Desalination and Water Treatment
www.deswater.com

doi: 10.5004/dwt.2023.29437

293 (2023) 100–106
May

Derivation of formulas for sediment carrying capacity and erosion and deposition strength applicable to channel siltation calculation

Xiaotian Dong[a,b], Zigui Jin[a], Wenjin Zhu[a,*], Feigang Liang[c]

[a]*Jiangsu Ocean University, Lianyungang 222005, China, email: dongxt@jou.edu.cn*
[b]*Observation and Research Station of East China Coastal Zone, Ministry of Natural Resources, Nanjing 210005, China*
[c]*Institute of Geochemical Exploration and Marine Geological Survey, East China Mineral Exploration and Development Bureau for Non-Ferrous Metals, Nanjing 210007, China*

Received 2 June 2022; Accepted 18 September 2022

ABSTRACT

Sediment transport on silty and muddy coast had been a research hotspot because of its uniqueness. Based on the cross-section characteristics of channel, and in view of the problem that most of the existing formulas are semi-empirical and semi-theoretical, with assumption and simplification of the motion of sediment-laden flow, this paper deduces the formulas for calculating the flow carrying force and seabed scouring and silting strength from the perspective of hydrodynamic equilibrium of sediment-laden flow.

Keywords: Sediment carrying capacity; Seabed erosion and deposition; Sediment-laden flow; Silty mud coast

1. Introduction

An important problem in waterway engineering construction is silt siltation. The shoal of silty mud coast has a gentle slope, and generally has a deep silt layer. Under the great excavation depth of channel, the channel slope has poor stability. For deepwater channel dredging in shoal, there is abundant sediment source on both sides of the channel. The shear strength of the channel slope is slightly lower because of mechanical damage on the seabed surface layer structure caused by dredging, which constitutes the main factor of silt siltation in deepwater channel of silty mud coast. Silt siltation in open channel is more complicated: smaller channel cover area, stronger wave power, worse extreme sea conditions, which usually trigger sediment deposition during long-term or sudden siltation, occurring in Tianjin Port, Huanghua Port, Jingtang Port, Lianyungang Port, Hangzhou Bay and Changjiang Estuary channel.

The research methods of sediment deposition mainly include the analysis of measured data, empirical and semi-empirical formula calculation, numerical simulation, physical model test and so on. Among them, the empirical and semi-empirical sediment calculation formula shows good applicability and ease of use in the application [1–3]. Many researchers have put forward semi-theoretical and semi-empirical formulas for channel silt siltation calculation and seabed erosion and deposition in silty mud coast, such as Liu Jiaju formula, Cao Zude formula, Yu Yi-jin formula, Le Pei-Jiu formula, Luo Zhao-sen formula and Li Wang-sheng formula, etc. Among them, Liu Jiaju's formula are widely applicable. Based on approach channel under equilibrium between scouring and deposition, considering the angle between channel trend and flow direction, assuming suspended sediment falls evenly from water body to the channel, Liu and Yu [3] established the widely used formula for channel silt siltation strength calculating, which became recommended formula on "Code of Hydrology for Harbour and Waterway".

* Corresponding author.

2. Derivation of dynamic balance equation of sediment-laden flow

It is assumed that the x-axis and the flow velocity of sediment-laden flow V are parallel to the interface of seawater and sediment-laden flow, the flow velocity of seawater is U_1^r, and the positive direction is consistent with the direction of sediment-laden flow, so a motion model of sediment-laden flow is constructed, as shown in Fig. 1.

Considering the sediment-laden flow with a width, Δx length and J_0 slope as the research object, the force on the flow direction of this section of sediment-laden flow includes the following parts:

Pressure on sediment-laden flow:

$$P_1 = a\int_0^h \left(\rho_0 g\xi + \int_z^h \rho_m g \mathrm{d}z\right)dz \tag{1}$$

$$P_2 = a\int_0^{h+\frac{\partial h}{\partial x}\Delta x}\left[\rho_0 g\left(\frac{1}{\Delta x}\int_{x_0}^{x_0+\Delta x}\xi dx\right) + \int_z^{h+\frac{\partial h}{\partial x}\Delta x}\rho_m g dz\right]dz \tag{2}$$

$$P_3 \sin\left(\beta_2-\beta_1\right) = \begin{bmatrix} \frac{1}{\Delta x}\int_{x_0}^{x_0+\Delta x}\rho_0 g\xi dx \\ +\frac{1}{\Delta x}\int_{x_0}^{x_0+\Delta x}\left(\frac{1}{h}\int_0^h \rho_m dz\right)ghdx \end{bmatrix} \times\frac{a\Delta x}{\cos\left(\beta_2-\beta_1\right)}\sin\left(\beta_2-\beta_1\right) \tag{3}$$

where ξ is the depth of seawater; h is sediment-laden flow depth; ρ_m, ρ_s and ρ_0 are the bulk density of sediment-laden flow, sediment and seawater, respectively.

The gravity of the research object is:

$$G\sin\beta_2 = a\frac{1}{h+\frac{\partial h}{\partial x}\Delta x}\int_0^{h+\frac{\partial h}{\partial x}\Delta x}\left[\int_{x_0}^{x_0+\Delta x}\rho_m ghdx\right]dz\sin\beta_2 \tag{4}$$

The total resistance of the research object T includes bed resistance T_b, seawater resistance at the interface T_1 and side resistance T_c.

$$T = T_b\cos\left(\beta_2-\beta_1\right) + T_1 + T_c = \frac{f_0'}{8}\rho_{\mathrm{ma}}V^2 a\Delta x + \frac{f_1'}{8}\rho_{m0}V^2 a\Delta x + 2\frac{f_c}{8}V^2\int_{x_0}^{x_0+\Delta x}hdx\frac{1}{h}\int_0^h \rho_m dz \tag{5}$$

where f_0' and f_1' are bed resistance coefficient, resistance coefficient of sediment-laden flow and seawater interface, respectively. f_c is side resistance coefficient, ρ_{ma} is sediment-laden flow density at reference height, and ρ_{m0} is sediment-laden flow density at sediment-laden flow and seawater interface.

The inertia force of sediment-laden flow due to acceleration or deceleration is:

$$I = \frac{1}{\Delta x}\int_{x_0}^{x_0+\Delta x}\int_0^h \rho_m dzdx\Delta xa\frac{dV}{dt} \tag{6}$$

The density of sediment-laden flow is $\rho_m = \rho_0 + \frac{\rho_s-\rho_0}{\rho_s}S$. Because the integration along water depth of suspended sediment distribution is very complex, in order to simplify the derivation process and reveal the movement law of sediment-laden flow model, the vertical variation of sediment-laden flow density is simplified as follows:

$$k_1 = \frac{\int_0^h\left(\int_z^h \rho_m dz\right)dz}{\rho_m\frac{h^2}{2}} = \frac{\int_0^{h+\frac{\partial h}{\partial x}\Delta x}\left(\int_z^{h+\frac{\partial h}{\partial x}\Delta x}\rho_m dz\right)dz}{\rho_m\frac{\left(h+\frac{\partial h}{\partial x}\Delta x\right)^2}{2}} \tag{7}$$

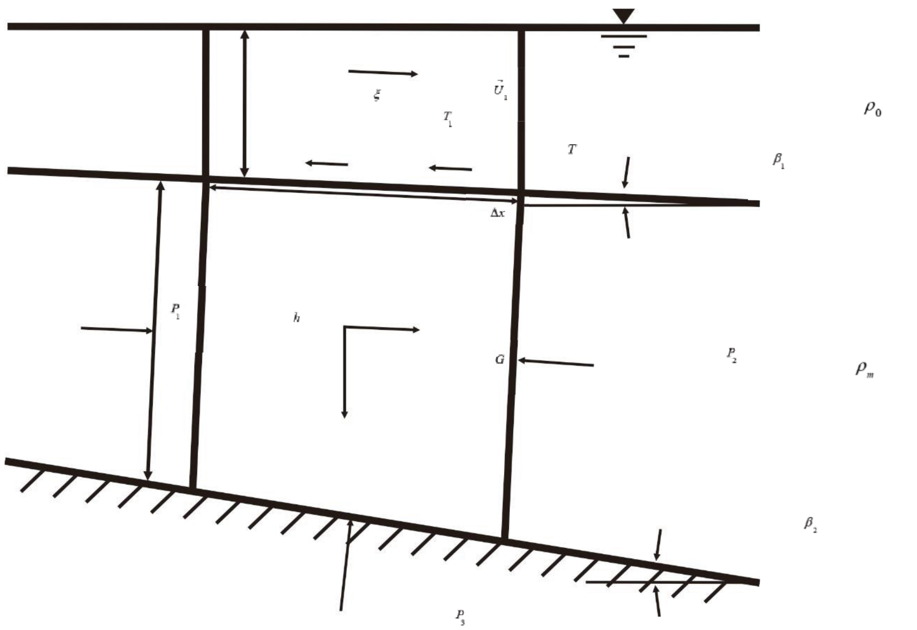

Fig. 1. Force analysis schematic diagram of sediment-laden flow.

$$k_1' = \frac{\int_0^h \rho_m dz}{h\rho_m} = \frac{\int_0^{h+\frac{\partial h}{\partial x}\Delta x} \rho_m dz}{\left(h + \frac{\partial h}{\partial x}\Delta x\right)\rho_m} \tag{8}$$

$$k_2 = \frac{\rho_{\mathrm{ma}}}{\rho_m} \tag{9}$$

$$k_2' = \frac{\rho_{m0}}{\rho_m} \tag{10}$$

where k_1, k_1', k_2 and k_2' are coefficients to correct the uneven distribution of sediment-laden flow density along the water depth.

Then Eqs. (1)–(6) can be reduced to:

$$P_1 = \left(\rho_0 g \xi h + k_1 \rho_m g \frac{h^2}{2}\right) a \tag{11}$$

$$\begin{aligned} P_2 &= a\left[\rho_0 g\left(\xi + \frac{\partial \xi}{\partial x}\Delta x\right)\left(h + \frac{\partial h}{\partial x}\Delta x\right) + k_1 \frac{\rho_m g}{2}\left(h + \frac{\partial h}{\partial x}\Delta x\right)^2\right] \\ &= \left[\rho_0 g \xi h + k_1 \rho_m g \frac{h^2}{2} + \left(\rho_0 g \xi + k_1 \rho_m g h\right)\frac{\partial h}{\partial x}\Delta x + \rho_0 g h \frac{\partial \xi}{\partial x}\Delta x\right] a \end{aligned} \tag{12}$$

$$\begin{aligned} P_3 \sin\left(\beta_2 - \beta_1\right) &= \left[\rho_0 g\left(\xi + \frac{\partial \xi}{\partial x}\frac{\Delta x}{2}\right) + k_1' \rho_m g\left(h + \frac{\partial h}{\partial x}\frac{\Delta x}{2}\right)\right] \\ &\times \frac{a\Delta x}{\cos\left(\beta_2 - \beta_1\right)} \sin\left(\beta_2 - \beta_1\right) = \left(\rho_0 g \xi + k_1' \rho_m g h\right)\frac{\partial h}{\partial x} a\Delta x \end{aligned} \tag{13}$$

$$G \sin\beta_1 = k_1' \rho_m g\left(h + \frac{\partial h}{\partial x}\frac{\Delta x}{2}\right)\Delta x \sin\beta_1 a = k_1' \rho_m g h \frac{\partial \xi}{\partial x}\Delta x a \tag{14}$$

$$\begin{aligned} T &= T_b \cos\left(\beta_2 - \beta_1\right) + T_1 + T_c = k_2 \frac{f_0'}{8}\rho_m V^2 a\Delta x \\ &+ k_2' \frac{f_1'}{8}\rho_m V^2 a\Delta x + 2k_1' \frac{f_c}{8}\rho_m V^2 h\Delta x \end{aligned} \tag{15}$$

$$I = k_1' \rho_m\left(h + \frac{\partial h}{\partial x}\frac{\Delta x}{2}\right)\Delta x \frac{\mathrm{d}V}{\mathrm{d}t} a = k_1' \rho_m h\Delta x a\left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}\right) \tag{16}$$

In addition, the additional stress T' caused by the reverse flow of the upper layer of sediment-laden flow at velocity V_1 can be written as:

$$T' = \tau' \Delta x a \tag{17}$$

where τ' is additional stress.

Therefore, the equilibrium equation of force on sediment-laden flow is:

$$P_1 - P_2 + P_3 \sin\left(\beta_2 - \beta_1\right) + G\sin\beta_1 - T - T' = I \tag{18}$$

By substituting Eqs. (11)~(17) into Eq. (18) and assuming $k_1 = k_1'$, the force balance equation is simplified as:

$$\begin{aligned} &\left(k_1\rho_m - \rho_0\right) g h \frac{\partial \xi}{\partial x} - \frac{\left(k_2 f_0' + k_2' f_1'\right)}{8}\rho_m V^2 - \frac{h}{a} k_1 \frac{f_c}{4}\rho_m V^2 - \tau' \\ &= k_1 \rho_m h\left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}\right) \end{aligned} \tag{19}$$

Because of the following relationship between bed elevation and sediment-laden flow depth.

$$\frac{\partial \xi}{\partial x} = -\frac{\partial\left(Z_0 + h\right)}{\partial x} = J_0 - \frac{\partial h}{\partial x} \tag{20}$$

By substituting Eq. (20) into Eq. (19) and making the comprehensive resistance coefficient $f = k_2 f_0' + k_2' f_1'$, Eq. (19) can be simplified as:

$$\begin{aligned} &J_0 - \frac{\partial h}{\partial x} - \frac{f}{8}\frac{V^2}{\frac{k_1\rho_m - \rho_0}{\rho_m} g h} - \frac{f_c}{4a}\frac{V^2}{\frac{k_1\rho_m - \rho_0}{k_1\rho_m} g} \\ &- \frac{\tau'}{h\left(k_1\rho_m - \rho_0\right) g} = \frac{1}{\frac{k_1\rho_m - \rho_0}{k_1\rho_m} g}\left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}\right) \end{aligned} \tag{21}$$

Eq. (21) is the dynamic balance equation of sediment-laden flow. In the next section, the sediment carrying capacity formula will be derived from the dynamic balance equation of sediment-laden flow under the condition of stable and uniform state.

3. Derivation of sediment carrying capacity formula

According to the derived dynamic balance equation of sediment-laden flow, if it is extended to the whole depth range, in Eq. (21), h is water depth and V is vertical average velocity of sediment-laden flow.

Under the condition of stable and uniform sediment-laden flow, Eq. (21) is transformed into:

$$J_0 - \frac{f}{8}\frac{V^2}{\frac{k_1\rho_m - \rho_0}{\rho_m} g h} - \frac{f_c}{4a}\frac{V^2}{\frac{k_1\rho_m - \rho_0}{k_1\rho_m} g} - \frac{\tau'}{h\left(k_1\rho_m - \rho_0\right) g} = 0 \tag{22}$$

Multiply both sides of the equation by $\frac{k_1\rho_m - \rho_0}{\rho_m}$, Eq. (22) is reduced to:

$$\frac{k_1\rho_m - \rho_0}{\rho_m} J_0 - \frac{f}{8}\frac{V^2}{gh} - k_1 \frac{f_c}{4a}\frac{V^2}{g} - \frac{\tau'}{\rho_m g h} = 0 \tag{23}$$

- Suppose $k_1 = 1$, sediment-laden flow density is $\rho_m = \rho_0 + \frac{\rho_s - \rho_0}{\rho_s} S$, ρ_0 is seawater density, ρ_s is sediment density, S is sediment concentration (kg/m^3), then Eq. (23) is transformed into:

$$J_0\left(\frac{\rho_m - \rho_0}{\rho_m}\right) = \frac{f}{8}\frac{V^2}{gh} + \frac{f_c}{4a}\frac{V^2}{g} + \frac{\tau'}{\rho_m g h} \tag{24}$$

Let $\chi_1 = \frac{f}{8}\frac{V^2}{gh} + \frac{f_c}{4a}\frac{V^2}{g} + \frac{\tau'}{\rho_m g h}$, then Eq. (24) can be written as:

$$J_0\left(\frac{\left(\rho_s - \rho_0\right) S}{\left(\rho_s - \rho_0\right) S + \rho_0 \rho_s}\right) = \chi_1 \tag{25}$$

Reduction to get:

$$S=\frac{\rho_0\rho_s\chi_1}{(\rho_s-\rho_0)(J_0-\chi_1)} \tag{26}$$

Eq. (26) is the sand carrying capacity formula applicable to rivers and nearshore sea area. When the sediment-laden flow is extended to the full depth, the resistance at the interface between sediment-laden flow and seawater is $\frac{f_1'}{8}\frac{V^2}{gh}=0$. In the nearshore sea area, if the side boundary is nearly infinite, the side resistance is $\frac{f_c}{4a}\frac{V^2}{g}=0$, there it is $\chi_1=\frac{k_2 f_0'}{8}\frac{V^2}{gh}$, Eq. (26) can be simplified as follows:

$$S=k_2C_D\frac{\rho_0\rho_s}{\rho_s-\rho_0}\frac{1}{(J_0-\chi_1)}\frac{V^2}{gh} \tag{27}$$

where the resistance coefficient is $C_D=\frac{f_0'}{8}$; k_2 is the coefficient to correct the non-uniform bulk density of sediment-laden flow along water depth. When it is assumed that the density of sediment-laden flow is uniform along water depth, there is $k_2 = 1$. Eq. (27) is sediment carrying capacity formula that suited for offshore waters.

It can be seen from Eq. (27) that when $\chi_1 = o(J_0)$, Eq. (27) can be written in a more concise form:

$$S=k_2\frac{C_D}{J_0}\frac{\rho_0\rho_s}{\rho_s-\rho_0}\frac{V^2}{gh} \tag{28}$$

Eq. (28) is consistent with the general formula of sediment carrying capacity [4].

- If $\chi_2=\frac{f}{8}\frac{V^2}{gh}+\frac{f_c}{4a}\frac{V^2}{g}$, then Eq. (23) can be written as:

$$J_0\left(\frac{(\rho_s-\rho_0)S}{(\rho_s-\rho_0)S+\rho_0\rho_s}\right)-\frac{\rho_s}{(\rho_s-\rho_0)S+\rho_0\rho_s}\frac{\tau'}{gh}=\chi_2 \tag{29}$$

Reduction to get:

$$S=\frac{\rho_0\rho_s\chi_2+\rho_s\frac{\tau'}{gh}}{(\rho_s-\rho_0)(J_0-\chi_2)} \tag{30}$$

where τ is the surface wind stress.

Eq. (30) is the sediment carrying capacity formula which can be applied to offshore waters considering the effect of surface wind stress. Similarly, if the sediment-laden flow is extended to the full depth, the resistance of the interface between sediment-laden flow and seawater is $\frac{f_1'}{8}\frac{V^2}{gh}=0$; in nearshore sea area, the side resistance is $\frac{f_c}{4a}\frac{V^2}{g}=0$ if the side boundary is nearly infinite, so $\chi_2=\frac{k_2 f_0'}{8}\frac{V^2}{gh}$, Eq. (26) can be simplified as:

$$S=k_2C_D\frac{\rho_0\rho_s}{\rho_s-\rho_0}\frac{1}{(J_0-\chi_2)}\frac{V^2}{gh}+\frac{\rho_s}{\rho_s-\rho_0}\frac{1}{(J_0-\chi_2)}\frac{\tau'}{gh} \tag{31}$$

In Eq. (31), the resistance coefficient is $C_D=\frac{f_0'}{8}$. Eq. (31) is sediment carrying capacity formula considering the effect of surface wind stress.

- In general, $k_1\neq 1$, it corresponds to the second case:

Let $\chi_3=\frac{1}{k_1}\left(\frac{f}{8}\frac{V^2}{gh}+k_1\frac{f_c}{4a}\frac{V^2}{g}\right)$, then Eq. (23) can be written as:

$$J_0\frac{k_1\left[(\rho_s-\rho_0)S+\rho_0\rho_s\right]-\rho_0\rho_s}{(\rho_s-\rho_0)S+\rho_0\rho_s}-\frac{\rho_s}{(\rho_s-\rho_0)S+\rho_0\rho_s}\frac{\tau'}{gh}=k_1\chi_3 \tag{32}$$

Reduction to get:

$$S=\frac{k_1\rho_0\rho_s\chi_3+\rho_s\frac{\tau'}{gh}-J_0\rho_0\rho_s(k_1-1)}{k_1(\rho_s-\rho_0)(J_0-\chi_3)} \tag{33}$$

Eq. (33) is sediment carrying capacity formula applicable to rivers and coastal waters, which considering the effect of surface wind stress. Similarly, if the sediment-laden flow is extended to all water depths, the resistance at the interface between sediment-laden flow and seawater is $\frac{f_1'}{8}\frac{V^2}{gh}=0$; in coastal waters, if the side boundary is nearly infinite, then the side resistance is $\frac{f_c}{4a}\frac{V^2}{g}=0$. Therefore $\chi_3=\frac{k_2 f_0'}{8k_1}\frac{V^2}{gh}$, Eq. (32) can be simplified as:

$$\begin{aligned}S=&\frac{k_2}{k_1}C_D\frac{\rho_0\rho_s}{\rho_s-\rho_0}\frac{1}{(J_0-\chi_3)}\frac{V^2}{gh}+\frac{1}{k_1}\frac{\rho_s}{\rho_s-\rho_0}\frac{1}{(J_0-\chi_3)}\frac{\tau'}{gh}\\&-\frac{(k_1-1)}{k_1}\frac{\rho_0\rho_s}{\rho_s-\rho_0}\frac{1}{(J_0-\chi_3)}J_0\end{aligned} \tag{34}$$

In Eq. (34), the resistance coefficient is $C_D=\frac{f_0'}{8}$. Eq. (34) is sediment carrying capacity formula suited in coastal waters, with the effect of surface wind stress is considered.

4. Key parameters

According to the three-sediment carrying capacity Eqs. (27), (31) and (34) which are applicable to the nearshore sea area, it can be seen that the relevant undetermined parameters of the three formulas are the flow resistance coefficient C_D and the slope J_0.

4.1. Slope

Generally, the channel section is designed as inverted trapezoid, and the channel slope is determined according to the soil properties and hydrodynamic conditions. The dredged slope will be adjusted to be a stable slope within a certain period of time. The stable slope is related to the thickness of dredged mud layer, dredging depth, wave, current, geological topography and time. In silty mud coast, the channel slope is generally between 1:3 and 1:50.

The energy dissipation rate of the system is the minimum under the condition of water flow and bed surface. Based on the theory and the measured data, Yang [5] deduced that:

$$\frac{VJ_0}{\omega_s} = \text{const} \tag{35}$$

where Φ is the flow rate.

Then, in the equilibrium state, the above Equation can be written as $J_0 = K\frac{\omega_s}{V} = K\frac{\upsilon}{Vh}\frac{\omega_s d}{\upsilon}\frac{h}{d} = K\frac{1}{d/h}\frac{\text{Re}_d}{\text{Re}}$, indicating that the energy slope is related to the relative roughness, the Reynolds number of sand particles and the Reynolds number of water flow.

According to minimum energy dissipation rate theory [6], Eq. (35) can be substituted into Eq. (36):

$$S = k_3 C_D \frac{\rho_0 \rho_s}{\rho_s - \rho_0}\frac{V^3}{\omega_s g h} \tag{36}$$

where k_3 is the coefficient.

Eq. (36) is consistent with Li Ruijie's general form of sediment carrying capacity formula [14] which also confirms the rationality of the derived sediment carrying capacity formula.

Through numerical flume simulation test, Chang and Xu [7] have concluded that under different inflow and sediment conditions, the flow and sediment conditions and riverbed boundary conditions before and after riverbed adjustment change, and the corresponding minimum energy dissipation rate is also different.

4.2. Drag coefficient

The resistance coefficient C_D is related to the turbulence intensity, relative roughness of bed surface and pressure density, especially with Reynolds number. It reflects the resistance of boundary to flow, which is an important part of the turbulent structure of open channel and pipeline, and also the foundation of building the mathematical model of tidal current sediment. Generally, C_D is difficult to be obtained by theoretical calculation, and it is determined by experiments.

Many scholars have obtained the calculation formula of resistance coefficient of pipeline and open channel from theory or experiment. Poisson, Posilex, Schiller and [8], Prandtl, think that it is only related to Reynolds number; Carmen and Soulsby [9] think it is related to bed roughness length. Dou et al. [10] analyzed the random structure of turbulent flow, combined with the near wall flow structure, theoretically derived the unified calculation formula of resistance coefficient of open channel flow in turbulent rough area, smooth area and transition area; Dai et al. [11] built the index of resistance coefficient and Reynolds number in different flow areas by fitting the experimental data of pipeline resistance, according to the turbulent characteristics of flow relationship.

5. Verification of sediment carrying capacity formula

In the verification process of Eq. (28), the concept of effective velocity is adopted, and the calculation formula is [12]:

$$V = \left(\frac{1}{T}\int_0^T \left(u^2 + v^2\right)dt\right)^{\frac{1}{2}} \tag{37}$$

In this paper, the sediment carrying capacity Eq. (28) is verified and analyzed by using river hydrological and sediment data, coastal water hydrological and sediment data. The comparative verification is shown in Fig. 2. The hydrologic and sediment data of the rivers used include the measured data of 3,570 stations in about 20 rivers of the Yellow River [13,14], the Yangtze River [15–23]; the hydrological and sediment data of coastal waters cover Liaoning, Shandong, Fujian, Jiangsu, Zhejiang, including TongZhou Bay, Sanmen Bay, Zhoushan, Dayu Bay, Taizhou and Oujiang Estuary, with a total of 70 bays. The grain size of river sediment is 0.01~8.0 mm, and that of nearshore is 0.0007~0.1 mm.

Based on the measured data of rivers and coastal waters, the derived sediment carrying capacity formula is verified and analyzed. The results show that the sediment carrying capacity Eq. (28) can be applied to the calculation of sediment carrying capacity of river and offshore area.

6. Derivation of seabed erosion and deposition formula

The basis of the calculation of the sediment deposition is the mechanism of sediment erosion and deposition in the bottom bed. Considering the suspended movement, the mechanism of sediment deposition can be understood as the difference between the sediment content of the water body and the sediment carrying capacity of the water. When the sediment content of water body is equal to the sediment carrying capacity of water body, the seabed is in the state of

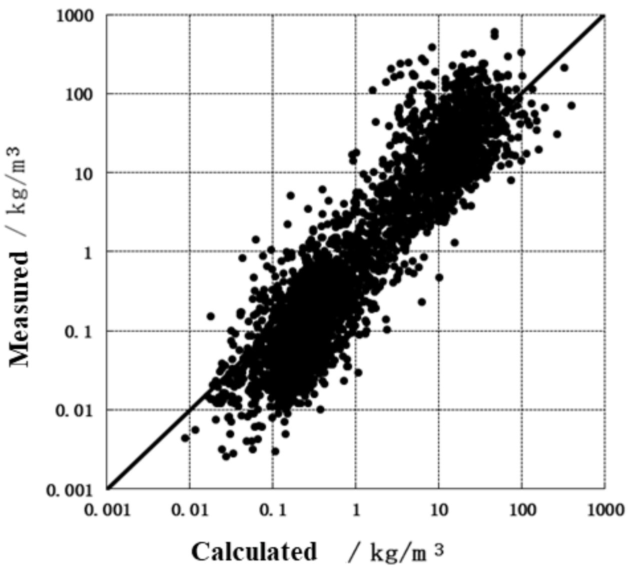

Fig. 2. Verification chart of sediment carrying capacity formula.

balance of scour and deposition; when the sediment content of water body is greater than the sediment carrying capacity of water body, the seabed will deposit; when the sediment content of water body is less than the sediment carrying capacity of water body, the seabed will be scoured. In this case, the fluxes of scour and deposition of seabed are related to the difference between sediment content and sediment carrying capacity of water. In this section, based on Eqs. (28) and (36) of water flow carrying capacity obtained by combining minimum energy dissipation rate, the formula of seabed erosion and deposition strength is derived.

The equation of sediment unit width motion can be expressed as follows:

$$\frac{\partial(qS)}{\partial x}+\alpha\omega\left(S-C_s\right)=0 \tag{38}$$

where x is the coordinate along the flow direction; q is the unit width discharge and α is the probability of sediment settlement.

The seabed deformation equation is:

$$\frac{\partial(qS)}{\partial x}+\gamma_0\frac{\partial Z}{\partial t}=0 \tag{39}$$

where Z is bed elevation and t is the time coordinate.

By subtracting Eq. (39) from Eq. (38), the calculation formula of seabed erosion and deposition can be obtained:

$$\Delta z_0=\frac{\alpha\omega_s\Delta t}{\gamma_0}\left(S-C_s\right) \tag{40}$$

where Δz_0 is the corresponding thickness of erosion and deposition, and Δt is the duration of erosion and deposition.

If the bed thickness change at time t is recorded as Δz_1 and that at time $t+\Delta t$ as Δz_2, the erosion and deposition intensity during Δt can be written as:

$$P=\Delta z_2-\Delta z_1=\frac{\alpha\omega_s\Delta t}{\gamma_0}\left[\left(S_2-C_{s2}\right)-\left(S_1-C_{s1}\right)\right] \tag{41}$$

In Eq. (41), the intensity of erosion and deposition is positive for sedimentation and negative for erosion.

Assuming that the local seabed erosion and deposition changes do not change the incoming sediment conditions, as $S_2=S_1$, there is:

$$P=\frac{\alpha\omega_s\Delta t}{\gamma_0}\left(C_{s1}-C_{s2}\right)=\frac{\alpha\omega_s C_{s1}\Delta t}{\gamma_0}\left(1-\frac{C_{s2}}{C_{s1}}\right) \tag{42}$$

By substituting Eqs. (28) and (36) into the above formula, two forms of seabed erosion and deposition strength formula can be obtained:

$$P=\frac{\alpha\omega_s C_{s1}\Delta t}{\gamma_0}\left[1-\frac{C_{D2}}{C_{D1}}\frac{J_1}{J_2}\left(\frac{V_2}{V_1}\right)^2\left(\frac{h_1}{h_2}\right)\right] \tag{43}$$

$$P=\frac{\alpha\omega_s C_{s1}\Delta t}{\gamma_0}\left[1-\frac{C_{D2}}{C_{D1}}\left(\frac{V_2}{V_1}\right)^3\left(\frac{h_1}{h_2}\right)\right] \tag{44}$$

Table 1
Resistance coefficient ratios for different flow regions

Flow area	C_{D2}/C_{D1}
Laminar flow region	$\frac{V_1h_1}{V_2h_2}$
Turbulent smooth region	$\left(\frac{V_1h_1}{V_2h_2}\right)^{1/4}$
Turbulent transition zone	$\left(\frac{V_1h_1}{V_2h_2}\right)^{m}, m\in\left(-\frac{1}{4},0\right)$
Turbulent rough area	$\left(\frac{h_1}{h_2}\right)^{2/7}$

Considering the relationship between seabed erosion and deposition and flow resistance, combined with the ocean resistance coefficient Eq. (45) obtained by Soulsby and the roughness height calculation Eq. (46) given by Colebrook, the resistance coefficient ratios of the following different flow regions (laminar flow area, turbulent smooth area, turbulent rough area and turbulent transition area) can be obtained, as shown in Table 1.

$$C_D=\left[\frac{1}{7}\left(\frac{12z_0}{h}\right)^{\frac{1}{7}}\right]^2 \tag{45}$$

$$z_0=\frac{k_s}{30}+\frac{\nu}{9u_*} \tag{46}$$

where k_s is the height of Nikuradse roughness.

The ratio of resistance coefficient in different flow areas is shown in Table 1, and the Eqs. (43) and (44) are used to calculate the seabed erosion and deposition formula for different flow areas.

α is the saturation coefficient of sediment recovery or sediment movement parameter, which is related to dynamic conditions and sediment settling velocity. In the application of tidal current and sediment model, the empirical values of sediment restoration saturation coefficient are mostly between 0.25 and 1.0. It is generally believed that the sediment restoration saturation coefficient during scouring is greater than that of sediment deposition. Li Ruijie, Dou Guoren and Li Wangsheng gave the empirical value of saturation coefficient.

7. Conclusions

In this paper, based on the derivation of the hydrodynamic balance equation of sediment-laden flow and considering the steady and uniform state, new formulas of sediment carrying capacity (27), (31) and (34) are derived, and two simplified forms of Eq. (27), as (28) and (36) are given. Eq. (27) is verified by a large number of measured data.

Eq. (27) is the formula of sand carrying capacity without considering the surface wind effect, and the Eqs. (31) and (34) are the formula of sand carrying capacity considering the surface wind effect, and the three formulas are applicable to the offshore area.

Based on the derived Eqs. (28) and (36), two kinds of seabed erosion and deposition Eqs. (43) and (44) under different flow regions are derived by combining the sediment single width motion Equation and the bed deformation equation.

Because the formula of sediment carrying capacity and seabed erosion and deposition intensity contain slope, there are some advantages in the calculation of channel sedimentation. The calculation formula of seabed erosion and deposition strength derived from this paper will be discussed in the next paper.

Acknowledgement

This paper was supported by Observation and Research Station of East China Coastal Zone, Ministry of Natural Resources (ORSECCZ2022204).

References

[1] Z. Cao, H. Xiao, Seabed evolution forecast by hydrologic data, Port Waterway Eng., 39 (2010) 20–22.
[2] Y. Wang, X. Lin, W. Feng, A simple method for calculating long-term deposition rate after construction of warping bank in estuary, J. Hohai Univ., 28 (2000) 100–102.
[3] J. Liu, G. Yu, Calculation and forecast of the beach protection and accretion acceleration on the siltbeach, The Ocean Eng., 8 (1990) 51–59.
[4] R. Li, J. Zheng, W. Zhang, H. Cao, A general formula of sediment carrying capacity and its analysis, J. Sediment Res., 39 (2014) 1–7.
[5] C.T. Yang, Unit stream power equation for gravel, J. Hydraul. Eng., 110 (1984) 1783–1797.
[6] G. Xu, J. Lian, Theories of the minimum rate of energy dissipation and the minimum entropy production of flow(II), J. Hydraul. Eng., 34 (2003) 43–47.
[7] M. Chang, G. Xu, Numerical simulation of fluid motion in flume based on theory of minimum rate of energy dissipation, J. Sediment Res., 38 (2013) 67–71.
[8] Y. Li, X. Yu, Numerical study on suspended sediment transport in oscillatory flows, J. Hydrodyn., 22 (2007) 420–426.
[9] R. Soulsby, Dynamics of Marine Sands: A Manual for Practical Applications, Thomas Telford, London, 1997.
[10] G. Dou, Generalized laws of turbulent flow in open channels and pipes for various regions, Hydro-Sci. Eng., 1 (1980) 1–12.
[11] L. Dai, R. Li, Q. Feng, W. Zhu, A general formula of resistance coefficient and its application in sediment carrying capacity computation, J. Sediment Res., 41 (2016) 7–13.
[12] R. Li, J. Zheng, Q. Feng, W. Zhang, Sediment carrying capacity and erosion rate of nearshore sediment, J. Sediment Res., 38 (2013) 21–25.
[13] K. Shihohara, T. Tsuhaki, On the Characteristic of Sand Waves Formed Upon Beds of the Open Channels and Rivers, Reports of Research Institute of Applied Mechanics, Kyushu University, 1959.
[14] S.V. Chitales, Hydraulics of Stable Channels, Government of India, Ministry of Irrigation and Power, Central Water and Power Commission, 1966.
[15] B.R. Cobly, C.H. Hembree, Computation of Total Sediment Discharge, Niobrara River Near Cody, Nebraska, U.S. Geol. Surv., Water-Supply Pap.1, 1955.
[16] D.B. Simons, Theory and Design of Stable Channels in Alluvial Materials, Ph.D. Thesis, Colorado State University, Fort Collins, Colorado, 1957.
[17] D.W. Hubbell, D.Q. Matejka, Investigations of Sediment Transportation, Middle Loup River at Dunning, Nebraska, U.S. Geol. Survey, Water-Supply Pap.1, 1968.
[18] F.B. Totffaleti, A Procedure for Computation of the Total River Sand Discharge and Detailed Distribution, Bed to Surface, Technical Report No. 5, Committee of Channel Stabilization, Corps of Engineering, U.S. Army, November, 1968.
[19] H.M. Chaudhry, K.V.H. Smith, H. Vigil, Computation of sediment transport in irrigation canals, Proc. Inst. Civ. Eng., 45 (1970) 79–101.
[20] J.K. Culbertson, C.H. Scott, J.P. Bennett, Summary of Alluvial-Channel Data From Rio Grande Conveyance Channel, New Mexico, 1965–1969, Professional Paper 562-J, United States Geological Survey, Washington D.C., 1972.
[21] K. Mahmood, M.I. Haque, A.M. Choudri, M.A. Malik, ACOP Canals Equilibrium Data, Vol. X: Summary of 1974–1980 Data, Civil, Mechanical and Environmental Engineering Department, George Washington University, 1984.
[22] T. Nakato, Tests of selected sediment-transport formulas, J. Hydraul. Eng., 116 (1990) 362–379.
[23] C.T. Yang, Sedimentation, River Morphology, and Modeling, Technical Service Center, U.S. Bureau of Reclamation, Denver Colorado, July, 1998.