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Slip flow effect on laminar convection inside micro-tubes with permeable walls

Nizar Loussif[a,b], Jamel Orfi[c,*], Ahmed Omri[b]

[a]*École Nationale d'Ingénieur de Monastir, Université de Monastir, Monastir, Tunisia*
[b]*Unité de Recherche Matériaux, Energie et Energies Renouvelables, Université de Gafsa, Gafsa, Tunisia*
[c]*Department of Mechanical Engineering, College of Engineering, King Saud University, P.O. Box 800, 11421 Riyadh, Saudi Arabia*
Tel. +966 1 467 9798; Fax: +966 1 467 6652; email: orfij@ksu.edu.sa

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ABSTRACT

In this study, a two-dimensional steady state simultaneously developing laminar flow along a permeable micro-tube is investigated numerically under slip flow conditions. The constant wall temperature boundary condition and the case of uniform suction at the entire tube wall were considered. The set of governing equations subjected to the appropriate boundary conditions for the hydrodynamic and thermal fields was solved by using the Finite Volume Method. The numerical model was validated using the available data for developing and fully developed continuum flow. The results show that increasing the Knudsen number reduces the axial velocity of the tube center and increases the streamwise fluid velocity at the wall, inducing a flattening of the velocity profiles. This leads to a reduced friction coefficient compared to the continuum case. Furthermore, the study reveals a significant effect on the rarefaction on the hydrodynamic and thermal fields especially for high values of the suction Reynolds number, Re_w. In fact, for values of Re_w close to zero, the impact on the apparent friction coefficient and Nusselt number was found to be negligible and the behavior of both parameters along the duct remains unchanged compared with the case of $\mathrm{Re}_w = 0$ (impermeable tube). The variation of the fully developed Nusselt number with Prandtl number, Knudsen number, and suction Reynolds number is presented and analyzed.

Keywords: Micro-tube; Suction; Slip flow; Ultra-filtration

1. Introduction

Laminar flows with suction/injection have been widely investigated because of their wide range of applications, such as in separation processes, food, and biomedical technologies. Uniform suction or injection in micro-ducts was studied by Terrill [1], where heat transfer in laminar flow between parallel porous plates and porous tubes was investigated. It was found that the fully developed Nusselt number increases linearly with the suction Reynolds number. Pederson and Kinney [2], Raithby [3], Kinney [4], and others studied the heat transfer for laminar flow inside the porous tubes. Large variations of the

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*Corresponding author.

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Nusselt numbers occur when considering suction or injection. Besides, Prandlt number has a significant effect on the fully developed Nusselt number.

Moussy and Snider [5] investigated the laminar flow over pipes with injection and suction through the porous wall at low Reynolds numbers. Analytical expressions describing the two-dimensional steady-state laminar flow over an array of porous pipes were developed from the solution of the Navier–Stokes equations for the case of low wall Reynolds numbers. Libby et al. [6] investigated the flow development in a tube with injection of a light or heavy gas.

On the other hand, flow field and heat transfer at micro-scale have attracted an extensive research interest in recent years due to the rapid development of micro electro mechanical systems (MEMS), biomedical applications, innovative cooling techniques for integrated circuits and separation systems.

Modeling the fluid flow with heat and mass transfer for micro-devices is different from that of the macro-scale systems. The ratio of the mean free path to characteristic length known as Knudsen number, $\mathrm{Kn} = \lambda/L$, defines the region where the continuum assumption is valid and where it becomes no longer valid for the case of gases. For small values of Kn, the fluid behavior can be analyzed using the Navier–Stokes equations with no-slip flow boundary conditions. For values of Kn varying between 0.001 and 0.1, the regime is called slip flow regime [7]. However, for Kn higher than 0.1, the continuum description is expected to fail [8]. Other methods using molecular simulations such as the direct simulation Monte Carlo are used for such ranges of Kn numbers.

Several studies, mainly analytical and numerical ones, investigated the effect of rarefaction and temperature jump condition on the hydrodynamic and thermal fields inside ducts and tubes. The majority of these studies have been interested to solve the energy equation when the hydrodynamic flow is considered fully developed for both cases constant wall temperature (CWT) and constant wall heat flux [7,9]. It is of interest to mention the analytical work of Barron et al. [10] where they extended the original Graetz problem of thermally developing heat transfer in laminar flow through a circular tube to slip flow. Relationships for Knudsen numbers ranging from 0 to 0.12 were developed to describe the effect of slip flow on heat transfer coefficient.

The effect of slip flow at a membrane surface of water treatment or desalination systems was studied by few authors. Singh and Laurence focused on the effect of slip velocity at the membrane surface of an ultra-filtration unit on the concentration polarization for tube flow [11] and channel flow systems [12]. The solution of the momentum and the diffusion equations for a uniform permeation rate was obtained analytically using the perturbation method. Soundalgekar et al. [13] studied laminar slip flow through a uniform circular pipe with small suction, and significant impact of slip velocity on the hydrodynamic flow is shown. Recently, Ramon et al. [14] presented a two-dimensional, boundary layer model describing the heat transfer in the feed channel of a vacuum membrane distillation (VMD) module. The model allows for variations of viscosity with temperature and considers the effect of slip velocity over the liquid–gas interface.

2. Formulation

A simultaneously developing laminar flow in a micro-tube with uniform suction is investigated under slip velocity conditions. The main assumptions used in this study are as follows: steady state, axisymmetric, and constant fluid properties flow with negligible gravitational forces and negligible viscous dissipation. The suction velocity V_w is supposed uniform.

The coordinate system and geometry for the duct are shown in Fig. 1.

For a flow inside an isothermal micro-tube, the nondimensional governing equations and the corresponding boundary conditions are given as:

$$\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\bar{V}) + \frac{\partial}{\partial \bar{z}}(\bar{U}) = 0 \tag{1}$$

$$\left(\bar{V}\frac{\partial \bar{V}}{\partial \bar{r}} + \bar{U}\frac{\partial \bar{V}}{\partial \bar{z}}\right) = -\frac{\partial \bar{P}}{\partial \bar{r}} + \frac{1}{\mathrm{Re}}\left[\frac{\partial}{\partial \bar{r}}\left(\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\bar{V})\right) + \frac{\partial^2 \bar{V}}{\partial \bar{z}^2}\right] \tag{2}$$

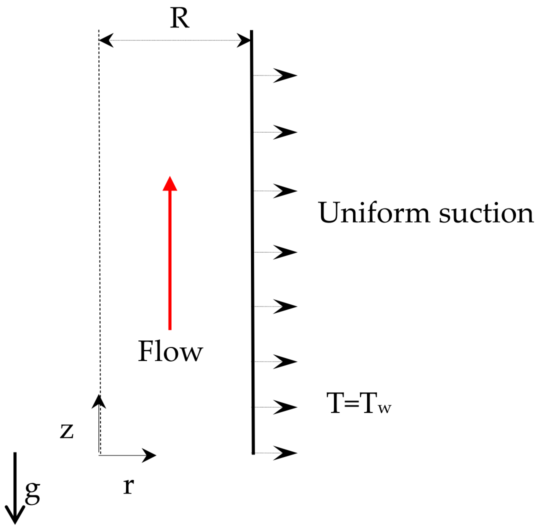

Fig. 1. Geometry and coordinate system of flow domain.

$$\left(\bar{V}\frac{\partial \bar{U}}{\partial \bar{r}} + \bar{U}\frac{\partial \bar{U}}{\partial \bar{z}}\right) = -\frac{\partial \bar{P}}{\partial \bar{z}} + \frac{1}{\mathrm{Re}}\left[\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial \bar{U}}{\partial \bar{r}}\right) + \frac{\partial^2 \bar{U}}{\partial \bar{z}^2}\right] \tag{3}$$

$$\left(\bar{V}\frac{\partial \bar{T}}{\partial \bar{r}} + \bar{U}\frac{\partial \bar{T}}{\partial \bar{z}}\right) = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left[\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial \bar{T}}{\partial \bar{r}}\right) + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right] \tag{4}$$

At the inlet,

$$\bar{z} = 0;\ \bar{U} = 1;\ \bar{V} = 0;\ \bar{T} = 1 \tag{5}$$

Symmetry condition at

$$\bar{r} = 0;\ \bar{V} = 0;\ \frac{\partial \bar{U}}{\partial \bar{r}} = 0;\ \frac{\partial \bar{T}}{\partial \bar{r}} = 0 \tag{6}$$

At the tube wall,

$$\bar{r} = 1;\ \bar{V} = V_{\mathrm{w}}/U_{\mathrm{in}} \tag{7a}$$

$$\bar{U} = -2\beta_{\mathrm{v}} Kn \left.\frac{\partial \bar{U}}{\partial \bar{r}}\right|_{\bar{r}=1} \tag{7b}$$

At the micro-tube outlet,

$$\bar{z} = L/R$$

$$\frac{\partial \bar{U}}{\partial \bar{z}} = 0;\ \frac{\partial \bar{V}}{\partial \bar{z}} = 0;\ \frac{\partial \bar{T}}{\partial \bar{z}} = 0 \tag{8}$$

where

$$\bar{r} = \frac{r}{R},\ \bar{z} = \frac{z}{R},\ \bar{V} = \frac{V}{U_{\mathrm{in}}},\ \bar{U} = \frac{U}{U_{\mathrm{in}}},\ \bar{P} = \frac{P}{\rho U_{\mathrm{in}}^2},\ \bar{T} = \frac{T - T_{\mathrm{w}}}{T_{\mathrm{in}} - T_{\mathrm{w}}} \tag{9}$$

and

$$\beta_{\mathrm{v}} = (2 - f_{\mathrm{v}})/f_{\mathrm{v}} \tag{10}$$

f_{v} is known as the tangential momentum accommodation coefficient. This parameter describes the gas–surface interaction and is function of the composition, temperature and pressure of the gas, the gas velocity over the surface, the solid surface temperature and roughness. Its value ranges from near 0 to 1. For most engineering applications, this coefficient is taken to be close to be unity [15,16]. In this work, β_{v} is considered as equal to unity.

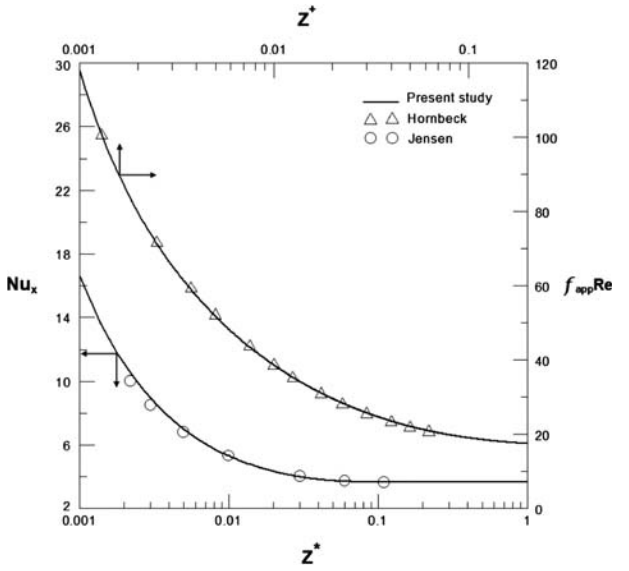

Fig. 2. Validation of the numerical model with the results of Jenson [19] and Hornbeck [20].

Eq. (7b) gives the first-order slip velocity condition as reported by Larrode et al. [17].

3. Numerical method and validation

The above set of 2D fully elliptic governing equations with the appropriate boundary conditions is solved numerically using the Control Volume Method and the *Simpler* algorithm [18]. Based on a grid dependence analysis, the number of nodes used for the simulations was chosen to be 1000 × 80 in the axial and radial directions, respectively. The proposed model was validated using the available data from the literature. In fact, the axial evolutions of the developing Nusselt number and friction factor corresponding to the continuum model ($\mathrm{Kn} = 0$) and for impermeable tube ($\mathrm{Re}_{\mathrm{w}} = 0$) were computed and compared with results provided by Jensen [19] and Hornbeck [20]. Fig. 2 shows an excellent agreement between our results and the previous results. In all of the performed validation tests, the agreements were found to be fairly good. Therefore, the numerical model is reliable and can be used for the analysis of confined forced convection in micro-tubes.

4. Results and discussion

In order to study the effects of rarefaction on the developing hydrodynamic and thermal fields of a laminar forced flow in permeable micro-tubes, several simulations were performed for fixed Prandtl number ($\mathrm{Pr} = 0.7$) and Reynolds number ($\mathrm{Re} = 100$). The length of the micro-tube equals to $100 \times R$.

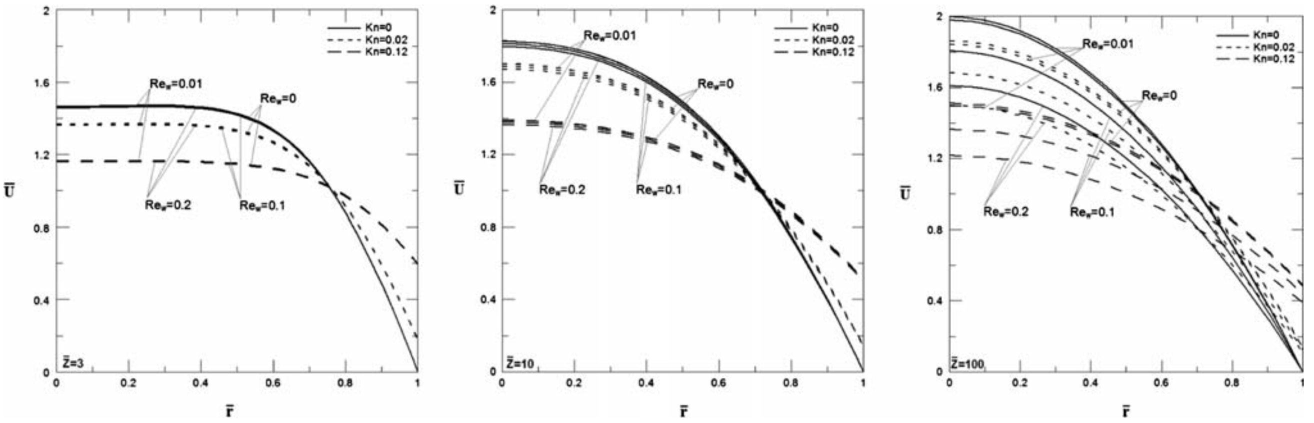

Fig. 3. Axial velocity evolutions at three axial positions and for different degrees of rarefaction and different suction Reynolds numbers.

The Knudsen number varies from 0 to 0.12, while the suction Reynolds number $\mathrm{Re_w}$ is kept small ($\mathrm{Re_w} \leq 0.5$).

Fig. 3 shows the axial velocity profiles as a function of $\mathrm{Re_w}$ and Kn for three axial positions, namely $z = 3\mathrm{R}$, $z = 10\mathrm{R}$ and $z = 100\mathrm{R}$. The profiles are parabolic with a maximum velocity located at the center of the tube as it is the case for the Poiseuille flow for an impermeable tube. One can see first that the effect of suction is very weak in the entrance region of the micro-tube. However, it becomes significant at the tube exit ($z = 100\mathrm{R}$). Increasing $\mathrm{Re_w}$ leads to a decrease in the maximum velocity. Small values of $\mathrm{Re_w}$ (<0.01) keep the axial velocity profiles almost unchanged.

On the other side and when the slip flow condition is applied (Kn is nonzero), the fluid particles adjacent to the solid surface of the tube wall no longer attain the velocity of the solid surface. In the core region of the tube, the fluid decelerates and its maximum velocity occurring at the centerline of the tube decreases significantly. The combined effect of suction and rarefaction leads to a more pronounced flattening of the axial velocity as shown at the micro-tube exit.

Fig. 4 presents the variation of the radial velocity profiles. These profiles that are independent of the axial position are affected by rarefaction. For the continuum case, the maximum radial velocity exceeds the imposed suction velocity (i.e. $V > V_\mathrm{w}$), while when increasing the Knudsen number, the maximum velocity decreases and reaches the unity at the wall surface.

Fig. 5 shows the evolution of the mean velocity along the tube. For an impermeable wall, U_m remains constant and equal to unity respecting the mass conservation principle. When suction occurs, the average velocity decreases linearly due to the imposed uniform suction effect.

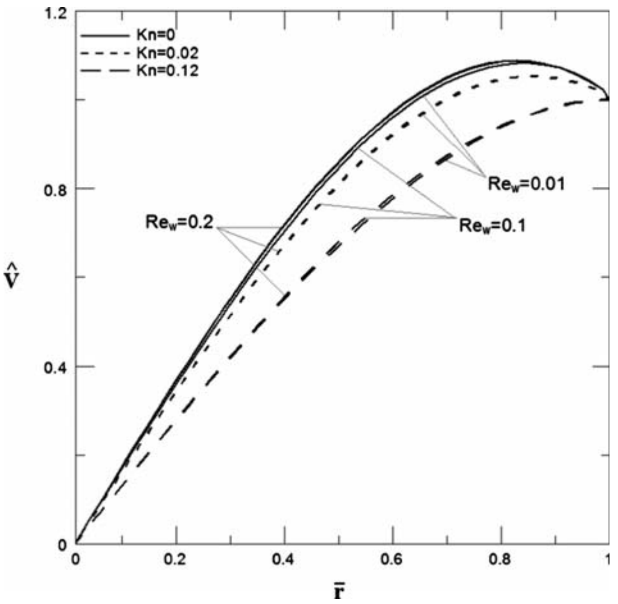

Fig. 4. Radial velocity evolution with suction and rarefaction effects.

Fig. 6 depicts the axial distribution of the friction coefficient where suction and rarefaction are taken into account. When a tangential slip velocity boundary condition is taken into consideration, a decrease in the apparent friction coefficient occurs along the tube. This decrease is very significant at the tube entrance and leads to almost a uniform distribution of this coefficient along the micro-tube length. This phenomenon is a direct consequence of the flattening of the velocity profiles observed previously. The particular case of $\mathrm{Kn} = 0.12$ is of interest since the friction coefficient is almost constant and lower to the well-known value of 16 corresponding to the Poiseuille flow.

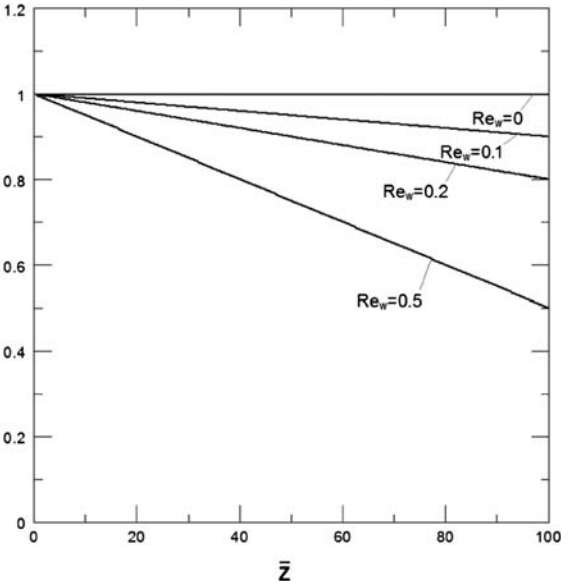

Fig. 5. Mean axial velocity evolution along the micro-tube.

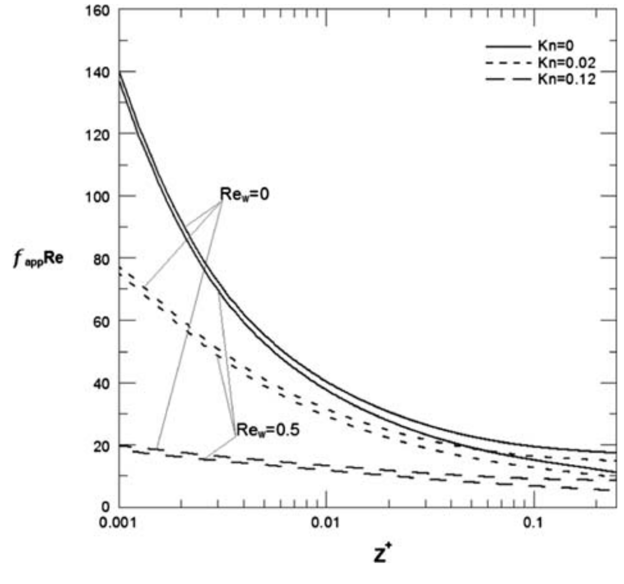

Fig. 6. Axial variation of the apparent friction coefficient for different rarefaction degrees.

One can also observe that the effect of Re_w is weak particularly in the entrance region of the tube.

Fig. 7 presents the local Nusselt number distribution along the micro-tube for the case of slip flow and uniform suction. Here again, the effect of Re_w is very weak while that of Kn is significant. The heat transfer coefficient is shown to be reduced as Kn increases.

Fig. 8 depicts the evolution of the fully developed Nusselt number as a function of Re_w for different rarefaction degrees. Two Prandtl number values are considered ($Pr = 0.5$ and $Pr = 1.0$). The results of Kinney [4] corresponding to the continuum case are shown and are in a good agreement with the present results. For the case of a continuum model ($Kn = 0$) and $Re_w = 0$ (impermeable tube wall), the Nusselt number corresponds to the constant and well known

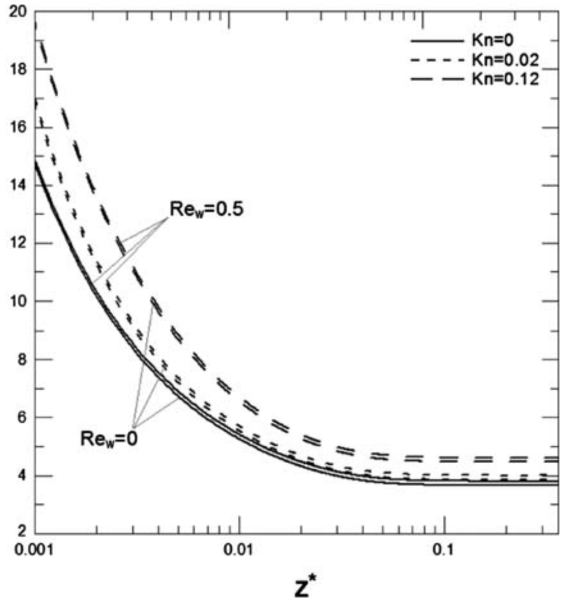

Fig. 7. Axial distribution of local Nusselt number with suction and rarefaction effects.

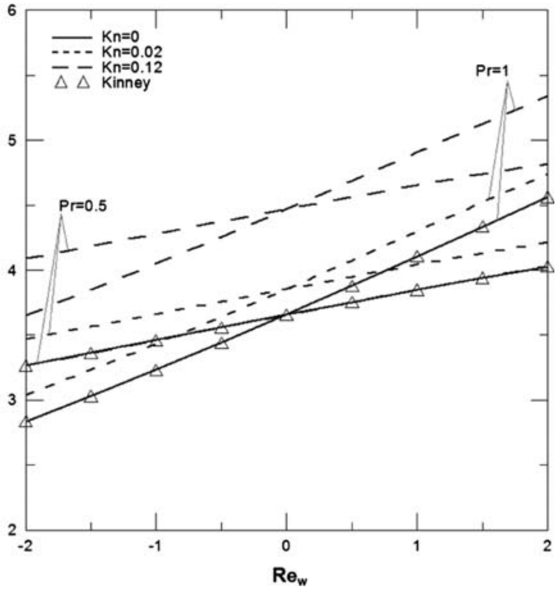

Fig. 8. Effect of the rarefaction, suction/injection, and Prandtl number on the fully developed Nusselt number.

value of 3.66. When increasing Pr, the heat transfer coefficient increases with suction and decreases with injection.

As observed in Fig. 7, the rarefaction enhances the heat transfer mechanism. Besides and for a fixed Pr, mass suction induces an increase in the fully developed Nusselt, number but when considering the mass injection, the opposite effect occurs and increasing absolute value of $\mathrm{Re_w}$ leads to a decrease in the fully developed number. For very small injection or suction rates, no significant variation on the heat transfer coefficient is observed.

5. Conclusion

A numerical study for a steady-state, two-dimensional, laminar convective heat transfer in a micro-tube is conducted using first-order slip velocity conditions. The wall of the micro-tube is considered permeable allowing the analysis of the injection and the suction effects. Results for a simultaneously developing flow in a micro-tube for the constant wall temperature boundary condition and the case of uniform suction rates are presented and analyzed. These results are expressed in terms of velocity profiles and axial variation of the friction factor as well as that of the Nusselt number. The main findings of this study can be summarized as following:

(1) The hydrodynamic flow was found to be dependent on the degree of rarefaction represented by the Knudsen number, Kn. Increasing Kn reduces the axial velocity of the tube center, increases the streamwise fluid velocity at the wall, and induces a flattening of the velocity profiles.
(2) The effect of suction is very weak in the entrance region of the micro-tube. However, it becomes significant at the tube exit ($z = 100\mathrm{R}$). Increasing $\mathrm{Re_w}$ leads to a decrease in the maximum velocity of the fluid.
(3) Increasing $\mathrm{Re_w}$ has a small influence on the overall quantities, that is, the friction factor and the Nusselt number.

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Symbols

c_p	specific heat at constant pressure
D	micro-tube diameter
fv	tangential momentum accommodation
f_{app}	apparent friction coefficient
h	convective heat transfer coefficient
k	thermal conductivity
Kn	Knudsen number, λ/D
L	micro-tube length
Nu_x	local Nusselt number, h_xD/k
Nu_∞	fully developed Nusselt
P	pressure
Pr	Prandtl number, $\mu c_p/k$
r	radial coordinate
R	micro-tube radius
Re	Reynolds number, $U_{\mathrm{in}}\ \rho R/\mu$
$\mathrm{Re_w}$	Reynolds wall, $V_{\mathrm{w}}\ \rho D/\mu$
T	temperature
U	velocity component in the z-direction, U/U_{in}
V	velocity component in the r-direction
$\breve{V}$	V/V_{w}
z	axial coordinate
z^*	dimensionless axial coordinate, $z/2D\mathrm{RePr}$
z^+	dimensionless axial coordinate, $z/2D\mathrm{Re}$

Greek symbols

β_{v}	$(2 - f_{\mathrm{v}})/f_{\mathrm{v}}$
λ	gas mean free path
μ	dynamic viscosity
ρ	fluid density

Subscripts

in	inlet
m	mean
w	wall

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