

Modeling and visualization of failure rate of a water supply network using the regression method and GIS

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ABSTRACT

In the study, the authors have attempted to develop a failure rate prediction model for a selected water supply network using linear regression, taking into account the simultaneous impacts of many factors on the failure rate; also, possible methods of failure rate visualization for the examined network were presented. In the regression model, the dependent variable is the unit failure frequency (fail/(km year)), whereas independent variables (explaining this frequency) are: pipe material, age and diameter, soil type (impermeable, permeable) and soil moisture content (dry, wet). Statistical analyses have shown that the failure rate of the examined water supply network is affected only by soil conditions. Spatial distribution of the failure number and frequency was visualized, paying special attention to the parts of the network where the spatial concentration of failures was the greatest.

Keywords: Water supply network; Failure rate; GIS

1. Introduction

The failure rate is a measure of the water supply network's reliability, which is defined [1–4] as a network's ability to carry out its functions under specific operating conditions over an assumed period of time. In the case of a water supply network, these functions involve supplying consumption sites with water of sufficient amount, proper quality, demanded pressure, and at times of day convenient for the user. Therefore, failure outcomes in a water supply network will be manifested in such adverse events, as:

- lack of water supply or supply with insufficient water amount to the consumer,
- distribution of water having inadequate quality,
- failure to comply with network pressure requirements,
- water supply at strictly determined times of day, unfavorable for consumers.

The failure rate is one of the most important criteria used to assess the technical condition of water supply pipes. In practical assessments, the failure rate is defined as a mean unit failure frequency expressing the number of failures along 1 km (or 10 km) long pipes over 1 year (or 10 years) [2–7]. This indicator is widely used to assess and compare failure rates of water supply pipes depending on their function (transit line, water main, distribution pipe, service line) and location (areas damaged after mining operations and locations outside such areas), material, age, network pressure, water and soil conditions, soil temperature and temperature of transmitted water.

An example of the failure frequency distribution for water supply pipes plotted based on the Local data base of the Central statistical office divided by Polish provinces (voivodships) is presented in Fig. 1.

As clearly shown in Fig. 1, the highest failure rate was reported for water supply networks in the Silesia Province,

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Fig. 1. Unit failure frequency of water distribution pipelines in polish provinces in 2015 [8].

where a great many pipes are laid in areas damaged by mining operations. Recent studies have shown that network failure rates within these areas (1.186 fail/(km year)) are nearly four times higher than in the networks laid outside these areas (0.259 fail/(km year) [9].

The existing research has mainly focused on assessing the impact of various factors on the failure rate of water supply networks, assuming however that these impacts are not interrelated. As a result of such analysis, the impact of soil instability, function, diameter and material of which pipes were made, soil moisture content, and pressure [1,4,10–13] was scrutinized the most thoroughly. A multi-criteria decision analysis was attempted in studies by [14,15], where the simultaneous effect of a number of factors on the failure rate of water supply networks was considered. In the study by [15], the author focused on pipes made of grey cast iron and steel, operating in Silesia province. Stepwise regression with backward elimination allowed us to identify the following factors exerting considerable impact on the failure rate: soil type (sandy, graveled), soil moisture content (low, high), pipe diameter, water corrosiveness (measured using the Langelier saturation index-LSI) and mining exploitation. Pipe water pressure, a period of use and material on the other hand, had no impact on pipe failure rate. Although numerous studies and analysis have been conducted to date [1,16,17], no operating standards for failure frequency prediction have been developed, which would enable one to compare and assess failure rates for different pipes comprising water supply networks. With the current stage of knowledge, one can only use to that end authors' original proposals presented in different scientific studies (Tables 1 and 2)

Here, a high failure rate (low reliability) indicates the need for pipe renewal after previous technical and economic analysis of the profitability of this undertaking for a given pipeline section.

Some experts are known to have claimed that a failure rate of 1 fail/(km year) should absolutely qualify a pipe for renewal. Literature, among numerous eligibility conditions for pipe renewal, also sets out failure frequency of 0.5 fail/ (km year) [19], as well as 5 fail/(km year) (2 fails per 100 m over 4 y) [20,21]. Many interesting Geographic Information Table 1

Suggested classifications of water supply pipes in terms of their reliability [18]

Failure rate/reliability category	Failure rate
	[fail/(km year)]
A. Low failure rate = high reliability	$\lambda_A \leq 0.1$
B. Average failure rate/reliability	$0.1 < \lambda_{\scriptscriptstyle B} \le 0.5$
<i>C</i> . High failure rate = low reliability	$\lambda_{_C} > 0.5$

Table 2

Suggested criterion values of the failure rate of water supply pipes depend on their function [1]

Pipe type	Failure rate [fail/(km year)]
Water mains	$\lambda \le 0.1; \lambda \le 0.3$
Distribution pipes	$\lambda \leq 0.5; \lambda \leq 0.8$
Service lines	$\lambda \le 0.7; \lambda \le 1.0$
Pipes irrespectively of their function	$\lambda \le 0.5$

System (GIS) applications can be found in decision-making processes employing failure rate assessment in water supply networks. One of them is a system detecting the impact of contaminants flowing out from sewage tanks and trenches on water pipes. At the same time, the system–using the IRA-WDS model–creates a risk map of pipe vulnerability to the intrusion of these contaminants Fig. 2.

In the paper by [23], it was shown how climate changes likely to result in the lowering of groundwater level and soil consolidation may consequently impact the failure rate of a water supply network Fig. 3. To forecast the failure rate, the Monte Carlo method was used which in fact may be successfully employed in a GIS environment. The use of the Monte Carlo method to predict the influence of a cascade failure of different networks also presented in [24], where the modeling considerations apply to the network infrastructure of cities considered as an interdependent network.



Fig. 2. Illustration of the possibilities of the IRA-WDS model (software and manual for risk assessment of contrast intrusion into water distribution systems)-to determine the level of risk of failure in a water supply network [22].



Fig. 3. Assessment of water network vulnerability to climate change-the probability of network failure [23].

Another example of using GIS in assessing the failure rate is a set of priorities for pipe renewal [25]. Research in this respect was conducted based on data involving such features as reliability (failure frequency), pressure and limit flow rates, age, diameter and number of consumers served by a given network section. The iterative feature selection method was applied, and the outcome was the network system with the assessed technical condition of marked network sections Fig. 4. Impact strength on the final outcome and sensitivity of specific features to changes were also determined.

In the paper by [26], special attention was paid to the mapping of areas vulnerable to network failures Fig. 5. Vulnerable areas are defined as urban areas where water



Fig. 4. Renewal priorities according to the technical condition of pipeline sections [25].

deficiency causes serious human consequences. These areas were encircled with rings with a 200 m radius (e.g. hospitals, patients undergoing dialysis, etc.). Heavy-traffic areas were also established (50 m buffer zone around roads with over 15,000 vehicle passages per day), important for keeping the city easily accessible.

Only a few examples of GIS applications in different decision-making processes were presented above. Possible GIS uses are however, practically unlimited in terms of visualization of different conditions of water distribution networks.

2. Material and method

2.1. Object of study

All data related to the water supply network were obtained from the GIS database maintained by the network



Fig. 5. Vulnerability of the area to network damage [26].

operator. The examined system was a water supply network with a total length of 178.4 km, including distribution pipes of 165 km, as well as water mains and transit pipes of 13.4 km. Currently, the pipeline supplies water to 72,962 residents, who account for 96.5% of the town's population. Supplied water is used for industrial and municipal purposes (domestic needs of the residents, services, general municipal applications-wetting of green areas, watering the streets, etc.) The water supply network is made primarily of polyethylene (PE) (32.7% of pipe length), grey cast iron (26.3%), polyvinyl chloride (PVC) (24.2%) and steel (11.3%). The remaining 5.5% includes ductile iron and asbestos cement pipes. Age structure analysis shows that the major part of the network was built in the last 25 years (58.6% of the pipes). Only about 1.3% of the pipes are more than 50 years old. Although the water supply network is not old, there is a regression in its development.

2.2. Method

The purpose of the research is to develop a failure rate prediction model using linear regression, taking into account the simultaneous impact of many factors on this failure rate and to show how the failure rate of the examined network may be visualized. Failure rate of a water supply network is represented in the model by the unit failure frequency, whereas variables determining this frequency are: material, age and pipe diameter, soil type (impermeable, permeable) and soil moisture content (dry-water level above 2 m below ground level, wet-up to 2 m below ground level). The study was based on data obtained for a period of over three and a half years, from the GIS database implemented in an enterprise servicing the network. The research method involved the following stages:

- Collection and establishment of a joint data set (emergency events, soil types, groundwater levels, pipe material, age and diameter) within the GIS database, and its verification.
- Calibration and digitization of maps presenting groundwater level and soil type.
- Joining data concerning failures with pipe types using GIS tools (Spatial Join).
- Export of essential data from the attribute table in the pipe and failure layer from ArcGIS software to Microsoft Excel (XIs format) and calculation of pipe failure rate considering the explaining variables, using a pivot table.
- Development of the regression model, where an important step was the preparation of data for the model, mainly of explaining variables-selection of pipes sharing the same features (diameter, age, water conditions, soil conditions, material).
- Visualization of pipe failure rates.

2.3. Regression model

To develop the regression model, a linear regression formula for *n*-variables determining *X* was used:

$$Y = B_0 + B_1 X_1 + B_2 X_2 + \dots + B_n X_n + \varepsilon$$
 (1)

where *Y* (dependent variable)-also referred to as the explained variable, predicted using the regression model (here: failure rate λ); *X* (independent variables)-also referred to as explanatory variables or regressors (here: pipe diameter, age, material, water conditions, soil conditions), assumed to impact the value of dependent variable *Y*; *B* (coefficients)-values calculated using regression tools, reflecting the direction and power of impact of each explanatory variable *X* on the value of the explained variable *Y*; ε (outliers)-part of the explained variable that failed to be explained using the model-it determines overestimation or underestimation of the model. These constitute differences between the observed *Y* value and the value calculated using the model.

An attempt was made to match the regression model with data obtained using linear regression lm() from *R* package. The lm() method assumes a normal distribution of random variables describing features of independent variables in the samples and uses typical significance tests: Student's *t*-test (*t*-test) as well as *F*-Snedecor test (*F*-test) [20,27,28]

2.4. Preparation of the dependent variable and independent variables

Sets of variables were prepared based on pipeline data comprehensively described in the GIS database. Completeness of data describing independent variables, that can be used in the regression model is illustrated in Fig. 6. In the figure, each square corresponds to sets containing specified data. Based on this record sheet, it may be concluded that comprehensive data concerning the material, length,



Fig. 6. Data ranges describing independent variables that can be used in the creation of the regression model.

diameter, age as well as water and soil conditions are available only for 11.8% of pipeline length.

Independent variables were grouped as follows:

- diameter: distribution pipes (from 90 to 250 mm), water mains (from 300 mm); in accordance with applicable provisions [29], distribution pipes and water mains form a water supply network.
- age: 100 to 51 years old, 50 to 26 years old, 25 to 11 years old, less than 10 years old;
- water conditions: up to 2 m, below 2 m.
- soil conditions: clays (impermeable), other than clays (permeable).
- material: PE, PVC, steel, grey cast iron, ductile iron, asbestos cement.

For selected groups of independent variables, pipe failure rate values were estimated, that is the values of the dependent (explained) variable. Due to the fact that independent variables have a different impact on the failure rate, it was necessary to assign relevant weights to these variables. Weight values for the materials (X_5) , water (X_3) and groundwater conditions (X_4) were determined subjectively in accordance with the algorithm presented in Fig. 7. Weight coefficients for diameters (X_1) and age (X_2) are presented by standardized data depending on the weight of specific features for pipe failure rate (the lesser the diameter and the longer the period of operation [age], the higher the failure rate). The highest values of these variables pertained to the most unfavorable features, the more the values approximated 1, the higher the impact they had on failure rate.

3. Modeling results

The results of the statistical analysis for the water supply network are presented in Table 3.

Unfortunately, *p*-values for both *F*-test and *t*-test are significantly higher than 0.1 (which is the assumed test significance level), therefore the model fails to exhibit a good fit with the data (*R*-squared = 0.2496).

Here Fig. 8, residuals appear to visually correlate with Y (failure rate), the majority approximating-2, that is, very near zero. Only one point differs significantly from other observations and approximates 6. We may suspect that this an outlier, that is a point which is not subject to relationships other points are subject to, or it occurs very rarely. This point was therefore rejected and the analysis was performed again.

Results of statistical analysis for the water supply network excluding the outlier (Table 4)

Table 4 shows that the outlier might have covered some effect, as the p-value in the t-test for the regressor (water conditions) X_3 is lower than the assumed 0.1. This proves that there is a 90% probability of the existence of a relationship between X_3 and Y (failure frequency).

Residuals, outlying regressors, in Fig. 9 are very evenly distributed around zero. This confirms that data are accurately represented in the model and that the model is likely to contain precisely determined coefficients, however, significance tests only confirm one parameter to be significant- X_3 (water conditions). We cannot form any reliable conclusions as regards other parameters, as we are short of some data



Fig. 7. The algorithm of assigning weight coefficients to specific independent variables.

and/or *Y* distribution is too disperse, or there is a too weak relationship to be detected using this amount of data and such *Y* dispersion. Results obtained using the lm() method were confirmed by using another regression method, that is rlm() method from the MASS library, which does not require to assume a normal distribution of random variables in the samples and which uses non-parametric equivalents of *F* test and *R* package sfsmisc, ETH Zürich, Rämistrasse 101, 8092 Zurich, Switzerland [29]. In this method, the test identified the same relationship between pipe failure frequency and soil conditions X_3 even with p = 0.05. Non-parametric tests sometimes have more statistical power. Therefore, in the event of a disturbance in data regularity, we may suspect that there is the relationship between X_3 (water conditions) and *Y* (failure frequency) at the probability level of 95%.

4. Failure rate visualization

Spatial distributions of failure number and intensity of failures of the analyzed water supply network were visualized, with special attention paid to the pipes for which there is the highest concentration of failures in the area. After selection of the number of failures using pivot tables, and entering data to the database, we used ArcGIS to render visualizations of the number of network failures Fig. 10. The highest number of failures (10 per 3.5 years) was identified along a section of 58 m, described using: street name, diameter-DN100, age-unknown, material-unknown, water conditions-groundwater level of 2m below ground level and soil type-clays (impermeable). All failures were removed by mounting of a corrective band, and in one case through replacement of a 10 m pipeline section. Failure frequency along this section was as much as $\lambda = 49.3$ fail/(km year).

Also, an analysis intended to identify areas with the highest failure concentration (in specific polygons) was performed. To that end, using "fishnet" tools [30] a grid was created for the network area divided into 400 rectangles (with meshes of approx. 369 m length and 331 m width). Then, based on the calculated data concerning the length and number of failures, a (double) column was developed where the spatial distribution map for the failure frequency was created in a polygon. The results are presented in Fig. 11.

	Coefficients					
		Estimate std.	Error	<i>t</i> value	$\Pr(> t)$	
	(Intercept)	4.4267	5.1825	0.854	0.421	
Diameter	x^1	-1.7483	3.2073	-0.545	0.603	
Age	<i>x</i> ²	0.2119	3.8820	0.055	0.958	
Water conditions	<i>x</i> ³	2.6061	3.1890	0.817	0.441	
Ground conditions	χ^4	-0.7269	7.2478	-0.100	0.923	
Material	<i>x</i> ⁵	-3.4229	3.8955	-0.879	0.409	
	Residual standard error: 2.873 on 7 degrees of freedom					
	Multiple R-squared: 0.2496, Adjusted R-squared: –0.2865					
	F-statistic: 0.4656	on 5 and 7 DF, <i>p</i> -value: 0.7	914			

Table 3 Statistical fit report of the regression model using the lm() method for the water supply network

Table 4

Statistical fit report of the regression model using the lm() method for the water supply network after outlier is rejected

	Coefficients					
		Estimate std.	Error <i>t</i>	value	Pr(> t)	
	(Intercept)	1.43135	1.73528	0.825	0.441	
Diameter	x^1	0.85734	1.09974	0.780	0.465	
Age	<i>x</i> ²	-0.08433	1.26756	-0.067	0.949	
Water conditions	<i>x</i> ³	-2.54750	1.23616	-2.061	0.085	
Ground conditions	x^4	1.07502	2.37698	0.452	0.667	
Material	x^5	0.81309	1.38453	0.587	0.578	
	Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 ' ' 0.1 ' ' 1					
	Residual standard error: 0.9376 on 6 degrees of freedom					
	Multiple R-squared: 0.431, Adjusted R-squared: -0.04321					
	<i>F</i> -statistic: 0.9089 on 5 and 6 DF, <i>p</i> -value: 0.5319					



Fig. 8. Estimated values of random dispersion in regression carried out using the lm() method for a water supply network.



Fig. 9. Estimated values of random dispersion in regression carried out using the lm () method for a water supply network after outlier is rejected.



Fig. 10. Spatial distribution of the number of failures in the water supply network in the period of 3.5 years.

A number of other analyses were carried out as well for example, a number of failures distribution in pipeline sections with the highest failure rate. However, due to the considerable space they occupy, the results of this analysis were not included in the body of the paper. All analysis in question was focused on preliminary identification of sections for renewal. Failure frequency for these sections was as many as more than 3 fail/(km year).

5. Summary

Despite the significant advancement of implementation works in the field of creation of GIS-like databases in water supply enterprises, considerable shortcomings may still be observed in resources essential to analyze and assess failure rates for water supply networks. And the failure rate comprises grounds for assessment of the technical condition



Fig. 11. Spatial distribution of failure rate for the water supply network in the period of 3.5 years.

and consequently, decisions on pipe renewal. The reason for this may be high costs of data entry related, among many others, to problems with access to data sources and the need for their verification, amendment and completion. Based on experiences collected to date and on analyses of data contained in GIS-like databases of other enterprises, it may be stated that the rules for determining the manner of using databases for assessment of water supply network failure rate have already been developed. They have not, however, been used in water supply enterprises. Since GIS databases offer extensive possibilities to streamline the network system management process, if only by supporting spatial assessment of the technical conditions of pipes and making decisions about their renewal, further attempts should be made in the field of ordering and completion of data with information about failures in GIS. It is important to define the goals of proceeding with database creation. Does this involve, for example, setting up of a numerical model (where data topology should be complied with), or only network inventory control, as data entry methods may differ considerably. Special attention should be paid to an in-depth analysis of data range and offered possibilities to update data. To that end, the best solution would be to develop a drop-down list presenting the most commonly entered data and the possible introduction of a more detailed and precise description. Data supplemented in this manner could contribute to the establishment of a more accurate and reliable model for failure rate determination and provide for identification of potential failure sites in the future, and consequently, prompt counteraction of unfavorable outcomes. While entering failure-related data, attention should also be paid to the occurrence of a number of failures over a short period of time along the same section, which may suggest that subsequent failures are caused by the ineffective removal of the previous ones. It is also very important to continually update data and prevent duplication. If for any reason, we wish to keep archive data, we should create separate folders with such data. However, the reliability and accuracy of entered data should be verified, depending on the manner of their spatial representation using coordinates, additional surveys or sketches. We must however, select the most recent data.

6. Final conclusion

The modeling results obtained by the lm() method were also confirmed by another rlm() method [31], which does not require to assume a normal distribution of random variables. The dependence of the failure rate of a water supply network to soil conditions X_3 was verified even at a significance level of 0.05.

Regression results could be more favorable, if the water supply failure data were complete and more uniform. This could be influenced also by water and soil condition data which were developed based on large-scale low-accuracy maps. It is also necessary to consider new parameters that might impact failure frequency.

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