16	65.5880	47.0385	1.1184
$(l = 1)W_0(s,l)$	–0.186	0.0239	0.8825
Bias $b(1,s)$	129.0939	17.7516	6.4906
$b(2,1)$	0.2427		