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# Controllable robust optimization of ship concept design for quantifying uncertainty of marine water quality forecasts

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**ABSTRACT**

This paper is concerned with the application of controlled robust optimization (CRO) methods in solving conceptual design problems of bulk carriers sailing in the ocean with many uncertainties. Although there are many publications in the literature on the optimization of uncertainties in ships sailing in the ocean environment, most of them have some limitations in deep engineering applications due to high computational costs or methodological limitations. The purpose of this paper is to demonstrate the feasibility of the CRO approach and to find robust solutions for optimizing ship design for long-term ocean voyages. In this paper, two problems in the conceptual design of bulk carriers for oceanic transportation are investigated. It should be noted that these two problems differ only in terms of the target number and uncertain parameters. Moreover, another robust optimization method is involved in the comparison of the single-objective problem. In order to make the Pareto solution uniformly distributed, a directed search domain approach is used in the optimization process. In this way, a sufficient number of solutions are provided for the designer's analysis at different levels of robustness. The results also show that with the CRO method, the ship designer is able to handle optimization problems with multiple uncertain parameters, which is a useful reference for studying the characteristics associated with this type of ships located in marine navigation.

*Keywords:* Robust design optimization; Ship conceptual design; Marine voyage; Marine characteristics; Uncertainties; Multi-objective optimization; Pareto solutions

## 1. Introduction

In engineering optimization, the complexity of multi-objective optimization has substantially increased in view of the involvement of uncertainties. Therefore, the application of optimization theory is restricted due to at least two factors: the multiple conflicting objectives and uncertainty of input data [1]. More specifically, the designer encounters various uncertain parameters even in the conceptual design. The design variables, objectives, and constraints have been assumed as deterministic optimization problems in most ship optimization problems. However, the uncertainty of those parameters and functions should also be considered in the optimization problem [2–4]. The study of robustness in multi-objective optimization has become a hot topic for the analysis of uncertainties in engineering problems [5,6].

There are two main approaches to deal with optimization problems with uncertainties. The first one is based on stochastic optimization. It gives a probabilistic description of uncertainties. Another approach, which has been recently developed, is robust optimization (RO). Generally, the uncertainty model in RO is set-based or deterministic [7]. In RO, the uncertainty is often taken into account using objective functions such as the expected revenue, the variance, or the risk.

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In the single objective optimization, robustness has been addressed by different researches [8–13]. In contrast to the other papers focusing on the mathematical modeling or strategy of the algorithm, Diez and Peri [14] gave a more detailed illustration of the application of RO in the ship design problem. In this paper, the single objective is divided into the expectation and variance of the objective via considering the uncertain parameters as probabilistically distributed. In addition, the expectation of constraints is also used to handle the uncertainties. Then, considering the single uncertain parameter each time, the robust solutions at various cases are analyzed. Thus, this paper shows a constructive way of combining the application and mathematical theory in RO. However, for method dividing the original objectives, there are some obstacles when extending the existing method for the multi-objective optimization. It can be computationally expensive for practical problems with multiple objectives.

In multi-objective optimization, Deb and Gupta [15] extended the existing approach of finding robust solutions from single objective optimization to multi-objective optimization via two methods. The first approach is letting the mean value of representative labour solutions be the objective. The paper implies that such an objective can represent the robust Pareto solution to some certain. The second method is adding a constraint for restriction of the objective variance. Although the second approach can control the robustness to some extent, it seems that it is still not clear for the designer to handle the extent of constraint. In addition, the connection between uncertain parameters and objectives is not considered. Therefore, the technique seems substantially restricted for engineering applications.

In another study, Ide and Schöbel [1] have presented several methodologies regarding robustness in the single objective and also multi-objective optimization. Additionally, some efforts have also been devoted to extending the RO methods in the single objective optimization for multi-objective optimization. Although this paper provides the guidance for application, its limitation lies in the lack of engineering practice, which is the further work in the next stage as explained in the paper.

In this regard, Jafaryeganeh et al. [16] applied an optimization procedure with uncertainties for the internal layout of oil tankers targeting safety and economic feedback considering the incorporated uncertainty. Similarly, Priftis et al. [17] addressed the multi-objective optimization problem of ships under uncertainty with Holistic Optimization Design Approach.

Overall, after years of effort, researchers have made significant advances in multi-objective optimization. However, there are still certain concerns about the limitations of existing methods regarding the applicability to engineering problems. These shortfalls include high computational cost and the lack of control of robustness. In fact, the robustness of different engineering problems, for example, warship design or merchant ship design, should be treated separately. Taking ship conceptual design as an example, the robustness of warship is of great importance to maintain survivability while the merchant ship design needs a trade-off between the cost and robustness. Therefore, a profound analysis is needed regarding not only the existence but also the level of robustness.

The present paper extends the control of robustness for multi-objective optimization application in ship conceptual design after comparing with the method by Diez and Peri [14]. Sen and Yang [18] suggested the ship design problem for the first time. Then, Parsons and Scott [19] modified this problem for multi-objective optimization. Afterwards, Diez and Peri [14] first introduced robustness to handle the uncertainty factors.

The reminder of this paper is organized as follows. First, an introduction to uncertainties and RO is presented in Section 2 – Robust optimization theory. Comparison between controlled robust optimization (CRO) and another robust optimization method is conducted by applying both methods in the bulk carrier conceptual design with single objective and single uncertain parameter in Section 3 – Single objective optimization comparison. A further application of CRO in the bulk carrier conceptual design with multi-objective and multiple uncertain parameters is presented in Section 4 – Multi-objective optimization application. Finally, the Conclusion is given in Section 5.

## 2. Robust optimization theory

### *2.1. Multi-objective optimization*

There exists more than one objective in engineering problems. Let $n$ be the number of objectives and $m$ be the number of variables. Then, $\{F_1(x), F_2(x), F_3(x), \ldots, F_n(x)\}$ is called the objective set. The multi-objective optimization can be formulated as follows:

$$\begin{aligned}&\mathrm{Min}\left\{F=\left[F_1(x),F_2(x),F_3(x),\cdots,F_n(x)\right]\right\}\\&\text{subject to } x\in D^*\subseteq R^m,\end{aligned} \tag{1}$$

where the set $D^*$ means the feasible space, in which the elements satisfy all the constraints.

### *2.2. Robust optimization*

#### *2.2.1. Uncertainties involved in engineering optimization problem*

When uncertainties are involved, the solutions to the optimization problem can vary with the variation of uncertain parameters. This relationship is illustrated in Fig. 1. Recognizing the effect from various uncertain parameters

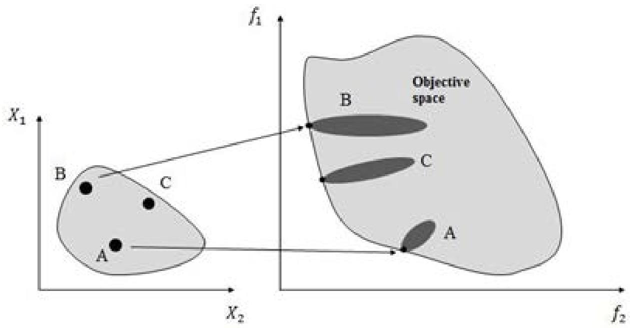


Fig. 1. Uncertainties involved in engineering (Deb and Gupta [15]).

on solutions in engineering problems, the research on uncertain parameters is needed.

According to Du and Chen [20], Marijt [21], Diez and Peri [14], uncertain parameters can be categorized as:

- *Controllable uncertain parameters*: This kind of uncertain parameters represents those can be minimized by the designer or consumer via certain ways, for example, increasing the precision of mathematical model, improving the algorithm to avoid calculation error or using high precision equipment to reduce manufacture tolerance.
- *Uncontrollable uncertain parameters*: When the designer or consumer cannot decide or change the extend of deviation of the parameter, it can be classified as an uncontrollable uncertain parameter. This kind of parameters generally derives from environment changes or operating conditions. Taking ship conceptual design as an example, for the designer and carrier, the variation of fuel price and port handling rate is definitely out of their reach.

For the controllable uncertain parameters, some efficient measures can be introduced to reduce the influence for the whole engineering solution. Therefore, researchers are more inclined to emphasize on the consequences caused by the uncontrollable uncertain parameters. Boulougouris and Papanikolaou [22] introduced risk-based approaches with the consideration of waves and external damage to the ship hull which are uncontrollable. The survivability under such conditions is studied. In turn in Diez and Peri [14], the influence of fuel price and port handling rate to the annual cost of future operation is discussed during the conceptual design. Similarly, when dealing with the application of robust optimization, the present paper mainly focuses on the uncontrollable uncertain parameters in ship conceptual design.

### 2.2.2. Dealing the uncertainties

Generally, there are three methods to quantify uncertain parameters. Namely the deterministic, probabilistic and possibilistic quantifications [21].

In the deterministic quantification, the variation is already known. In turn, the probabilistic quantification introduces probability density functions, whilst the possibilistic quantification is often applied in those parameters with little cognition, and normally the fuzzy theory is involved to insure the rationality.

For the ship design considered in the present paper, uncertain parameters are unpredictable. Taking fuel price for example, the general range of variation is known, whereas the details such as probability density cannot be exactly determined by the designer. In Diez and Peri [14], the assumption of certain distribution is taken as the premise of dealing with uncertainties. While in the present paper, the fuzzy theory is adopted to deal with uncertainties [23].

### 2.2.3. Measurement of robustness

To get a robust solution, we need to re-define the multi-objective problem. Generally, the re-define work can be outlined as follows:

- *Re-defining original objectives*: The former objectives are re-defined as the variance or standard deviation of them [24], or expectation of objectives [25], or both of them at the same time [14,26].
- *Adding constraints to problem formulation*: The original objectives are maintained, while some extra constraints are attached after that. One possible method is based on a restriction of the search domain. More specifically, the objective functions are minimized at the worst conditions, as considered the well-known min-max method [27,28]. Another method is based on adding a robustness measure, which is introduced by Erfani and Utyuzhnikov [29], as a new objective at premise of maintaining the original objectives and search domain.
- *Changing constraints*: One method of this type gives a probabilistic formulation with uncertainties involved instead of a deterministic description. It is generally used in the reliability-based design or risk-based design [22]. Another one is based on the comparison of the objective function $f$ with a fixed function value $f_D$. A constraint of the variance between $f$ and $f_D$ is added, by which the authors expressed that the robust solutions can be obtained [15]. The handling of constraints is not the focal point, so there is no further discussion here.

A typical method is addressed for each kind of measurements except the third one:

*(1) Re-defining original objectives*

The method of taking the expectation and variance of objective function as the new objective functions is used by Diez and Peri [14] in ship design. A brief introduction of the method is as follows:

$$\begin{aligned}
&\mathrm{Min}\left\{f_1(x)=\mu\left[f(x,y)\right]\right\}\\
&\mathrm{Min}\left\{f_2(x)=\sigma\left[f(x,y)\right]\right\}\\
&\mathrm{s.t.}\, x\in A, y\in B\\
&\sup\left[g_p(x,y)\right]\le 0, p=1,\ldots,N\\
&\mu\left[h_q(x,y)\right]=0, q=1,\ldots,M
\end{aligned}\tag{2}$$

where $x \in A$ is the deterministic design component vector and $y \in B$ is the uncertain design component vector (e.g., environmental conditions), and the original objective function $f$ is split into two objective functions, namely, the expectation $f_1$ and the variance $f_2$ of $f$. The original inequality constraint $g_p$ is converted to sup($g_p$), and the equality constraint $h_q$ to the expectation $\mu(h_q)$ of $h_q$.

It is noted that in the application problem in Diez and Peri [14], the uncertain parameters are assumed evenly distributed.

*(2) Adding constraints to problem formulation*

As mentioned above, re-defining the original objective can cause the situation that is very time consuming as the number of objectives increase. We can maintain the objectives and add the robustness measure as another objective [29,30].

For this purpose, the generic multi-objective optimization problem (1) is converted into the possibilistic multi-objective optimization problem using the following formulation:

$$\begin{aligned}&\mathrm{Min}\left\{\tilde{F}-\left[\tilde{F}_1(x),\tilde{F}_2(x),\ldots,\tilde{F}_n(x)\right]\right\}\\&\mathrm{s.t.}\tilde{g}_l(x)\le\tilde{b}_l\quad i=1,\ldots,k\\&x\in D^*\end{aligned}\tag{3}$$

The tilde indicates that the model is constructed within uncertainty parameters and the uncertainty is considered using the fuzzy theory. Then, to convert the problem in Eq. (3) into a deterministic formulation, the definitions from the fuzzy theory are used (Appendix). Thus, the problem in Eq. (3) is converted into:

$$\begin{aligned}&\mathrm{Min}\left\{F_{pm}=\left[F_1^{pm}(x),F_2^{pm}(x),\ldots F_n^{pm}(x)\right]\right\}\\&\mathrm{s.t.}g_i^{pm}(x)\le b_i^{pm}\quad i=1,\ldots,k\\&x\in D^*\end{aligned}\tag{4}$$

where the pm implies the fuzzy possibilistic mean value of an uncertain parameter. The optimal solution of problem in Eq. (4) is called the possibilistic mean Pareto optimality [29–31].

Finally, we add the robustness measure as another objective function. The general formulation was originally constructed by Erfani and Utyuzhnikov [31]. Let $r$ be the number of uncertain design components. The robustness measure has the following form:

$$R_F=\frac{1}{nr}\sum_{i=1}^{n}\sum_{j=1}^{r}\frac{\sigma F_i}{\sigma P_J}\tag{5}$$

where $\sigma P_j$ denotes the variance of the uncertain parameter $P_j$ and it can be calculated by the definition of variance of the fuzzy number in Appendix. Here $\sigma F_i$ can be estimated by first-order Taylor series:

$$\sigma F=\sqrt{\sum_{i=1}^{r}\left(\frac{\partial F}{\partial P_i}\right)^2\left(\sigma P_i\right)^2}\tag{6}$$

where $P$ is the uncertain parameter of the model. Thus, a set of robust solutions can be obtained *via* the optimization of the original objective functions and measure of robustness.

### *2.3. Controllable robust optimization*

The requirement to robustness varies as different engineering problems have different trade-offs. For the designer, it is important to keep robustness controllable.

Fig. 2 illustrates the relationship between the design variable and objective function. Point A represents the solution that is most sensitive to the uncertainties, while point C represents the solution that is most insensitive to the uncertainties. Generally, point A can be obtained by most optimization methods, while point B can be derived from most robust optimization methods. It is CRO that can provide solutions represented by both three of the points.

An approximation method was first introduced by Utyuzhnikov et al. [32] and then applied to the construction of a dimensionless positively increasing convex function [29,32].

The convex function called tunable robust function (TRF) is constructed as follows:

$$\mathrm{TRF}(R)=\begin{cases}T^{(0)}(R) & R\le\mathrm{LR}-d\\T^{(1)}(R) & \mathrm{LR}-d\le R\le\mathrm{LR}-d\\T^{(2)}(R) & \mathrm{LR}\le R\le\mathrm{LR}+d\\T^{(3)}(R) & \mathrm{LR}+d\le R\le 100\\T^{(4)}(R) & R>100,\end{cases}\tag{7}$$

where $T^{(i)}(R)$ ($l$ = 0, 1, 2, 3, 4) is short for TRF ($R$) and LR (between 0 to 100) is the desired level of robustness. The free parameter $d$ is defined by the designer.

Thus, the scaled function $R$ (between 0 and 100) of robust measure [Eq. (7)] is mapped onto a dimensionless positively increasing function $T^{(i)}(R)$. The calculation of $T^{(i)}(R)$ is approximated piece wisely as [29]:

$$T^{(l)}=T_l+\Delta_l T\frac{e^{a_l}\xi^{(l)}-1}{e^{a_l}-1}\quad\left(R_l\le R\le R_{l+1}\right),\tag{8}$$

where $T_l = T(R_l)$ and $R_l$ is the value of the scaled robustness measure at the boundary of the region as shown in Eq. (7). For the region $R < R_0$, $T^{(0)} = 1$. The value of $\Delta_l T$ is fixed as the boundaries of each region in Eq. (7) does not depend on the value of $R$. Thus, it can be obtained as follows:

$$\begin{aligned}&\Delta_l T=\beta n\Delta_{l-1}T\\&\beta>1,\Delta_0 T=1\end{aligned}\tag{9}$$

where $n$ is the number of original objectives.

For $e^{a_l}$ and $\xi^{(l)}$, the following formulations is given:

$$\begin{aligned}&a_l=\frac{A\Delta_l\mathrm{Re}^{b_k}\left(e^{a_l}-1\right)}{\Delta_l T}\\&b_l=a_{l-1}+b_{l-1}\\&\xi^{(1)}=\frac{R-R_l}{R_{l+1}-R_l},\end{aligned}\tag{10}$$

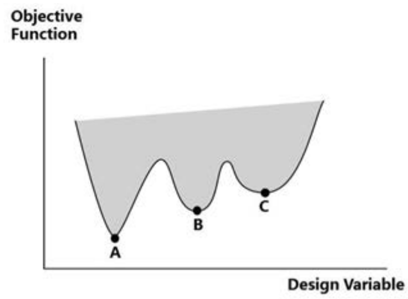


Fig. 2. Different level of robust.

where $A$ can be chosen as $1/(R_4 - R)$to have the same dimensionality with $R^{-1}$, and for the region of $R < R_0$, $a_0 = 0$, $b_0 = 0$.

Then, the CRO can be summarized in the following form:

$$\begin{aligned}
&\text{Min}\left\{F_{pm}=\left[F_1^{pm}(x),F_2^{pm}(x),\ldots,F_n^{pm}(x),\right]\right\}\\
&\text{Min TRF}(R)\\
&\text{s.t.}\,g_i^{pm}(x)\le b_i^{pm}\;\; i=1,\ldots,k\\
&x\in D^*
\end{aligned}\tag{11}$$

By given the LR and the d, the designer can control the robustness of the problem and at the same time obtain the robust solution.

The above multi-objective optimization problem can be solved using the DSD method [30]. It can generate a well distributed Pareto set, which is very useful for the decision maker to choose an appropriate solution according to their requirements.

## 3. Single objective optimization comparison

### 3.1. Optimization case

The CRO approach is applied to a ship conceptual design problem by Diez and Peri [14] for a comparison with the re-defined objective method.

The problem is dealing with the bulk carrier design during the conceptual design phase. The design variables indicating the solution are the length $L$ (m), width $B$ (m), depth $D$ (m), draft $T$ (m), block coefficient $C_b$ and the cruising speed $V_k$ (m/s). The main particular vector of the model is $x = (L, B, D, T, C_b, V_k)^T$.

Here, we consider the unit transportation cost $C_U$ (£/ton) as the only objective for the sake of comparison with the method by Diez and Peri [14]. The multi-objective optimization problem is further explained in Section 4 – Multi-objective optimization application. Using CRO method, the problem is formulated as follows:

$$\begin{aligned}
&\text{Min}\left\{\frac{F_{pm}=C_U=C_{\text{cost}}}{C_{\text{cargo}}}\right\}\\
&\text{Min TRF}(R)\\
&\text{s.t.}\,100\le L\le 600\quad 10\le B\le 100\quad 5\le D\le 30\\
&5\le T\le 30\quad 0.63\le C_b\le 0.75\quad 14\le V_k\le 18\\
&\frac{L}{B}\ge 6\quad \frac{L}{D}\le 15\quad \frac{L}{T}\le 19\quad T\le 0.7D+0.7\\
&T\le 0.45\text{DWT}^{0.31}\quad 0.63\le C_b\le 0.75\quad F_n\le 0.32\\
&\text{GM}_T=\text{KB}+\text{BM}_T-\text{KG}\ge 0.07B
\end{aligned}\tag{12}$$

where $C_{\text{cost}}$ indicates the annual cost of the ship and $C_{\text{cargo}}$ implies the annual cargo of the ship. Both of them can be obtained *via* a model in Diez and Peri [14]. The calculation of the Froude number $F_n$, metacentric height $\text{GM}_T$ and deadweight DWT is also referred to Diez and Peri [14]. During the design phase, the uncontrollable uncertain parameters are port handling rate $H_u$ (ton/d), round trip distance $T_u$ (nm) and fuel price $P_u$ (£/ton). Unlike the assumption by Diez and Peri [14], these parameters are uniformly distributed. We construct them in a more practical way using the fuzzy theory in CRO (Table 1). The same upper and lower bound of the parameter are maintained without limiting the variance of the parameter.

For the test case shown in Table 1, one uncertain parameter is considered each time. Similar cases from Diez and Peri [14] are also shown in Table 2 for the sake of comparison, where case *1 uses the robust solution introduced in Diez and Peri [14], and the case *2 exploits the deterministic solution also mentioned in Diez and Peri [14].

A comparison between CRO, robust method in paper by Diez and Peri [14] and the deterministic method is shown in Table 3. The main differences are in the formulation of objectives and the construction of uncertainty.

### 3.2. Comparison of results

After setting the LR as 40, 60 and 80, separately, the solutions of case *0 under requirements of 40% robustness, 60% robustness and 80% robustness are calculated.

In Table 4, the data of six variables ($L$, $B$, $D$, $T$, $C_b$, $V_k$) in cases *1 and *2 are derived from Diez and Peri [14], which indicate the solutions obtained by them.

In order to compare the results on each level of robustness, the solutions from the configuration characterized by a minimum unit cost expectation in case *1 and the solutions in case *2 are displayed. The former configuration implies the robust solution as illustrated in Diez and Peri [14] while the latter one represents the deterministic solution. Similarly, the solutions in case *0 from the configuration characterized by the minimum TRF and unit transportation cost are chosen, which are referred to as the robust solution in CRO. This configuration represents a compromise between the robustness and unit transportation cost.

The comparison of each solution in the configuration characterized by TRF and unit transportation cost is shown in the last line in Table 4.

As can be seen from the data in the last line of Table 4, the final results of cases *0 and *1 perform better than the result in case 3. This is obvious since both methods used in cases *0 and *1 involve robustness in the optimization process. It is also worth noting that the solution in case *0 performs slightly better than the solution in case *1.

Fig. 3 shows the entire Pareto set of case *0 under requirements of 40% robustness, 60% robustness and 80% robustness. Thus, the result enables the designer to choose an appropriate solution from the entire set according to the requirements.

Table 1
Optimization case in controlled robust optimization

| Case id | $P_j$ | Unit | Lower bound | Upper bound | Distribution | Objectives(s) |
|---|---|---|---|---|---|---|
| *0 | $H_u$ | ton/d | 1,000 | 11,000 | (8,000; 7,000; 3,000) | $F_{\text{pm},T}$ $R_F$ |

Table 2
Optimization case in Diez and Peri [14]

| Case id | $y$ | Unit | Lower bound | Upper bound | Distribution | Objectives(s) |
|---|---|---|---|---|---|---|
| *1 | $H_u$ | ton/d | 1,000 | 11,000 | Uniform | $\mu(f)$, $\sigma(f)$ |
| *2 | $H_u$ | ton/d | 1,000 | 11,000 | Uniform | $f(x,\mu(y))$ |

Table 3
Three methods for optimization

| Approach | Objective(s) | Construction of robustness |
|---|---|---|
| Controlled robust optimization | $F_{pm}$, TRF | Possibilistic |
| Robust method | $\mu(f)$, $\sigma(f)$ | Probabilistic |
| Deterministic method | $f(x,\mu(y))$ | Probabilistic |

Table 4
Optimization case in controlled robust optimization

| Variable | 40% robustness | | | 60% robustness | | | 80% robustness | | |
|---|---|---|---|---|---|---|---|---|---|
| | Case *0 | Case *1 | Case *2 | Case *0 | Case *1 | Case *2 | Case *0 | Case *1 | Case *2 |
| $L$ (m) | 162.69 | 165.70 | 182.79 | 163.34 | 165.70 | 182.79 | 164.14 | 165.70 | 182.79 |
| $B$ (m) | 27.04 | 27.75 | 30.60 | 27.22 | 27.75 | 30.60 | 27.23 | 27.75 | 30.60 |
| $D$ (m) | 13.85 | 14.25 | 15.89 | 13.87 | 14.25 | 15.89 | 13.85 | 14.25 | 15.89 |
| $T$ (m) | 10.39 | 10.76 | 11.90 | 10.38 | 10.76 | 11.90 | 10.36 | 10.76 | 11.90 |
| $C_b$ | 0.66 | 0.65 | 0.67 | 0.65 | 0.65 | 0.67 | 0.65 | 0.65 | 0.67 |
| $V_k$ (m/s) | 14.01 | 14.00 | 14.00 | 14.03 | 14.00 | 14.00 | 14.04 | 14.00 | 14.00 |
| TRF | 10.83 | 11.13 | 13.04 | 9.51 | 11.13 | 13.04 | 2.91 | 3.58 | 12.65 |
| $F_{pm}$ (£/ton) | 9.49 | 9.26 | 8.72 | 9.49 | 9.26 | 8.72 | 9.49 | 9.26 | 8.72 |
| TRF & $F_{pm}$ | 20.32 | 20.39 | 21.04 | 19.00 | 20.39 | 21.76 | 12.4 | 12.84 | 21.37 |

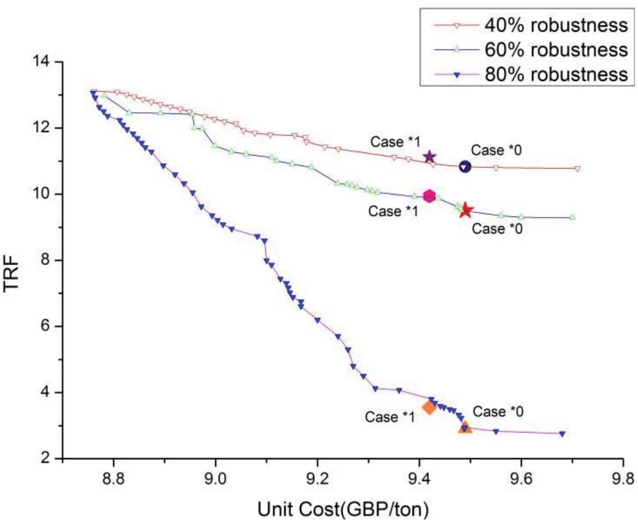


Fig. 3. Result of case *0 under different level of robust.

In addition, the results in cases *0 and *1 in Table 4 are also displayed in Fig. 3 for comparison. It is expected that the points representing the results of case *1 are all situated on the same line. Namely, the robust solution in case *1 is also a part of the robust Pareto set of case *0. This phenomenon implies that the CRO method can generate the robust solutions precisely.

## 4. Multi-objective optimization application

### 4.1. Multi-objective optimization case

It is important to emphasize that the ability of handling multi-objective problems with multiple uncertain parameters is an important element for a robust optimization method. In this section, the CRO is applied to the same engineering problem with two objectives and three uncertain parameters, which are introduced as the port handling rate $H_u$ (ton/d), round trip distance $T_u$ (nm) and fuel price $P_u$ (£/ton) in Diez and Peri [14]. Here, the light ship mass is considered as another objective. The new problem is formulated as:

$$\begin{aligned}
&\mathrm{Min}\left[F_1^{pm} = W_{\mathrm{ls}}\right] \\
&\mathrm{Min}\left\{\frac{F_2^{pm} = C_U = C_{\mathrm{cost}}}{C_{\mathrm{cargo}}}\right\} \\
&\mathrm{Min}\,\mathrm{TRF}\,(R) \\
&\mathrm{s.t.}\,100 \le L \le 600 \quad 10 \le B \le 100 \quad 5 \le D \le 30
\end{aligned} \tag{13}$$

$$5 \le T \le 30 \quad 0.63 \le C_b \le 0.75 \quad 14 \le V_k \le 18$$
$$\frac{L}{B} \ge 6 \quad \frac{L}{D} \le 15 \quad \frac{L}{T} \le 19 \quad T \le 0.7D + 0.7$$
$$T \le 0.45\mathrm{DWT}^{0.31} \quad 0.63 \le C_b \le 0.75 \quad F_n \le 0.32$$
$$\mathrm{GM}_T = \mathrm{KB} + \mathrm{BM}_T - \mathrm{KG} \ge 0.07B$$

where $W_{ls}$ indicates the light ship weight of the bulk carrier.

The multi-objective problem shown as case *3 is summarized in Table 5.

For uncertain parameters, the triangular fuzzy parameters are substituted by their possibilistic mean value $H_u$ = 5,000 ton/d, $T_u$ = 5,000 nm and $P_u$ = 5,000 £/ton. The data is derived from Parsons and Scott [19], which deals with the deterministic optimization. The robustness measure in Eq. (5) can be obtained by:

$$R_F = \frac{1}{2 \times 3}\left(\left(\frac{\sigma F_1}{\sigma H_u} + \frac{\sigma F_1}{\sigma T_u} + \frac{\sigma F_1}{\sigma P_u}\right) + \left(\frac{\sigma F_2}{\sigma H_u} + \frac{\sigma F_2}{\sigma H_u} + \frac{\sigma F_2}{\sigma H_u}\right)\right)$$

where $\sigma F_1$ and $\sigma F_2$ are calculated using Eq. (6), while $\sigma H_u$, $\sigma T_u$ and $\sigma P_u$ are calculated using definitions from the Appendix.

After scaling $R_F$ into $R$, $T^{(i)}$ ($i$ = 1, 2, 3, 4) in Eq. (7) needs to be constructed by Eq. (8) for obtaining TRF, where $\beta = 1.5$, $n = 2$ and $A = 1/100$ [29].

In this section, three levels of robustness are considered, which is LR = 40, 60, and 80. The results for different levels of robustness are shown in Fig. 3. Only two objectives are displayed for the clarity and emphasis, excluding the TRF.

Fig. 4 implies that with higher requirements on robustness, the solutions perform worse on the other objectives. There is a compromise between robustness and performance that the designer needs to consider according to the circumstances.

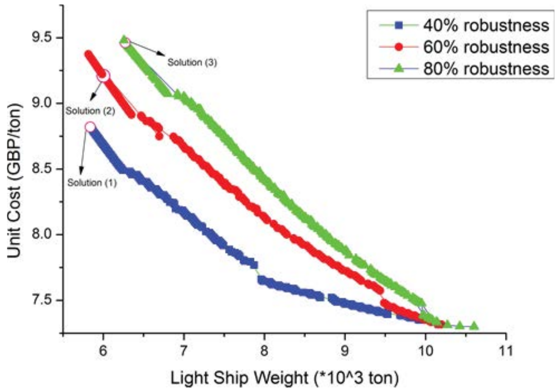


Fig. 4. Results of case *3 with different levels of robustness.

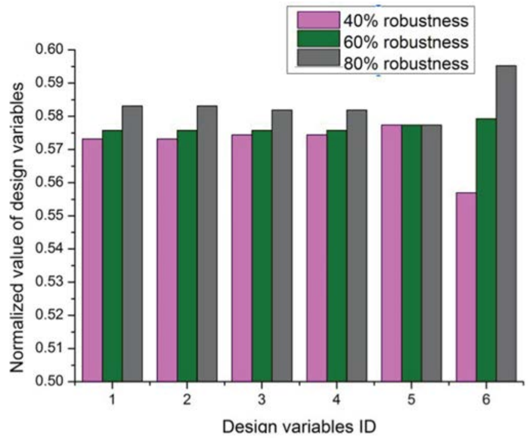


Fig. 5. Difference of design variables under different level of robust.

Table 5
Multi-objective optimization problem with uncertainties

| Case id | $y$ | Unit | Lower bound | Upper bound | Distribution | Objectives(s) |
|---|---|---|---|---|---|---|
| | $H_u$ | ton/d | 1,000 | 11,000 | (8,000; 7,000; 3,000) | |
| *3 | $T_u$ | nm | 1,000 | 5,000 | (4,000; 3,000; 1,000) | $F_1^{pm}$, $F_2^{pm}$, TRF |
| | $P_u$ | £/ton | 50 | 150 | (100; 50; 50) | |

Table 6
Results of Min (TRF + $F_1^{pm}$ + $F_2^{pm}$) in case *3

| Variable id | Variable | 40% robustness solution (1) | 60% robustness solution (2) | 80% robustness solution (3) |
|---|---|---|---|---|
| 1 | $L$ (m) | 153.7157 | 154.4043 | 156.3888 |
| 2 | $B$ (m) | 25.6193 | 25.734 | 26.0648 |
| 3 | $D$ (m) | 15.0224 | 15.0579 | 15.2193 |
| 4 | $T$ (m) | 10.5157 | 10.5405 | 10.6535 |
| 5 | $C_b$ | 0.75 | 0.75 | 0.75 |
| 6 | $V_k$ (m/s) | 15.6897 | 16.3166 | 16.7664 |
| | TRF | 8.3397 | 5.0022 | 3.8474 |
| | $F_1^{pm}$ (ton × $10^3$) | 5.84 | 6.00 | 6.27 |
| | $F_2^{pm}$ (£/ton) | 8.82 | 9.21 | 9.46 |

For better comparison, the solutions in case *3 from the configuration characterized by minimum TRF, $F_1^{pm}$, $F_2^{pm}$ are chosen. This configuration implies the comprise between the robustness and two other objectives. The details of these solutions with different level of robustness are presented in Table 6.

Next, the variation of six variables is considered as the requirement changes. The difference is shown in Fig. 5. As can be seen from the figure, while the requirement of robustness increases, most variables of the solution also increase. This is sensible because the larger size of the ship hull, the more space remains for the cargo.

## 5. Conclusion

In this paper, a CRO approach has been introduced to tackle bulk carrier conceptual design problems. The comparison of CRO with another robust optimization approach proposed by Diez and Peri [14] in the singe objective case has been carried out. Three sets of solutions with different levels of robustness have been obtained. There is a good correspondence between the solutions obtained with both approaches. This confirms the ability of CRO in generating the robust solutions with good quality.

The application of CRO in handling the multi-objective optimization problem in bulk carrier conceptual design has also been conducted. This is possible because in CRO the number of objectives does not exponentially increase and the CRO is able to process the multiple uncertain parameters simultaneously. The solutions under requirements of 40%, 60% and 80% robustness have been obtained. Thus, the ship designers can analyze the results and choose the appropriate solutions according to their preferences.

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## References

[1] J. Ide, A. Schöbel, Robustness for uncertain multi-objective optimization: a survey and analysis of different concepts, OR Spectrum, 38 (2016) 235–271.
[2] A. Priftis, A. Papanikolaou, T. Plessas, Parametric design & multi-objective optimization of containerships, Ocean Eng., 156 (2018) 347–357.
[3] Z. Zhang, FM.A. Altalbawy, M. Al-Bahrani, Y. Riadi, Regret-based multi-objective optimization of carbon capture facility in CHP-based microgrid with carbon dioxide cycling, J. Cleaner Prod., 384 (2023) 135632, doi: 10.1016/j.jclepro.2022.135632.
[4] Z. Zhang, B. Wu, L. Wu, W. Liu, L. Liu, N. Li, L. Cui, Optimization of the bionic wing shape of tidal turbines using multi-island genetic algorithm, Machines, 11 (2023) 43, doi: 10.3390/machines11010043.
[5] Z. Zhang, W. Liu, X. Zheng, H. Liu, N. Li, Numerical simulation of hydrodynamic oscillation of side-by-side double-floating-system with a narrow gap in waves, Open Phys., 19 (2021) 188–207.
[6] Z. Zhang, X. Chen, H. Wu, W. Liu, L. Cui, Numerical study of a novel hybrid system with the Wavestar wave energy converter array and a SPIC semi-submersible floating platform, J. Cleaner Prod., 407 (2023) 137178, doi: 10.1016/j.jclepro.2023.137178.
[7] D. Bertsimas, D.B. Brown, C. Caramanis, Theory and applications of robust optimization, SIAM Rev., 53 (2011) 464–501.
[8] A. Ben-Tal, L. El Ghaoui, A. Nemirovski, Robust Optimization, Princeton University Press, Princeton, NJ, 2009.
[9] M. Fischetti, M. Monaci, Light Robustness, R.K. Ahuja, R.H. Möhring, C.D. Zaroliagis, Eds., Robust and Online Large-Scale Optimization, Lecture Notes in Computer Science, Vol. 5868, Springer, Berlin, Heidelberg, 2009, pp. 61–84. https://doi.org/10.1007/978-3-642-05465-5_3
[10] M. Goerigk, A. Schöbel, Algorithm Engineering in Robust Optimization, L. Kliemann, P. Sanders, Eds., Algorithm Engineering, Lecture Notes in Computer Science, Vol. 9220, Springer, Berlin, Heidelberg, 2015.
[11] H. Wu, Z. Zhang, Effects of one mooring line failure on the behavior of a work ship, Proc. Inst. Civ. Eng. - Mar. Eng., 175 (2022) 65–77.
[12] H. Wu, Z. Zhang, W. Liu, Dynamic behavior analysis of an engineering ship with mooring line failure, Mar. Technol. Soc. J., 56 (2022) 20–34.
[13] H. Wu, L. Cui, W. Liu, Z. Zhang, J. Kang, Performance changes of an engineering ship with one mooring line failure, Eng. Fail. Anal., 137 (2022) 106233, doi: 10.1016/j.engfailanal.2022.106233.
[14] M. Diez, D. Peri, Robust optimization for ship conceptual design, Ocean Eng., 37 (2010) 966–977.
[15] K. Deb, H. Gupta, Introducing robustness in multi-objective optimization, Evol. Comput., 14 (2006) 463–494.
[16] H. Jafaryeganeh, M. Ventura, C. Guedes Soares, Robust-based optimization of the hull internal layout of oil tanker, Ocean Eng., 216 (2020) 107846, doi: 10.1016/j.oceaneng.2020.107846.
[17] A. Priftis, E. Boulougouris, O. Turan, G. Atzampos, Multi-objective robust early stage ship design optimization under uncertainty utilizing surrogate models, Ocean Eng., 197 (2020) 106850, doi: 10.1016/j.oceaneng.2019.106850.
[18] P. Sen, J.-B. Yang, Multiple Criteria Decision Support in Engineering Design, Springer, London, 1998.
[19] M.G. Parsons, R.L. Scott, Formulation of multi-criterion design optimization problems for solution with scalar numerical optimization methods, J. Ship Res., 48 (2004) 61–76.
[20] X. Du, W. Chen, Methodology for managing the effect of uncertainty in simulation-based design, AIAA J., 38 (2000) 1471–1478.
[21] R. Marijt, Multi-objective Robust Optimization Algorithms for Improving Energy Consumption and Thermal Comfort of Buildings, Ph.D. Thesis, Technical University of Eindhoven, Holland, 2009.
[22] E. Boulougouris, A. Papanikolaou, Risk-based design of naval combatants, Ocean Eng., 65 (2013) 49–61.
[23] C. Carlsson, R. Fullér, On possibilistic mean value and variance of fuzzy numbers, Fuzzy Sets Syst., 122 (2001) 315–326.
[24] G. Taguchi, M.S. Phadke, Quality engineering through design optimization, K. Dehnad, Ed., Quality Control, Robust Design, and the Taguchi Method, Springer, Boston, MA, 1989, pp. 77–96. https://doi.org/10.1007/978-1-4684-1472-1_5
[25] M.W. Trosset, N.M. Alexandrov, L.T. Watson, New Methods for Robust Design Using Computer Simulation, Proceedings of the Section on Physical and Engineering Sciences, American Statistical Association, 2003.
[26] C. Zang, M.I. Friswell, J.E. Mottershead, A review of robust optimal design and its application in dynamics, Comput. Struct., 83 (2005) 315–326.
[27] D. Kuroiwa, G.M. Lee, On robust multi-objective optimization, Vietnam J. Math., 40 (2012) 305–317.
[28] M. Ehrgott, J. Ide, A. Schöbel, Minmax robustness for multi-objective optimization problems, Eur. J. Oper. Res., 239 (2014) 17–31.

[29] T. Erfani, S.V. Utyuzhnikov, Control of robust design in multi-objective optimization under uncertainties, Struct. Multidiscip. Optim., 45 (2012) 247–256.
[30] T. Erfani, S.V. Utyuzhnikov, Directed search domain: a method for even generation of the Pareto frontier in multi-objective optimization, Eng. Optim., 43 (2011) 467–484.
[31] T. Erfani, S. Utyuzhnikov, Handling Uncertainty and Finding Robust Pareto Frontier in Multi-objective Optimization Using Fuzzy Set Theory, 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference 18th AIAA/ASME/AHS Adaptive Structures Conference 12th, 2010, p. 3092.
[32] S. Utyuzhnikov, M.D. Guenov, P. Fantini, Numerical Method for Generating the Entire Pareto Frontier in Multi-objective Optimization, Proceedings of Eurogen, Munich, 2005, pp. 12–14.

## Appendix A1: Elements in fuzzy theory

Suppose $A$ is a triangular fuzzy number shown by membership function $A = (a, b, c)$, where $b$ and $c$ are the left-width and right-width of the fuzzy number centered at $a$.

To dealing with uncertain parameters via the fuzzy theory, the following definition needs to be applied.

*Definition 1*

(Possibilistic mean value of fuzzy number) Following Carlsson and Fullér [23], if $A$ is a fuzzy number, its possibilistic mean value $M(A)$ is the arithmetic mean of its lower and upper possibilistic mean value.

For triangular fuzzy number $A = (a, b, c)$, its possibilistic mean value is given by:

$$M\left(A\right) = a + \frac{c-b}{6} \tag{A1}$$

*Definition 2*

(Variance of fuzzy number) If $A$ is a fuzzy number, the variance Var($A$) of it is given by Carlsson and Fullér [23].

$$\mathrm{Var}\left(A\right) = \frac{\left(b+c\right)^2}{24} \tag{A2}$$