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A fuzzy dependent-chance interval multi-objective stochastic expected value programming approach for irrigation water resources management under uncertainty

Youzhi Wang[a], Zhong Li[b], Liu Liu[a], Ping Guo[a,c,*]

[a]*Center for Agricultural Water Research in China, China Agricultural University, Beijing 100083, China, emails: guop@cau.edu.cn (P. Guo), yzwang@cau.edu.cn (Y.Z. Wang), liuliu@cau.edu.cn (L. Liu)*
[b]*Department of Civil Engineering McMaster University, Ontario L8S4L8, Canada, email: zoeli@mcmaster.ca*
[c]*Wuwei Experimental Station for Efficient Water Use in Agriculture, Ministry of Agriculture and Rural Affairs, Wuwei 733000, China*

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ABSTRACT

In this study, a fuzzy dependent-chance interval multi-objective stochastic expected value programming model is developed for irrigation water resources management under uncertainties. It incorporates fuzzy dependent-chance programming, stochastic expected value programming, interval programming into multi-objective programming. Compared with conventional programming methods, it can quantify the relationship between the expected values of stochastic variables and the fuzzy goals of expected values set by decision-makers through the satisfactory degrees, and trade-off the relationship amid multiple satisfactory degrees selected as objective functions. Besides, it can cope with uncertainties expressed as interval numbers, fuzzy numbers, and stochastic variables. Moreover, the fairness of water allocation constraints formulated by the GINI coefficient can achieve the interactions between fair water allocation and satisfactory degrees. The model is applied to a real case study of irrigation water resources management of different water types (i.e., surface water and groundwater) under different water flow levels (high, medium, and low flow levels) in the midstream region of the Heihe River basin, northwest China. The results reveal that: (1) maximum water demands of wheat and economic crop are satisfied while that of corn is not met under three flow levels; (2) the expected economic benefit and water shortages of crops have positive relationships with water allocation while the expected canal water loss has a negative relationship with water allocation; (3) the bigger expected economic benefit results in the higher satisfactory degree of the expected economic benefit while the lower expected water shortage and canal water loss lead to higher satisfactory degrees of expected water shortage and canal water loss. It shows that the developed model can overcome the disadvantages of the single-objective programming of putting attention to the satisfactory degree of a kind of expected value, and neglecting the satisfactory degree of other associated expected benefit. It also can overwhelm the drawbacks of the two-objective programming model of more focus on the satisfactory degree of the expected canal water loss. The results can provide different water allocation schemes for decision-makers.

Keywords: Irrigation water allocation; Interval programming; Stochastic expected value programming; Fuzzy dependent-chance programming; Multi-objective programming; Uncertainty

* Corresponding author.

1. Introduction

The water resource is an essential factor for agricultural production, especially for semi-arid and arid districts where the agricultural production is obtained mainly from irrigation. In recent years, the problems of the water shortage caused by the unbalanced relationships between water supplies and water demands in these places have become more serious because of the variations of climatic conditions, and human activities [1–4]. Besides, a large amount of canal water loss in the process of water transportation aggravates these problems [5]. Therefore, it is vital to optimizing irrigation water resources with consideration of agricultural production, water shortage, and canal water loss simultaneously to reach sustainable agricultural development.

The deterministic mathematical models have been widely used in irrigation water resources management, but they are not applicably owing to the uncertain parameters in the irrigation water resources system [6,7]. In reality, the runoff owns the randomness because of the interactions of hydro-meteorological elements, and its caused variabilities of benefits aggravate the uncertainties of decisions. In order to deal with the stochastic variables and quantify the benefits covering all kinds of possible scenarios, the stochastic expected value programming (SEP) is adopted through calculating the expected value of stochastic variables [1,8]. In general, the decision-makers usually require that the expected value satisfies the goal set by themselves, for example, the expected economic benefit is larger than the economic goal of 200×10^8 Yuan. And the measurement of this kind of relationship can recognize the decision attitudes of managers. However, the above relationship is transformed into the relationship that the expected economic benefit is larger than the fuzzy economic goals of $[180, 200, 230] \times 10^8$ Yuan because of the fuzziness of goals caused by the subjectivity of decision-makers. This kind of problem can be dealt with by fuzzy dependent-chance programming (FDCP) [9]. It can quantify the relationship between the expected value and fuzzy goals of the expected value as a satisfactory degree based on the credibility distribution function, at the same get the optimal water allocation schemes corresponding to the maximum satisfactory degree. The SEP model can deal with the stochastic variables, while the FDCP model is able to cope with the fuzzy numbers, but the single model is unable to handle both the stochastic variables and fuzzy numbers simultaneously. Therefore, the fuzzy dependent-chance stochastic expected value programming (FDCSEP) is developed by integrating the SEP with the FDCP model for irrigation water resources management in this paper. Nevertheless, there are few studies developing the FDCSEP model to conduct irrigation water resources management.

The FDCSEP model can address the stochastic variables and fuzzy numbers but is incapable of coping with the contradictory multiple objectives. In reality, the managers are usually encountered with multiple objectives, such as maximizing the expected economic benefit, minimizing the expected water shortages of crops, and the trade-off amid multiple objectives can be addressed by the multi-objective programming (MOP) approach by searching comprised optimal solutions [10–13]. Therefore, in order to improve the robustness of the FDCSEP model, a fuzzy dependent-chance multi-objective stochastic expected value programming (FDCMOSP) model is developed by integrating the MOP model with the FDCSEP model. Therefore, maximizing expected economic benefit is transformed into the maximizing the satisfactory degree than expected economic benefit is larger than the fuzzy goals of the expected economic benefit, and minimizing expected water shortage is converted into maximizing the satisfactory degree that the expected water shortage is smaller than the fuzzy goals of expected water shortage. That is the trade-off amid multiple expected values is converted into the trade-off amid multiple satisfactory degrees. The nonlinear format of the satisfactory degree can be transformed into the linear format based on the credibility distribution function that can be formulated by referring to Zhang et al. [8,14–20]. Nevertheless, the previous studies can hardly deal with the relationship between the expected values and fuzzy goals of expected values, and trade-off the relationships amid multiple satisfactory degrees. Except for the randomness of runoff and fuzziness of set goals, the other associated parameters have uncertainties because of the variations of water and market demands-supplies environments, such as water demand, market price. These uncertain parameters can be addressed by interval programming (IP) through formulating its upper and lower boundary values of interval numbers when the data availability is limited [6,21–24]. Therefore, in order to address the conflictions amid multiple satisfactory degrees, and deal with the uncertainties presented as interval numbers, fuzzy numbers, and stochastic variables, the fuzzy dependent-chance interval multi-objective stochastic expected value programming (FDCIMOSP) model is developed by integrating the IP approach with the FDCMOSP model. Nevertheless, the related studies formulating the FDCIMOSP to manage agricultural water resources have not been reported.

The water allocations of crops amid different subareas are usually determined based on the irrigation benefits, which may lead to the problem of unfair water allocation among subareas. To address the above problem, the fair-water allocation constraints measured by the GINI coefficient is widely used in water resources management, which can achieve the fairness of water allocation by minimizing the deviation of water allocation per area of one crop amid different subareas, at the same time reach the interactions between fair water allocation and satisfactory degree [25,26]. Therefore, the fair constraints quantified by the GINI coefficient are added into the FDCIMOSP model to guide irrigation water resources management in this paper.

Therefore, the purpose of this study is to develop a FDCIMOSP model to optimize irrigation water resources under uncertainties. It integrates the FDCP, stochastic expected value programming, interval programming within a MOP framework. It has advantages in: (1) dealing with the uncertainties expressed as interval numbers, fuzzy numbers, and stochastic variables simultaneously; (2) measuring the relationships between expected values and fuzzy goals of expected values as the satisfactory degree, and achieving the trade-off amid multiple satisfactory degrees; (3) achieving the interactions between fairness of water allocation, and satisfactory degrees; (4) generating the optimal interval water allocation schemes. The FDCIMOSP model is applied to a case study for optimizing water allocation of crops in the middle reaches of the Heihe River basin, northwest China. And the optimal water allocation will be obtained from applying and solving the FDCIMOSP model. Besides, the robustness of the FDCIMOSP model is verified by comparing it with three single-objective programming models, and three two-objective programming models.

2. Methodology

The framework of the FDCIMOSP model is shown in Fig. 1. The model integrates FDCP, stochastic expected value programming, interval programming with MOP. It can deal with the uncertainties presented as interval numbers, fuzzy numbers, and stochastic variables, and quantify the relationship between the expected value of stochastic variables and fuzzy goals as the satisfactory degree, and trade-off the relationships amid multiple satisfactory degrees. The detailed process of the model building can be seen below:

2.1. Stochastic expected value programming (SP)

To deal with the stochastic variable and its trigged uncertain outputs, such as economic benefit, and canal water loss, the stochastic expected value programming (SP) is adopted in this paper. The discrete stochastic variable is used here because the probability distributions of stochastic variable are difficult to be acquired. Let the value of stochastic variable x is equal to ξ_1 at the probability $p_l\left(p_l, \sum_l^n p_l = 1\right)(l = 1,2,...,n)$, when $x \geq 0$, the expected value of discrete stochastic variable can be rewritten as $\max f = E\left[Q(x,\omega)\right] = \sum_{l=1}^{n} p_l Q(x,\xi_l)$ [22]. And thus the SP model is formulated as follows:

$$\max f = \max E\left[Q(x,\omega_1)\right] = \max \sum_{l=1}^{n} p_l Q(x,\omega_1)$$
$$\text{s.t.} \tag{1}$$
$$h_s(x) \leq 0, \quad s = 1,2,...,S$$

where f is an objective function; $E[Q(x,\omega_1]$ are the expected values of discrete stochastic variables; ω_1 is the stochastic variable; x is the decision variable; $h_s(x)$ are the constraints.

2.2. Fuzzy dependent-chance stochastic expected value programming

To address the relationship between the expected value of stochastic variables and fuzzy goals of the expected value set by decision-makers, the FDCSP is formulated by incorporating the dependent-chance programming (FDCP) into the SP model, which can be rewritten as follows [8,27]:

$$\begin{cases} \max \mathrm{Cr}\left\{E\left[Q(x,\omega_1)\right] \geq \tilde{f}_{\mathrm{pd}}\right\} \\ \text{s.t.} \\ g_j(x,\xi) \leq 0, \quad j = 1,2,...,J \\ h_s(x) \leq 0, \quad s = 1,2,...,S \end{cases} \tag{2}$$

where $E[Q(x_i^{\pm},\omega_i^{\pm})]$ is the expected value of stochastic variables; $\tilde{f}_{\mathrm{pd}}$ is the fuzzy goals of expected value; ξ denotes fuzzy vectors; $\mathrm{Cr}^{\pm}\left\{E\left[Q(x_i^{\pm},\omega_i^{\pm})\right] \geq \tilde{f}_{\mathrm{pd}}\right\}$ is the credibility (satisfactory) degree that expected value satisfies the fuzzy goals, and it changes from 0 to 1, and the higher value corresponds to a higher satisfactory degree. The 0.5 is defined as the allowable minimum satisfactory degree, and 1.0 is regarded as the allowable maximum satisfactory degree. Through the above transformations, maximizing expected value is transformed into maximizing the satisfactory degree that expected value fulfills the fuzzy goals, where the name can be simplified called the satisfactory degree of expected value.

2.3. Fuzzy dependent-chance interval multi-objective stochastic expected value programming

In order to address the uncertain parameters with limited data availability, and trade-off the contradictory relationship amid multiple satisfactory degrees, the FDCIMOSP model is formed by combining the FDCSP model with the interval multi-objective programming approach. The expression of the FDCIMOSP model is as follows:

$$\begin{cases} \max \begin{bmatrix} f_1^{\pm}(x_1^{\pm}) = \mathrm{Cr}^{\pm}\left\{E\left[Q(x_1^{\pm},\omega_1^{\pm})\right] \geq \tilde{f}_{p1}\right\} \\ f_2^{\pm}(x_2^{\pm}) = \mathrm{Cr}^{\pm}\left\{E\left[Q(x_2^{\pm},\omega_2^{\pm})\right] \leq \tilde{f}_{p2}\right\} \\ \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \\ f_t^{\pm}(x_t^{\pm}) = \mathrm{Cr}^{\pm}\left\{E\left[Q(x_t^{\pm},\omega_t^{\pm})\right] \geq \tilde{f}_{pt}\right\} \end{bmatrix} \\ \text{s.t.} \\ g_j^{\pm}(x_i^{\pm}) \leq 0 \quad j = 1,2,...,p \\ h_l^{\pm}(x_i^{\pm}) \leq 0 \quad l = 1,2,...,q \end{cases} \tag{3}$$

where $E[Q(x_1^{\pm},\omega_1^{\pm})]$, $E[Q(x_2^{\pm},\omega_2^{\pm})]$ and $E[Q(x_t^{\pm},\omega_t^{\pm})]$ are the expected value$_1$, expected value$_2$, and expected value$_t$, respectively; $\tilde{f}_{p1}$, $\tilde{f}_{p2}$, and $\tilde{f}_{pt}$ are the fuzzy goals of expected value$_1$, expected value$_2$, and expected value$_t$ set by decision-makers, respectively; $f_1^{\pm}$, $f_2^{\pm}$, and $f_t^{\pm}$ are objective functions, representing the interval satisfactory degrees of expected value$_1$, expected value$_2$, and expected value$_t$, individually. The objective functions in Eq. (3) are non-linear formats, and they can be converted into linear formats by building corresponding credibility distribution functions.

When the satisfactory degree is expressed as a format that expected value is larger than the fuzzy goals, such as $\mathrm{Cr}^{\pm}\left\{E\left[Q(x_1^{\pm},\omega_1^{\pm})\right] \geq \tilde{f}_{p1}\right\}$, the credibility distribution function is shown in Eq. (4). When the satisfactory degree is expressed as a format that expected value is smaller than the fuzzy goals, like $\mathrm{Cr}^{\pm}\left\{E\left[Q(x_2^{\pm},\omega_2^{\pm})\right] \leq \tilde{f}_{p2}\right\}$, the credibility distribution function is shown in Eq. (5). Let $\tilde{b} = \left(\underline{b}, b, \overline{b}\right)$, where $\tilde{b}$ indicates triangular fuzzy numbers of goals; $\underline{b}, b, \overline{b}$ delegate minimum possible value, possible value, and maximum possible value of fuzzy goals, respectively, which cover variation ranges of goals; Ax represents the expected

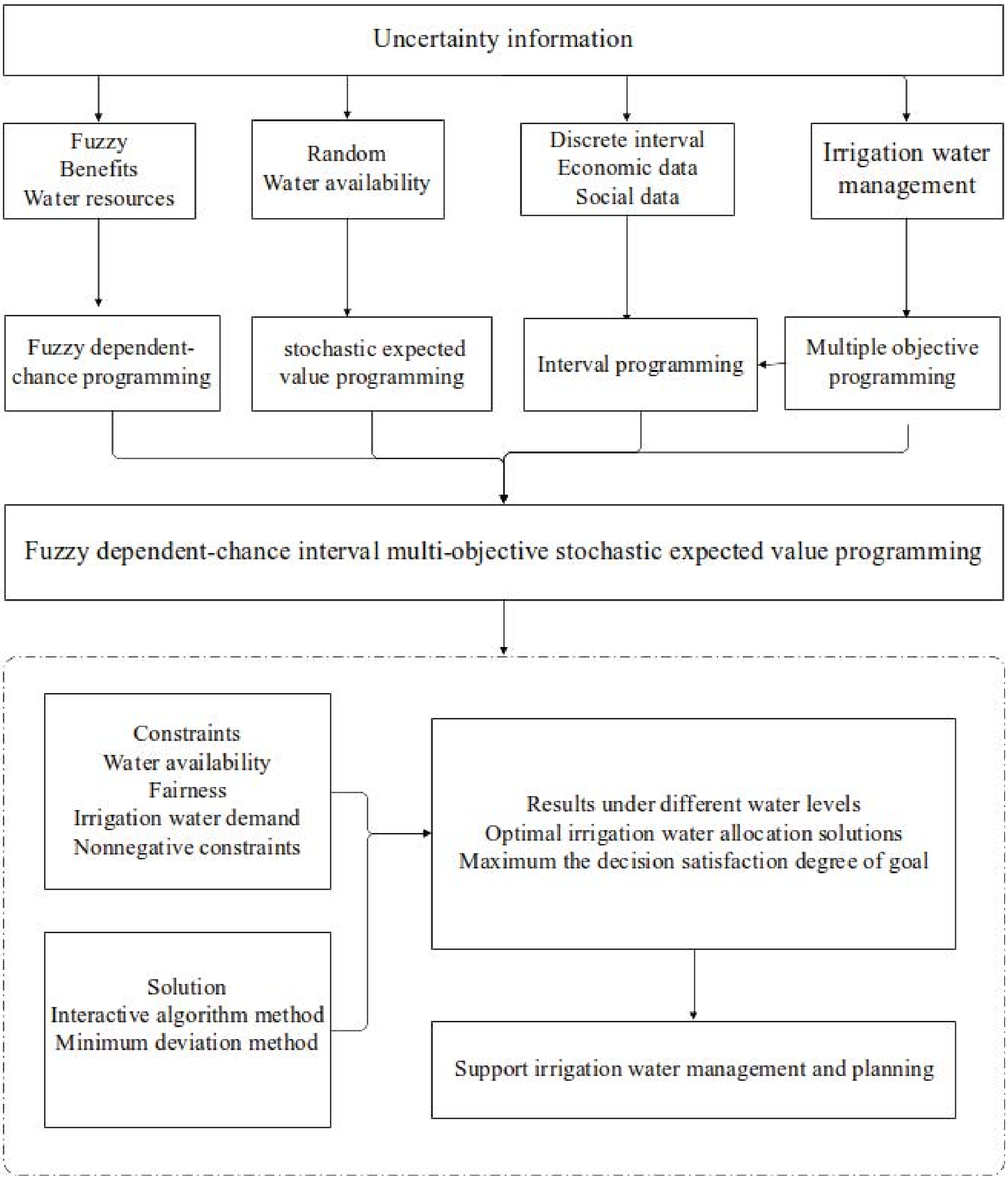

Fig. 1. The model framework of the FDCIMOP model.

value. The credibility distribution function of $\mathrm{Cr}\{\mathrm{Ax} \geq \tilde{b}\}$ and $\mathrm{Cr}\{\mathrm{Ax} \leq \tilde{b}\}$ can be defined as the following expressions according to the definition of credibility measure [16,17].

$$\mathrm{Cr}\left(\mathrm{Ax} \geq \tilde{b}\right) = \begin{cases} 0 & \mathrm{Ax} \leq \underline{b} \\ \dfrac{\mathrm{Ax} - \underline{b}}{2\left(b - \underline{b}\right)} & \underline{b} \leq \mathrm{Ax} \leq b \\ \dfrac{\bar{b} - 2b + \mathrm{Ax}}{2\left(\bar{b} - b\right)} & b \leq \mathrm{Ax} \leq \bar{b} \\ 1 & \mathrm{Ax} \geq \bar{b} \end{cases} \tag{4}$$

$$\mathrm{Cr}\left(\mathrm{Ax} \leq \tilde{b}\right) = \begin{cases} 1 & \mathrm{Ax} \leq \underline{b} \\ \dfrac{2b - \underline{b} - \mathrm{Ax}}{2\left(b - \underline{b}\right)} & \underline{b} \leq \mathrm{Ax} \leq b \\ \dfrac{\mathrm{Ax} - \bar{b}}{2\left(\bar{b} - b\right)} & b \leq \mathrm{Ax} \leq \bar{b} \\ 0 & \mathrm{Ax} \geq \bar{b} \end{cases} \tag{5}$$

The above two kinds of credibility distributions are shown in Fig. 2.

Fig. 2a represents the satisfactory degree of the expected benefit, and Fig. 2b shows the satisfactory degree of the expected canal loss. The higher expected benefit and lower

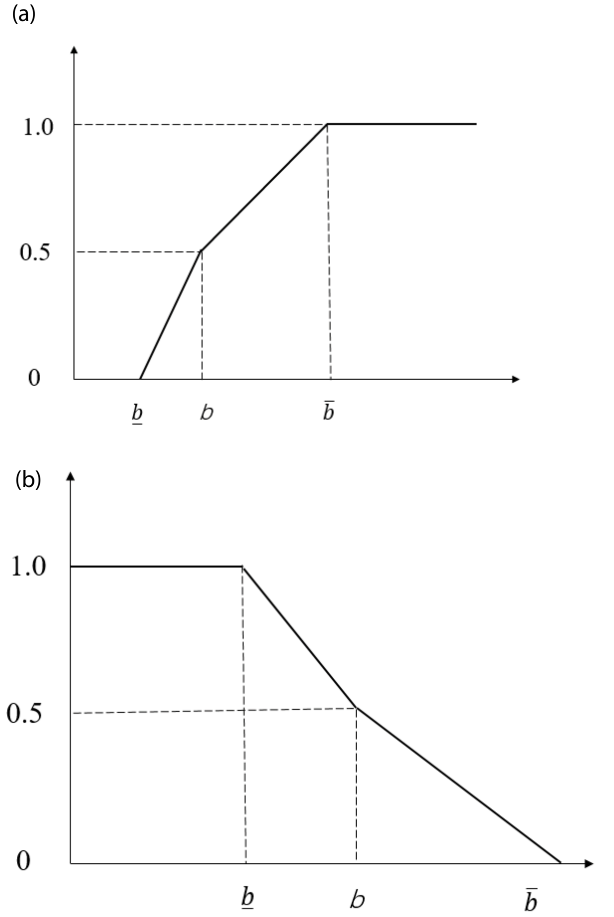

Fig. 2. Credibility distributions of triangular fuzzy variable: (a) $\mathrm{Ax} \geq \tilde{b}$ and (b) $\mathrm{Ax} \leq \tilde{b}$.

expected loss are desired by managers, and thus the higher expected benefit value will result in a higher satisfactory degree of expected benefit, and the lower expected loss value will lead to a higher satisfactory degree of the expected loss. The detailed relationships between the expected value, and the satisfactory degree of expected value can be seen from Figs. 2a and b.

The satisfactory degree is required to be greater than 0.5 to avoid the unsuitable satisfactory degree and violation risk in management. And thus the satisfactory degree can be rewritten as follows based on Eqs. (4) and (5):

$$\max\gamma = \left\{\frac{\bar{b} - 2b + \mathrm{Ax}}{2\left(\bar{b} - b\right)}\right\} \text{ when } \mathrm{Ax} \geq \tilde{b} \tag{6}$$

$$\max\gamma = \left\{\frac{2b - \underline{b} - \mathrm{Ax}}{2\left(b - \underline{b}\right)}\right\} \text{ when } \mathrm{Ax} \leq \tilde{b} \tag{7}$$

where γ is the satisfactory degree. The non-linear objective function in Eq. (3) could be converted to linear forms based on Eqs. (6) and (7). After the above transformation, the FDCIMOSP model can be rewritten as follows:

$$\begin{cases} \max \begin{bmatrix} f_1^{\pm}\left(x_1^{\pm}\right) = \dfrac{f_{p1}^{\max} - 2f_{p1}^{p} + E\left[Q\left(x_1^{\pm}, \omega_1^{\pm}\right)\right]}{2(f_{p1}^{\max} - f_{p1}^{p})} \\ f_2^{\pm}\left(x_2^{\pm}\right) = \dfrac{2f_{p2}^{p} - f_{p2}^{\min} - E\left[Q\left(x_2^{\pm}, \omega_2^{\pm}\right)\right]}{2\left(f_{p2}^{p} - f_{p2}^{\min}\right)} \\ \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \\ f_t^{\pm}\left(x_t^{\pm}\right) = \dfrac{f_{pt}^{\max} - 2f_{pt}^{p} + E\left[Q\left(x_t^{\pm}, \omega_t^{\pm}\right)\right]}{2\left(f_{pt}^{\max} - f_{pt}^{p}\right)} \end{bmatrix} \\ \text{s.t.} \\ g_j^{\pm}\left(x_i^{\pm}\right) \leq 0 \quad j = 1,2,\ldots,p \\ h_l^{\pm}\left(x_i^{\pm}\right) \leq 0 \quad l = 1,2,\ldots,q \end{cases} \tag{8}$$

where the $\tilde{f}_{p1} = \left[f_{p1}^{\min}, f_{p1}^{p}, f_{p1}^{\max}\right]$ is the fuzzy goals of expected value$_1$, and $f_{p1}^{\min}, f_{p1}^{p}, f_{p1}^{\max}$, are the minimum possible, possible and maximum possible values of fuzzy goals.

2.4. Solve methods

A method for integrating the interactive algorithm method [21] and the minimum deviation method [28] is used to solve the FDCIMOSP model. It can solve the FDCIMOSP model through solving the upper and lower submodels and minimizing the deviations between objectives and respect superior and inferior values. The integrated solved method is as follows:

2.4.1. Lower objective

$$\min F^{-}\left(X_i^{\pm}\right) = \sum_{i=1}^{I} \frac{f_i\left(X_i^{-}\right) - f_i^{*}}{f_i' - f_i^{*}} + \sum_{j=i+1}^{m} \frac{f_j^{*} - f_j\left(X_i^{+}\right)}{f_j^{*} - f_j'} \tag{9a}$$

2.4.2. Upper objective

$$\min F^{+}\left(X_i^{\pm}\right) = \sum_{i=1}^{I} \frac{f_i\left(X_i^{+}\right) - f_i^{*}}{f_i' - f_i^{*}} + \sum_{j=i+1}^{m} \frac{f_j^{*} - f_j\left(X_i^{-}\right)}{f_j^{*} - f_j'} \tag{9b}$$

where $f_i(X)$ (i = 1, 2, …, I) denotes the minimum objective function; $f_j(X)$ ($j = I + 1$, $I + 2$, …, m) presents the maximum objective function; $X_i^{\pm}$ is decision variable, X_i^{+} and X_i^{-} denote upper bound/lower bound of $X_i^{\pm}$, respectively; f_i', f_i^{*} are superior and inferior values of $f_i(X)$, individually; f_j^{*}, f_j' indicate superior and inferior values of $f_j(X)$ separately. The superior and inferior values of $f_i(X)$ and $f_j(X)$ are obtained by solving corresponding single objective programming. Eqs. (9a) and (b) concerns an equal transformation, and the detailed process is as follows: the objective function can be transformed into the equation of minimizing the deviation between the superior value of the objective function and objective function for maximizing the objective function. And the higher objective function value results in a lower deviation, and thus the minimizing the deviation is used to reach the bigger objective function value.

The detailed process for solving the FDCIMOSP method is shown as follows:

- *Step* 1: Acquire the uncertain input information and quantify the uncertain parameters, which includes the interval numbers, fuzzy numbers, and stochastic variables;
- *Step* 2: Form the FDCIMOSP model;
- *Step* 3: Convert the FDCIMOSP model into an equivalent linearization form based on Eq. (8);
- *Step* 4: Transform the linearization of the FDCIMOSP model into two sets of submodels based on the interactive algorithm and minimum deviation method;
- *Step* 5: Solve the F^- submodel and get the corresponding solution alternatives based on Eq. (9a);
- *Step* 6: Solve the F^+ submodel and get the corresponding solution alternatives based on Eq. (9b);
- *Step* 7: Get the optimal interval objectives value $F = [F^-, F^+]$, and corresponding optimal water allocation schemes;
- *Step* 8: End.

3. Case study

3.1. Study area

The study area is located at Zhangye City, Gansu Province, midstream of the Heihe River basin (HRB), northwest China, shown in Fig. 3. It is divided into Ganzhou District (denoted as GZ), Linze County (denoted as LZ), and Gaotai County (denoted as GT). The study crops include wheat (denoted as WC), corn (denoted as CC), and economic crop (denoted as EC), and the high crop productions make it become the main grain production base of China. The mean annual precipitation is about 140 mm while the mean annual evapotranspiration is around 1,400 mm, and thus the crop yield is mainly gained from irrigation. The irrigation water resources are derived from surface water from HRB, and groundwater. However, the unbalanced relationship between water supply and water demand becomes severe under the impact of climate changes and human activities. Therefore, it is essential to allocate limited water resources to crops reasonably to achieve sustainable agricultural development.

3.2. Problem statement

In the irrigation water resources system, the runoff owns randomness, and its trigged variabilities of outputs (e.g., economic benefit, water shortage, and canal water loss) make the water allocation full of uncertainties. Besides, the decision-makers usually require the expected value fulfills the fuzzy goals of the expected value which is caused by the subjectivity of managers, and measurement of this kind of relationship can recognize the decision attitudes of managers. Nevertheless, previous studies can hardly deal with this kind of problem. Moreover, the managers are usually faced with contradictory multiple objectives, such as the expected economic benefit, expected water shortage, and expected canal water loss. And trade-offs among multiple expected values are transformed into the trade-offs between multiple the expected values and respective fuzzy goals. Therefore, an applicable optimization model under uncertainties is desired for the above problems. In reality, the water allocations of crops among different subareas are operated based on the irrigation benefit, which may lead to the unfairness of water allocation amid different subareas, and thus fairness of water allocation is needed to be considered in the uncertain optimization model.

3.3. Data collection

Table 1 shows the triangular fuzzy numbers of the expected economic benefit, expected water shortage and expected canal water loss. The higher expected economic benefit and lower expected water shortage and lower canal water loss are aspired by managers, and thus the fuzzy goals for the above three expected values are different. For the expected economic benefit, the fuzzy numbers are set as $\tilde{f} = \left[f_{\min}^{+}, f_{\max}^{-}, f_{\max}^{+} \right]$. For the expected water shortage and canal water loss, the fuzzy numbers are arranged as $\tilde{f} = \left[f_{\min}^{-}, f_{\min}^{+}, f_{\max}^{-} \right]$. Where $f_{\min}^{-}$ and $f_{\min}^{+}$ denote the lower and upper bounds of minimizing the expected value, respectively; the $f_{\max}^{-}$ and $f_{\max}^{+}$ represent the lower and upper bound of maximizing the expected value separately, which are attained through solving the single objective programming model. Table 2 displays the groundwater,

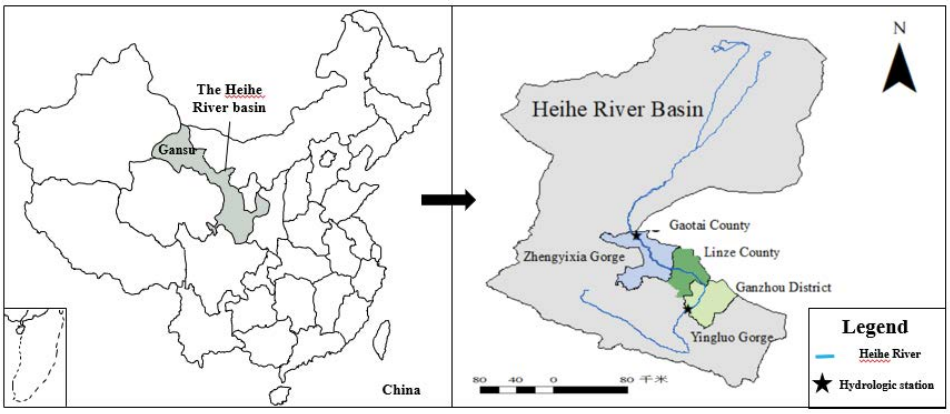

Fig. 3. Study area.

Table 1
Triangular fuzzy numbers of economic objective, social objective

	Minimum possible value	Possible value	Maximum possible value
Economic object (10^4 Yuan)	456,073	658,154	1,407,088
Social object (10^4 m^3)	18,209	69,421	71,524
Resource object (10^4 m^3)	18,652	50,171	51,691

Table 2
Irrigation water availability and occurrence probabilities of flow levels

Water level	Surface water (10^4 m^3)	Ground water (10^4 m^3)	Probability
High	[85,500; 104,500]	[20,500; 22,700]	0.25
Medium	[71,900; 78,400]	[25,400; 28,600]	0.5
Low	[45,700; 62,800]	[30,500; 32,900]	0.25

and surface water with known probabilities. The surface water resources are divided into high, medium, and low flow levels with probabilities of 25%, 50%, and 25% based on the P-III hydrographic curve. Table 3 displays the irrigated benefit coefficients of crops and effective precipitation, where the former is calculated by multiplying the unit price by unit yield and then dividing irrigation quota, and the latter is calculated by multiplying total effective precipitation with respective planting areas. Table 4 shows the water demands of crops under three flow levels, which are computed by multiplying the potential evapotranspiration by the crop coefficient. The interval numbers are all formulated by parameters estimation method with a 95% confidence level. The time series of surface water and groundwater is collected from the Water Conservancy Annual Report of Zhangye City. The water price and operation cost of surface water and groundwater refer to Jiang [29]. The precipitation data is attained from weather stations of Zhangye Station, Linze Station, and Gaotai Station.

3.4. Agricultural irrigation water allocation optimization model

The initial objective functions of the FDCIMOSP include maximizing the expected economic benefit, minimizing the expected water shortages of crops, and minimizing the expected canal water loss, and they are transformed into the maximizing the satisfactory degrees of the expected economic benefit, expected water shortage, and expected canal water loss, which are expressed as follows:

Objective 1: maximizing the expected economic benefit

$$\max f_{1m}^{\pm} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{p=1}^{P} p_h \left(\mathrm{BT}_{ij}^{\pm} - \mathrm{BS}_{k1} - \mathrm{BP}_{k1}\right)\mathrm{WS}_{ijh}^{\pm} + \left(\mathrm{BT}_{ij}^{\pm} - \mathrm{BS}_{k2} - \mathrm{BP}_{k2}\right)\mathrm{WG}_{ijh}^{\pm}$$

Objective 2: minimizing the expected water shortages of crops

$$\min f_{2m}^{\pm} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{p=1}^{P} p_h \left(\mathrm{ET}_{\max,ijh}^{\pm} - \mathrm{WS}_{ijh}^{\pm} - \mathrm{WG}_{ijh}^{\pm}\right)$$

Objective 3: minimizing the expected canal water loss

$$\min f_{3m}^{\pm} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{p=1}^{P}\left(p_h\left(1-p_{k1}\right)\mathrm{WS}_{ijh}^{\pm} + \left(1-p_{k2}\right)\mathrm{WG}_{ijh}^{\pm}\right) \quad (10a)$$

In order to address the relationship between the expected value and fuzzy goals of the expected value, the satisfactory degree is adopted, and thus the above objective functions are transformed into the following formats:

Objective 1: maximizing the satisfactory degree of the expected economic benefit

$$\max f_1^{\pm} = \mathrm{Cr}^{\pm}\left\{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{p=1}^{P} p_h \begin{pmatrix}\left(\mathrm{BT}_{ij}^{\pm} - \mathrm{BS}_{k1} - \mathrm{BP}_{k1}\right)\mathrm{WS}_{ijh}^{\pm} + \\ \left(\mathrm{BT}_{ijh}^{\pm} - \mathrm{BS}_{k2} - \mathrm{BP}_{k2}\right)\mathrm{WG}_{ijh}^{\pm}\end{pmatrix} \geq \widetilde{f_{\mathrm{eco}}}\right\}$$

Objective 2: maximizing the satisfactory degree of the expected water shortages of crops

$$\max f_2^{\pm} = \mathrm{Cr}^{\pm}\left\{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{p=1}^{P} p_h \begin{pmatrix}\mathrm{ET}_{\max,ijh}^{\pm} - \mathrm{WS}_{ijh}^{\pm} - \\ \mathrm{WG}_{ijh}^{\pm} - P_{ij}^{\pm}\end{pmatrix} \leq \widetilde{f_{\mathrm{soc}}}\right\}$$

Objective 3: maximizing the satisfactory degree of the expected canal water loss

$$\max f_3^{\pm} = \mathrm{Cr}^{\pm}\left\{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{p=1}^{P} p_h \begin{pmatrix}\left(1-p_{k1}\right)\mathrm{WS}^{\pm}{}_{ijh} + \\ \left(1-p_{k2}\right)\mathrm{WG}_{ijh}^{\pm}\end{pmatrix} \leq \widetilde{f_{\mathrm{res}}}\right\} \quad (10b)$$

Table 3
Irrigation benefits of crops and effective precipitation

Subarea	Wheat	Corn	Economic crop
	Irrigation benefits (Yuan/m^3)		
GZ	[3.76; 4.14]	[3.27; 3.50]	[24.84; 27.32]
LZ	[2.88; 3.17]	[2.23; 2.46]	[27.47; 30.22]
GT	[3.34; 3.67]	[2.14; 2.36]	[28.89; 31.78]
	Effective precipitation (10^4 m^3)		
GZ	[1,123; 1,400]	[3,939; 4,927]	[751; 941]
LZ	[794; 996]	[1,531; 1,914]	[418; 527]
GT	[374; 465]	[1,213; 1,515]	[898; 1,122]

Table 4
Water demands of crops under different flow levels (10^4 m^3)

District	Flow level	ET	Wheat	Corn	Economic crop
GZ	High	ET_{max}	[4,112; 8,390]	[19,487; 39,769]	[7,155; 14,601]
		ET_{min}	[3,397; 4,111]	[16,106; 19,487]	[5,914; 7,157]
	Medium	ET_{max}	[3,795; 7,744]	[17,988; 36,710]	[6,604; 13,478]
		ET_{min}	[3,136; 3,995]	[14,867; 17,988]	[5,459; 6,604]
	Low	ET_{max}	[3,479; 7,099]	[16,489; 33,651]	[6,053; 12,355]
		ET_{min}	[2,875; 3,479]	[13,628; 16,489]	[5,004; 6,054]
LZ	High	ET_{max}	[4,329; 8,833]	[11,178; 22,814]	[4,534; 9,253]
		ET_{min}	[3,577; 4,328]	[9,240; 11,179]	[3,748; 4,534]
	Medium	ET_{max}	[3,996; 8,153]	[10,319; 21,059]	[4,185; 8,541]
		ET_{min}	[3,302; 3,995]	[8,528; 10,319]	[3,459; 4,185]
	Low	ET_{max}	[3,663; 7,474]	[9,459; 19,304]	[3,837; 7,829]
		ET_{min}	[3,027; 3,662]	[7,818; 9,459]	[3,171; 3,836]
GT	High	ET_{max}	[1,794; 2,172]	[10,360; 21,144]	[11,119; 22,693]
		ET_{min}	[2,875; 3,479]	[8,564; 10,360]	[9,191; 11,120]
	Medium	ET_{max}	[1,656; 2,004]	[9,563; 19,518]	[10,264; 20,947]
		ET_{min}	[3,027; 3,662]	[7,905; 9,564]	[8,483; 10,264]
	Low	ET_{max}	[1,838; 3,749]	[8,766; 17,891]	[9,409; 19,202]
		ET_{min}	[1,518; 1,837]	[7,246; 8,767]	[7,777; 9,409]

Eq. (10b) could be converted into the following equivalent expressions (10c) based on Eqs. (6)–(8).

Objective 1: maximizing the satisfactory degree of the expected economic benefit

$$\max f_1^{\pm} = \frac{f_{\text{eco}}^{*} - 2f_{\text{eco}}^{\text{mv}} + p_h\left(\text{BT}_{ij}^{\pm} - \text{BS}_{k1} - \text{BP}_{k1}\right)\text{WS}_{ijh}^{\pm} + \left(\text{BL}_{ij}^{\pm} - \text{BS}_{k2} - \text{BP}_{k2}\right)\text{WG}_{ijh}^{\pm}}{2\left(f_{\text{eco}}^{*} - f_{\text{eco}}^{\text{mv}}\right)}$$

Objective 2: maximizing the satisfactory degree of the expected water shortages of crops

$$\max f_2^{\pm} = \frac{2f_{\text{soc}}^{\text{mv}} - f_{\text{soc}}' - p_h\left(\text{ET}_{\max,ijh}^{\pm} - \text{WS}_{ijh}^{\pm} - \text{WG}_{ijh}^{\pm} - P_{ij}^{\pm}\right)}{2\left(f_{\text{soc}}^{\text{mv}} - f_{\text{soc}}'\right)}$$

Objective 3: maximizing the satisfactory degree of the expected canal water loss

$$\max f_3^{\pm} = \frac{2f_{\text{res}}^{\text{mv}} - f_{\text{res}}' - p_h\left(p_{k1}\text{WS}_{ijh}^{\pm} + p_{k2}\text{WG}_{ijh}^{\pm}\right)}{2\left(f_{\text{res}}^{\text{mv}} - f_{\text{res}}'\right)} \tag{10c}$$

Constraints

- Surface water availability constraint

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\text{WS}_{ijh}^{\pm} \le \text{QS}_h^{\pm} \quad \forall h \tag{10d}$$

- Groundwater availability constraint

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\text{WG}_{ijh}^{\pm} \le \text{QG}_h^{\pm} \quad \forall h \tag{10e}$$

- Minimum water demand constraint

$$\text{WS}_{ijh}^{\pm} + \text{WG}_{ijh}^{\pm} + P_{ij}^{\pm} \ge \text{ET}_{\min,ijh}^{\pm} \quad \forall i,j,h \tag{10f}$$

- Maximum water demand constraint

$$\text{WS}_{ijh}^{\pm} + \text{WG}_{ijh}^{\pm} + P_{ij}^{\pm} \le \text{ET}_{\max,ijh}^{\pm} \quad \forall i,j,h \tag{10g}$$

- Fairness constraint

$$\frac{\sum_{i1}^{I}\sum_{i2}^{I}\left|\begin{array}{l}\left(\text{WS}_{i_1j}^{\pm} + \text{WG}_{i_1j}^{\pm} + P_{i_1j}^{\pm}\right) - \\ \left(\text{WS}_{i_2j}^{\pm} + \text{WG}_{i_2j}^{\pm} + P_{i_2j}^{\pm}\right)\end{array}\right|}{2n\sum_{i=1}^{I}\left(\text{WS}_{ij}^{\pm} + \text{WG}_{ij}^{\pm} + P_{ij}^{\pm}\right)} \le G_0 \quad \forall j,h \text{ and } i_1,i_2 \in I \tag{10h}$$

- Non-negative constraint

$$\text{WS}_{ijh}^{\pm} \ge 0 \quad \forall i,j,h$$

$$\text{WG}_{ijh}^{\pm} \ge 0 \quad \forall i,j,h \tag{10i}$$

where $f_1^{\pm}$, $f_2^{\pm}$, and $f_3^{\pm}$ denote satisfactory degrees of the economic benefit, water shortage, and canal water loss, respectively. They change from 0 to 1, and the higher value denotes a higher satisfactory degree. The $f_{1m}^{\pm}$, $f_{2m}^{\pm}$, and $f_{3m}^{\pm}$ are the expected economic benefit, water shortage, and canal water loss, respectively. The $\widetilde{f_{\text{eco}}} = \left[f_{\text{eco}}', f_{\text{eco}}^{\text{mv}}, f_{\text{eco}}^{*}\right]$, $\widetilde{f_{\text{soc}}} = \left[f_{\text{soc}}', f_{\text{soc}}^{\text{mv}}, f_{\text{soc}}^{*}\right]$, and $\widetilde{f_{\text{res}}} = \left[f_{\text{res}}', f_{\text{res}}^{\text{mv}}, f_{\text{res}}^{*}\right]$ are fuzzy goals of the expected economic benefit, water shortage, and canal water loss, individually. f_{eco}', f_{soc}', and f_{res}' are the minimum possible values of fuzzy goals of the economic benefit, water shortage, and canal water loss, individually; $f_{\text{eco}}^{\text{mv}}$, $f_{\text{soc}}^{\text{mv}}$, and $f_{\text{res}}^{\text{mv}}$ are the possible values of fuzzy goals of the economic benefit, water shortage, and canal water loss, separately; f_{eco}^{*}, f_{soc}^{*}, and f_{res}^{*} are the maximum possible values of fuzzy goals of the economic benefit, water shortage, and canal water loss, respectively; p_h is occurrence probability of surface flow level, and h represents the subscript of flow level. i declaims the subscripts of subareas, i = 1 denotes GZ, i = 2 presents LZ and i = 3 is GT. j delegates subscripts of crops, j = 1 is WC, j = 2 is CC and j = 3 is EC. $k1$ and $k2$ are subscripts of surface water and groundwater, separately. $\text{BT}_{ij}^{\pm}$ is the irrigation benefit coefficient (Yuan/m^3). BS and BP declaim irrigation water price and operation cost (Yuan/m^3), separately. p is the irrigation water-use efficiency. $\text{ET}_{ijh}^{\pm}$ denotes crop water demand (10^4 m^3). $P_{ij}^{\pm}$ is effective precipitation (10^4 m^3). $\text{WS}_{ijh}^{\pm}$ and $\text{WG}_{ijh}^{\pm}$ are gross water allocation of surface water and groundwater (10^4 m^3), decision variables; $\text{QS}_{h}^{\pm}$ and $\text{QG}_{h}^{\pm}$ denote available gross surface water and groundwater under different flow levels respectively, (10^4 m^3); G_0 is the GINI coefficient, which changes from 0 to 1, and the bigger value represents higher unfairness [30]. The 0.2 is used to promise the absolute fairness of water allocation for each crop among three subareas in this paper.

4. Results analysis

The results of optimal water allocation and satisfactory degrees of the expected economic benefit, water shortage, and canal water loss can be attained by solving the FDCIMOSP model. The result analysis includes two aspects: (1) optimal water allocation schemes analysis and (2) satisfactory degree analysis.

4.1. Optimal water allocation schemes

Fig. 4 shows the total water allocation of crops in three subareas. To evaluate the water shortage states of crops, the difference between the critical water demand and water allocation is compared, where the critical water demand is calculated by maximum water demand subtracting effective precipitation. If the difference is 0, indicating the maximum water demands of crops are satisfied, else a higher difference value means more serious water shortages of crops. Fig. 4 shows that the maximum water demands of wheat and economic crops are satisfied while the maximum water demand for corn is not met in three subareas under three flow levels. This is because the water resources are firstly allocated to crop with a higher irrigated benefit to reach the higher expected economic benefit and less water shortages of crops under the condition of limited water availability. It indicates that the maximum water demand for corn is not satisfied, and the difference between critical water demand and total water allocation increases with the reduction of flow levels, illustrating that the water shortage state of crop aggravates with the decline of flow levels. In other words, the water shortage of corn under high flow level is lowest, followed by medium flow level and low flow level. This is because the reduction range of available water resources is larger than the decline degree of critical water demands with variations of flow levels. It also discloses that the total water allocation of the crop in GZ is biggest, followed by LZ and GT, which is consistent with the water demands of crops in three subareas.

The surface water allocation and groundwater allocation of each crop present different variations for three subareas and three flow levels, and thus the differences in surface water allocation and groundwater allocation for each crop and each subarea are compared and analyzed, which is shown in Fig. 5. In three subareas, the surface water allocations of three crops decrease with the reduction of flow levels, while the groundwater presents different variations. Taking wheat as an example, the surface water allocations of wheat in GZ are [6,151.02; 7,266.87] × 10^4 m^3, [5,065.28; 5,895.58] × 10^4 m^3, and [1,107.62; 1,682.74] × 10^4 m^3 for high, medium, and low flow level, respectively, performing a decreasing tendency with variations of flow levels, and the groundwater allocation of wheat in GZ 0, [504.89; 725.90] × 10^4 m^3 and [3,881.70; 4,293.36] × 10^4 m^3 respectively, presenting an increasing tendency with the variations of flow levels. The groundwater allocation of wheat in LZ denotes first increasing and then decreasing tendency, while the groundwater allocation of wheat in GT expresses first decreasing and then increasing trend with the variations of flow levels. The above regulations illustrate that the groundwater is selected as supplementary sources to

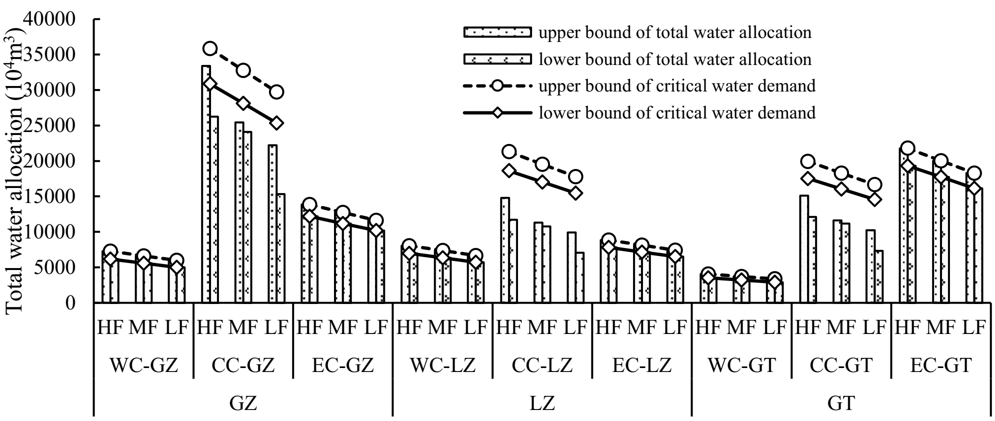

Fig. 4. Interval critical water demand and total water allocation.

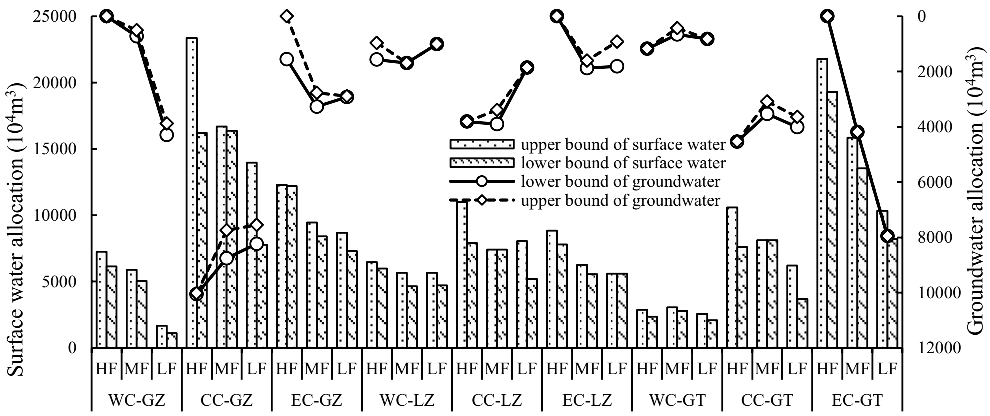

Fig. 5. Variation of surface water and groundwater allocation. *Note*: WC-GZ, CC-GZ, EC-GZ are wheat, corn, and economic crops in Ganzhou District, respectively; WC-LZ, CC-LZ, EC-LZ denote wheat, corn, and economic crops in Linze County separately; WC-GT, CC-GT, WC-GT refer to wheat, corn, and economic crops in Gaotai County, respectively. WS and WG are surface water allocation and groundwater allocation, individually. HF, MF, and LF are high, medium, and low flow levels, separately.

meet the respective maximum water demands of the wheat in three subareas. It is remarkable that the surface water allocation of the economic crop in GT is relatively high, which is because the total water demand of economic crops in GT is higher than other two subareas, and operation and price costs of surface water are lower than groundwater, and thus the more surface water resources are allocated to the economic crops in GT to satisfy its maximum water demand and achieve higher economic benefit. The groundwater allocation of corn in three subareas show different variation tendencies with the changes of flow levels, performing that the groundwater allocation of corn in GZ and LZ decreases with the reduction of flow levels, while that in GT presents first increasing and then decreasing trend. The differences in groundwater allocation amid three flow levels are caused by the various unbalanced relationship between water demand and water supply, and the reduction of available groundwater with the variation of low levels. The differences in groundwater allocation among three subareas are caused by the irrigation benefit, and the water resources are more allocated to crops of subareas with higher irrigation benefits. For example, the irrigation benefit of corn is [3.27; 3.50] × Yuan/m^3, [2.23; 2.46] × Yuan/m^3, and [2.14; 2.36] × Yuan/m^3 for GZ, LZ and GT, respectively. Thus the groundwater allocation of the corn in GZ is the biggest, followed by LZ and GT.

4.2. Optimal satisfactory degrees objective

The 0.5 is set as the minimum satisfactory degree of the expected economic benefit, water shortage, and canal water loss, and the 1.0 is selected as the maximum satisfactory degree of the expected economic benefit, water shortage, and canal water loss. If the satisfactory degree is between the minimum satisfactory degree and maximum satisfactory degree, which means the managers are satisfied with the current expected economic benefit, water shortage, and canal water loss, and the corresponding optimal water allocation schemes can be adopted by the managers. Because the bigger economic benefit, lower water shortage of crops, and smaller canal water loss are desired by managers, and thus bigger economic benefit, lower water shortage of crops, and smaller canal water loss will result in the bigger satisfactory degrees of the economic benefit, water shortage, and canal water loss. The above regulations are reflected in the optimal results. For example, the obtained interval economic benefit, water shortage, and canal water loss are [111.9; 140.7] × 10^8 Yuan, [18,208.8; 19,973.8] × 10^4 m^3, and [43,180; 50,170] × 10^4 m^3, respectively. And the corresponding relationships between expected values and satisfactory degree of the expected values are as follows: the maximum value of the economic benefit of 140.7 × 10^8 Yuan corresponds to its maximum satisfactory degree of 1.0, and the minimum value of water shortage of 18,208.8 × 10^4 m^3 corresponds to its maximum satisfactory degree of 1.0, and the minimum value of canal water loss of 43,180 × 10^4 m^3 corresponds to its maximum satisfactory degree of 0.61. Conversely, a higher satisfactory degree of the economic benefit means the bigger economic benefit, and a higher satisfactory degree of the water shortage indicates the lower water shortages of crops, and a higher satisfactory degree of the canal water loss donates the lower canal water loss. Compared with expected values, the satisfactory degrees of expected values can not only quantify the expected values but also measure the relationship between expected values and fuzzy goals of the expected values. Therefore, optimal satisfactory degrees of expected values are analyzed in this section. The results show that the satisfactory degrees of economic benefit, water shortage, and canal water loss are [0.81; 1], [0.98; 1], and [0.50; 0.61], respectively, between the minimum satisfactory degree and maximum satisfactory degree, and thus the expected values and corresponding optimal water allocation schemes can be adopted by managers. Moreover, it illustrates that satisfactory degrees of economic benefit and water shortage are contradictory with the satisfactory degree of canal water loss, which is caused by the relationship between water allocation and expected values. In general, the expected economic benefit increases, and the water shortage decreases, and the canal water loss enlarges with the increasing water allocation, and thus the satisfactory degrees of economic benefit and water shortage increase, the satisfactory degree of canal water loss decreases with the rise of water allocation. Therefore, if the managers require to reduce the canal water loss, fewer water resources should be allocated; if the managers want to achieve higher economic benefits and alleviate the water shortage of crops, more water resources should be allocated. And the decision-makers can select the corresponding water allocation schemes based on their emphasis and preferences.

5. Discussion

5.1. Comparison with single objective programming

The FDCIMOSP model is compared to three single objective programming models to verify its robustness. The first model is the economy-oriented fuzzy dependent-chance programming (EFDCP) model. The second is the society-oriented fuzzy dependent-chance programming (SFDCP) model. The third model is the resource-oriented fuzzy dependent-chance programming (RFDCP) model. The results are shown in Fig. 6.

Figs. 6a–e are the results of satisfactory degrees of the expected economic benefit, water shortage, and canal water loss of four models, and the economic benefit, water shortage, and canal water loss, and water allocation of four models. Fig. 6a indicates that the satisfactory degrees of the economic benefit, water shortage, and canal water loss derived from the single objective programming model can reach its respective maximum satisfactory degree, meanwhile, the satisfactory degrees of the economic benefit, water shortage and canal water loss obtained from the FDCIMOSP model are between the minimum satisfactory

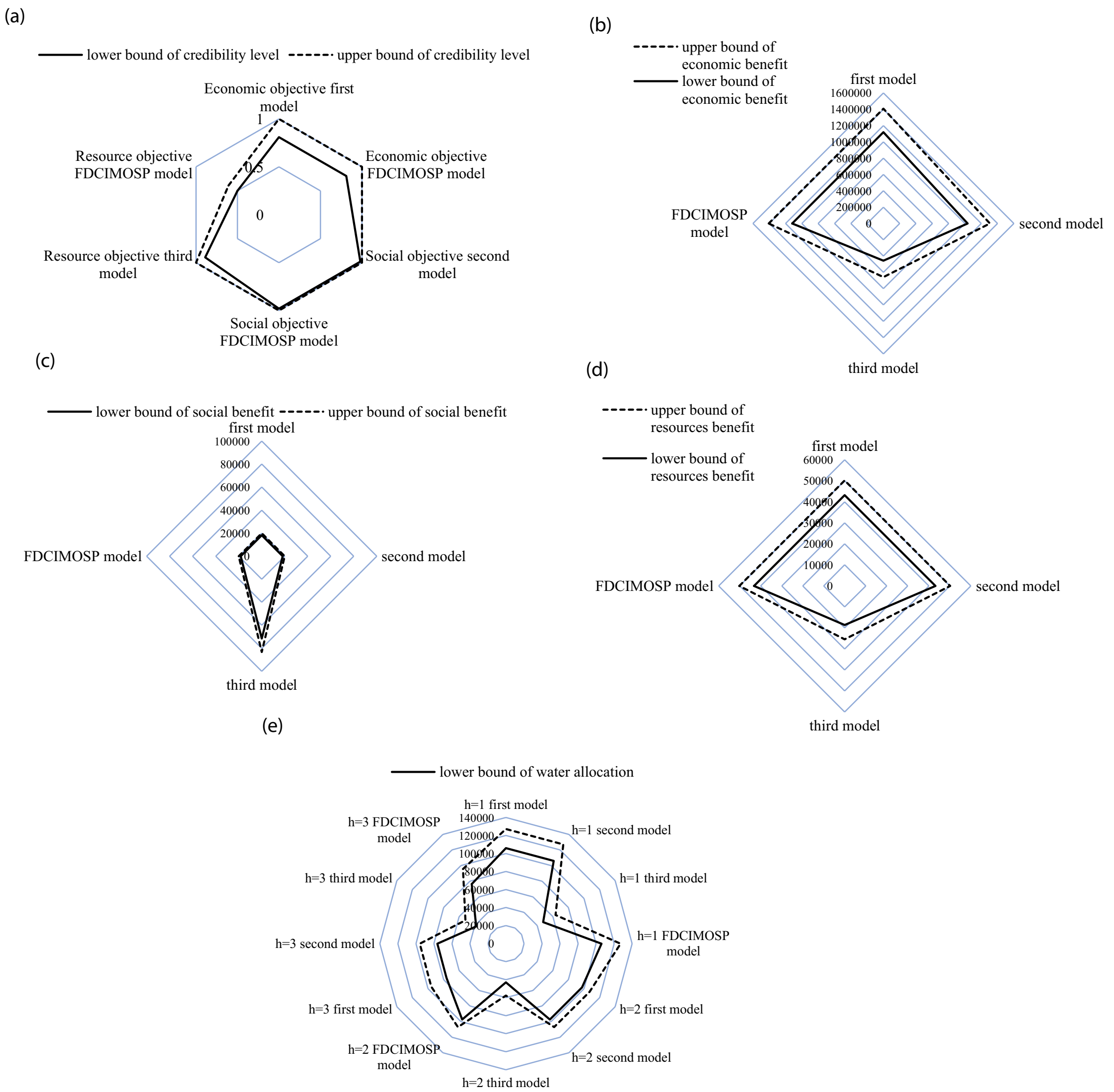

Fig. 6. Comparison amid FDIMOSEP model and three models with a single objective. (a) A satisfactory degree of economic, social, and resources objectives under four alternative models, (b) economic objective of four alternative models, (c) social objective of four alternative models, (d) resources objective of four alternative models, and (e) total water allocation of four alternative models under different water flow levels.

degree and maximum satisfactory degree. This is because the single objective programming model only optimizes the satisfactory degree of the single expected value not taking the satisfactory degrees of other associated expected values into account, but the FDCIMOP model optimizes and tradeoffs the multiple satisfactory degrees of the economic benefit, water shortage, and canal water loss simultaneously. Fig. 6b shows the upper bound of the economic benefit obtained from the FDCIMOSP model is the same as the EFDCP model, disclosing that the FDCIMOSP model can cover the economic benefit obtained from the EFDCP model. Fig. 6c shows that the FDCIMOSP model has the same interval water shortage ([18,208.8; 19,773.8] × 10^4 m^3) with the SFDCP model, indicating that the FDCIMOSP model can represent the SFDCP model to optimize irrigation water resources. Fig. 6d shows that the FDCIMOSP model has higher canal water loss than the RFDCP model, but at the same, it has the higher economic benefit and lower water shortage, and thus it can trade-off the relationship amid the economic benefit, water shortage, and canal water loss, and overcome the disadvantages of the RFDCP model of neglecting the economic benefit and water shortage. Fig. 6e indicates that the total water allocation among the FDCIMOSP model, SFDCP model, and EFDCP model are the same while the total water allocation of the RFDCP model is lowest. In summary, the FDCIMOSP model can get the compromised water allocation schemes with consideration of satisfactory degrees of economic benefit, water shortage, and canal water loss, and thus has higher robustness.

5.2. Comparison with two-objectives programming

The FDCIMOSP model is compared to three two-objective programming models. The objective functions of the three alternative models are structured by arbitrary two objective functions of the FDCIMOSP model. The objective functions of the first model are maximizing satisfactory degrees of the expected economic benefit and water shortage. The objective functions of the second model are maximizing the satisfactory degrees of the expected economic benefit and canal water loss. And the third objective functions are maximizing the satisfactory degrees of the expected water shortage and canal water loss. The comparisons among four alternative models about satisfactory degrees of the expected economic benefit, water shortage, and canal water loss, and water allocation are shown in Fig. 7.

It shows that the interval satisfactory degrees of the expected benefit attained from the FDCIMOSP model is larger than other models, indicating the FDCIMOSP model can effectively express the satisfactory degree of the expected economic benefit and has more robustness compared to the other three models. As for the satisfactory degree of water shortage, the FDCIMOSP model, first model, and third model have the same upper bound, but the lower bound of the FDCIMOSP model is lower than the other two models, disclosing that the FDCIMOSP model can cover wider ranges of the satisfactory degree of water shortage, and thus has higher robustness for representing the satisfactory degree of water shortage. With regard to the satisfactory degree of canal water loss, the third model has the biggest value, but the corresponding satisfactory degrees of economic benefit and water shortage are lowest, and thus it is not beneficial to the development of economic benefit and alleviating the water shortage of crops. Relatively, the satisfactory degree of canal water loss obtained from the FDCIMOSP model can give consideration to the satisfactory degrees of the economic benefit, water shortage, and canal water loss. The water allocation of the FDCIMOSP model is the same as the first model and the third model, and higher than the second model. This is because that the satisfactory degrees of economic benefit and water shortage are positive with water allocation while the satisfactory degree of canal water loss is negative with water allocation. It also indicates that the water allocation schemes from the FDCIMOSP model can give consideration to the satisfactory degrees of the expected economic benefit, water shortage, and canal water loss.

Consequently, the FDCIMOSP model can get the optimal irrigation water allocation schemes taking the

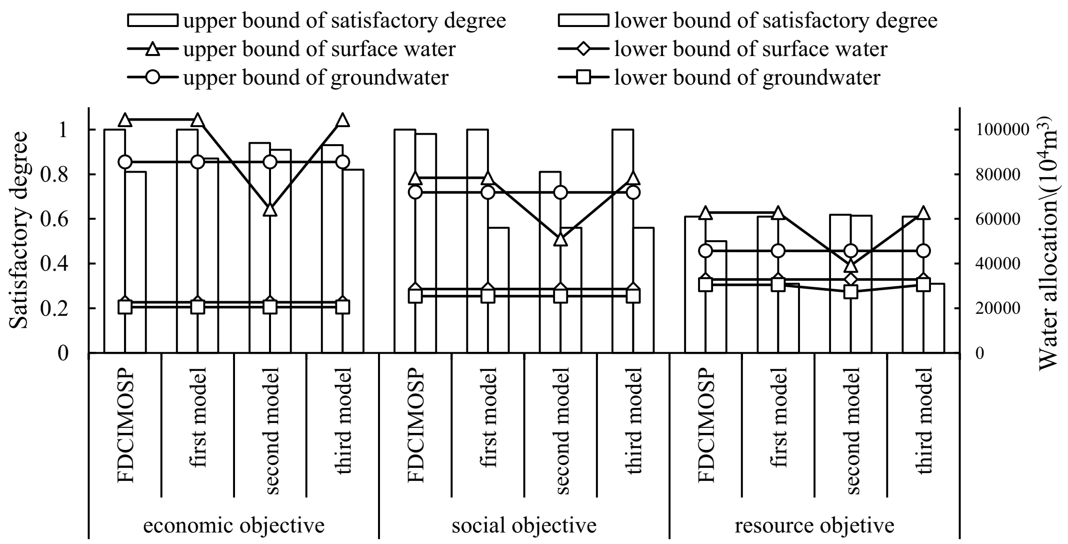

Fig. 7. The difference in water allocation and satisfactory degree among the four models.

satisfactory degrees of the economic benefit, water shortage, and canal water loss into account, and provides the different water allocation schemes for decision-makers. Besides, the FDCIMOSP model can overcome the disadvantages of paying more attention to the satisfactory degree of single expected value (e.g., economic benefit, water shortage, canal water loss) existed in the single objective programming, and excessive focus on the satisfactory degree of canal water loss existed in two objectives programming approach. Therefore, the FDCIMOSP model has more robustness compared with other alternative models. Based on results calculated by the FDCIMOSP model, a number of findings can be disclosed: the satisfactory degree of the expected economic benefit has a positive relationship with the expected economic benefit, while the satisfactory degrees of the expected water shortage and canal water loss have negative relationships with the expected water shortage and canal water loss, respectively; Secondly, the satisfactory degrees of the expected economic benefit and water shortage are positive with water allocation, while the satisfactory degree of the expected canal water loss is opposite with water allocation. Thirdly, the water resources are more allocated to the crop with higher irrigation benefits.

The optimal water allocation schemes obtained from the FDCIMOSP model can provide different water allocation schemes for decision-makers. And the decision-makers can select the corresponding water allocation schemes based on their focus and preferences.

6. Conclusion

This article proposed a fuzzy dependent-chance interval multi-objective stochastic expected programming (FDCIMOSP) model for irrigation water resources management under uncertainties. The model integrates FDCP, stochastic expected value programming, interval programming within the multi-objectives programming. The satisfactory degrees of economic benefit, water shortage, and canal water loss, and surface water allocation and groundwater allocation is optimized. Besides, the GINI coefficient is used to promise fair water allocations of crops among three subareas. The model has the following advantages:

- It can deal with the relationship between economic benefit, water shortage, and canal water loss and respective fuzzy goals based on the credibility degree, and trade-off relationships amid multiple satisfactory degrees.
- It can address the uncertainties expressed as interval numbers, fuzzy numbers, and stochastic variables.
- It can promise the fairness of water allocation, and achieve the interactions amid fairness of water allocation, and satisfactory degree.

The developed model was applied to a real case study, and the results showed that it is applicable for irrigation water resources in the arid-semi-arid district, and it can provide multiple groups of optimal water resource schemes of crops for decision-makers. However, the more uncertain parameters caused by the spatial difference in soil types, topography should be considered to make the water allocation schemes more robustness, which will be studied further.

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