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# MHD mixed convection of a viscous dissipating fluid about a vertical slender cylinder

## Orhan Aydin<sup>a</sup>, Ahmet Kaya<sup>b,\*</sup>

<sup>a</sup>Department of Mechanical Engineering, Karadeniz Technical University, 61080 Trabzon, Turkey <sup>b</sup>Department of Mechanical Engineering, Kahramanmaras Sutcu Imam University, 46100 Kahramanmaras, Turkey Tel: +90 344 219 1685; Fax: +90 344 219 1052; email: ekaya38@gmail.com

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#### ABSTRACT

The problem of steady laminar magnetohydrodynamic (MHD) mixed convection heat transfer about a heated/cooled vertical slender cylinder is studied numerically, taking into account the effects of transverse curvature and viscous dissipation. A uniform magnetic field is applied perpendicular to the cylinder. The resulting governing equations are transformed into the nonsimilar boundary layer equations and solved using the Keller box method. The velocity and temperature profiles as well as the local skin friction and the local heat transfer parameters are determined for different values of the governing parameters, mainly the magnetic parameter, the Richardson number, and the Eckert number. For some specific values of the governing parameters, the results agree very well with those available in the literature. Generally, it is determined that the local skin friction coefficient and the local heat transfer coefficient increase, increasing the Richardson number and the magnetic parameter Mn or decreasing the Eckert number.

Keywords: Vertical slender cylinder; Buoyancy effect; MHD flow; Viscous dissipation

## 1. Introduction

Mixed convection flow along vertical cylinders is important in situations encountered in the areas of geothermal power generation and drilling operation when free stream velocity and induced buoyancy velocity are of comparable order and received much less attention. Chen and Mucoglu [1] analyzed the buoyancy and transverse curvature effects on forced convection of Newtonian fluid flow along an isothermal vertical cylinder using the local nonsimilarity method. The same problem for a uniform surface heat flux case was conducted by Mucoglu and Chen [2]. Lee et al. [3] studied the problem of mixed convection along a vertical cylinder with uniform surface heat

flux for the entire mixed convection regime, ranging from pure forced convection to pure free convection by employing the buoyancy and curvature parameters. Hossain and Alim [4] studied a free convection flow of an optically dense viscous incompressible fluid along a vertical thin circular cylinder with effect of radiation. Takhar and Nath [5] investigated the unsteady flow and heat transfer of a viscous incompressible electrically conducting fluid in the forward stagnation point region of a rotating sphere in the presence of a magnetic field. Takhar et al. [6] studied the mixed convection flow over a continuous moving vertical slender cylinder under the combined buoyancy effect of thermal and mass diffusion. El-Amin [7] studied the effects of both first- and second-order resistance, Joule heating, and viscous dissipation on

<sup>\*</sup>Corresponding author.

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forced convection flow from a horizontal circular cylinder under the action of a transverse magnetic field. Kumari and Jayanthi [8] obtained steady state solutions for nonDarcy free convection flow along a horizontal cylinder in nonNewtonian fluid saturated porous medium. Kumari and Nath [9] studied the effects of localized cooling/heating and injection/suction on the mixed convection flow on a thin vertical cylinder. Chang [10] numerically investigated the flow and heat transfer characteristics of natural convection in a micropolar fluid flowing along a vertical slender hollow circular cylinder with conduction effects. Datta et al. [11] obtained the nonsimilar solution of a steady laminar-forced convection boundary layer flow over a horizontal slender cylinder including the effect of nonuniform slot injection (suction). The effects of transverse curvature and viscous dissipation are also included in the analysis. Roy et al. [12] developed general analysis for the influence of nonuniform double slot injection (suction) on the steady nonsimilar incompressible laminar boundary layer flow over a slender cylinder. Anwar et al. [13] numerically studied the steady mixed convection boundary layer flow of a viscoelastic fluid over a horizontal circular cylinder in a stream flowing vertically upwards for both cases of heated and cooled cylinders. Singh et al. [14] studied unsteady mixed convection flow over a rotating vertical slender cylinder under the combined effects of buoyancy force and thermal diffusion with injection/ suction where the slender cylinder is inline with the flow. The effect of surface curvature is also taken into account, especially for the applications such as wire and fiber drawing, where accurate predictions are desired. Molla et al. [15] investigated natural convection boundary layer laminar flow from a horizontal circular cylinder with uniform heat flux in presence of heat generation. Patil et al. [16] investigated effects of buoyancy force and thermal diffusion in presence of surface mass transfer unsteady mixed convection boundary layer flow over a permeable nonlinearly stretching vertical slender cylinder. Patil and Pop [17] studied the combined effects of buoyancy force and thermal diffusion on an unsteady mixed convection flow of a viscous incompressible fluid over a nonpermeable linear stretching vertical slender cylinder. They assumed that the slender cylinder was in line with the flow. The unsteadiness in the flow and temperature fields was caused due to the impulsive change in the wall velocity and wall temperature of linearly stretching vertical slender cylinder. Patil et al. [18,19] investigated effects of chemical reaction and viscous dissipation on unsteady mixed convection flow from a moving vertical slender cylinder.

However, the study of magnetohydrodynamic viscous flow has important industrial, technological, and geothermal applications such as cooling of nuclear reactors, liquid metal fluids, MHD accelerators, and power generation systems. Nasser and Hassan [20] have been investigated to thermal radiation effect on magnetohydrodynamic (MHD) unsteady mixed convection flow over a moving vertical cylinder with constant heat flux through a porous medium in the presence of transversal uniform magnetic field. El-Amin [7] has been studied the effects of Joule heating and viscous dissipation on forced convection flow from a horizontal circular cylinder under the action of a transverse magnetic field. Ishak et al. [21] have investigated a steady two-dimensional flow of an electrically conducting incompressible fluid due to a stretching cylindrical. Similarity solutions were obtained for a linearly stretching tube with a constant surface temperature.

In the present paper, the specific problem selected for study is the flow and heat transfer in an electrically conducting fluid adjacent to an isothermal vertical slender cylinder, with viscous dissipation and joule heating effects accounted for. The effects of mixed convection (aiding and opposing flow conditions), viscous dissipation, and joule heating on MHD mixed convection flow over a vertical slender cylinder with uniform surface temperature has been analyzed. The boundary layer equations governing the flow are reduced to local nonsimilarity equations which are solved using the implicit finite difference method (Keller box). Numerical results for the velocity and temperature profiles as well as local skin friction and local heat transfer parameters are presented.

## 2. Analysis

Consider the steady, incompressible, laminar, two-dimensional, boundary layer flow with viscous dissipation over a vertical slender cylinder of length L and outer radius  $r_o$  ( $L \gg r_o$ ). The physical model and coordinate system are shown in Fig. 1. The gravitational acceleration, g, acts in the downward direction. The temperature and velocity at a distance remote from the cylinder are given by  $T_{\infty}$  and  $u_{\infty}$ , respectively, and the body has a uniform temperature  $T_w$  ( $T_w > \text{or} < T_{\infty}$ , i. e. the cylinder is either heated or cooled). A uniform magnetic field is assumed to apply in the *r*-direction causing a flow resistive force in the x-direction. It is assumed that the induced magnetic field, the external or imposed electric field, and the electric field due to the polarization of charges (i.e. Hall effect) are negligible. The plate is considered to be electrically nonconduct. Under foregoing assumptions and taking into account the Boussinesq approximation and the



Fig. 1. The schematic of the problem.

boundary layer approximation, the system of continuity, momentum, and energy equations can be written:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \left(\frac{v}{r}\right)\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \mp g\beta(T - T_{\infty}) - \frac{\sigma B_0^2}{\rho}(u - u_{\infty})$$
(2)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \left(\frac{v}{\Pr}\right)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \left(\frac{\mu}{\rho C_p}\right)\left(\frac{\partial u}{\partial r}\right)^2 + \frac{\sigma B_0^2}{\rho C_p}(u - u_\infty)^2$$
(3)

Here *u* and *v* are the velocity components in the *x* and *r* direction, respectively, *T* is the temperature of the fluid,  $\beta$  is the coefficient of thermal expansion, *v* is the kinematic viscosity,  $\rho$  is the fluid density, *g* is the acceleration due to gravity, and *B*<sub>0</sub> is the magnetic flux density.

The appropriate boundary conditions for the velocity and temperature of this problem are:

$$\begin{array}{ll} r = r_0; & u = v = 0, & T = T_w \\ r \to \infty; & u \to u_\infty, & T \to T_\infty \end{array}$$

$$(4)$$

To seek a solution, the following dimensionless variables are introduced:

$$\begin{aligned} \xi &= \left(\frac{x}{r_0}\right), \quad \eta = \left[\frac{r^2 - r_0^2}{2r_0}\right] \left(\frac{u_\infty}{vx}\right)^{1/2}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \\ \psi(x, r) &= r_0 (vxu_\infty)^{1/2} f(\xi, \eta), \quad \frac{r^2}{r_0^2} = [1 + \lambda\eta], \end{aligned} \tag{5}$$
$$\lambda &= 2 \left(\frac{\xi}{\text{Re}_x}\right)^{1/2} \end{aligned}$$

where  $\psi(x, r)$  is the free stream function that satisfies Eq. (1) with  $u = \left(\frac{1}{r}\right)\frac{\partial \psi}{\partial r}$  and  $v = -\left(\frac{1}{r}\right)\frac{\partial \psi}{\partial x}$ .

In terms of these new variables, the velocity components can be expressed as:

$$u = u_{\infty}f', \quad v = -\left(\frac{r_0}{r}\right)\left(\frac{u_{\infty}}{vx}\right)^{1/2}\left[\frac{1}{2}f + \xi\frac{\partial f}{\partial\xi} - \frac{\eta}{2}f'\right] \tag{6}$$

The transformed momentum and energy equations together with the boundary conditions, Eqs. (2)–(4) 234, can be written as

$$(1 + \lambda \eta)f''' + \lambda f'' + \frac{1}{2}ff'' \mp Ri\theta - Mn\xi \left[\frac{1}{2}f' - 1\right]$$
$$= \xi \left[f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right]$$
(7)

$$\frac{1}{\Pr}(1+\lambda\eta)\theta'' + \frac{\lambda}{\Pr}\theta' + \frac{1}{2}f\theta' + \operatorname{Ec}(1+\lambda\eta)(f'')^{2} + \operatorname{EcMn}\xi[f'-1]^{2} = \xi\left[f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\right]$$
(8)

with the boundary conditions:

$$\eta = 0; \quad f + \xi \frac{\partial f}{\partial \xi} = 0, \quad f' = 0, \quad \theta = 1$$
  
$$\eta \to \infty; \quad f' = 1, \quad \theta = 0$$
(9)

The corresponding dimensionless groups that appeared in the governing equations defined as:

$$Pr = \frac{v}{\alpha}, \quad Ri = \frac{Gr}{Re^2}, \quad Gr = \frac{g\beta(T_w - T_\infty)r_0^3}{v^2},$$
$$Re = \frac{u_\infty r_0}{v}, \quad Mn = \frac{Ha}{Re}, \quad Ha = \frac{\sigma B_0^2 r_0^2}{\mu},$$
$$Ec = \frac{u_\infty^2}{c_p(T_w - T_\infty)}$$
(10)

where Pr is the Prandtl number, Ri is the Richardson number (Ri > 0; aiding flow, Ri < 0; opposing flow), Gr is the Grashof number, Re is the Reynolds number, Mn is the magnetic parameter, Ha is the Hartman number, and Ec is the Eccert number.

## 3. Numerical solution

The system of transformed equations together with the boundary conditions, Eqs. (7)–(9), have been solved numerically using the Keller box scheme, an efficient and accurate finite-difference scheme, similar to that described in Cebeci and Bradshaw [22]. For the sake of brevity, details of the numerical method are not described, referring the reader to Cebeci and Bradshaw [22]. This is a very popular implicit scheme, which demonstrates the ability to solve systems of differential equations of any order as well as featuring second-order accuracy (which can be realized with arbitrary nonuniform spacing), allowing very rapid x or  $\xi$  variations [22,23].

A set of nonlinear finite-difference algebraic equations derived are then solved by using the Newton quazi-linearization method. The same methodology as that followed by Takhar and Beg [23] is followed. Therefore, for the finite-difference forms of the equations, we have referred the reader to Takhar and Beg [23] for the brevity of the article.

In the calculations, a uniform grid of the step size 0.01 in the  $\eta$ -direction and a nonuniform grid in the  $\xi$ -direction with a starting step size 0.001 and an increase in 0.1 times the previous step size were found to be satisfactory in obtaining sufficient accuracy within a tolerance better than  $10^{-6}$  in nearly all cases. The value of  $\eta_{\infty} = 16$  is shown to satisfy the velocity to reach the relevant stream velocity.

In order to verify the accuracy of the present method, the present results were compared with those of Chen and Mucoglu [1], Takhar et al. [6], Chang [10], Datta et al. [11] and Singh et al. [14]. The comparisons are found to be in good agreement as shown in Tables 1–3.

## 4. Results and discussion

In this study, the effects of transverse curvature and viscous dissipation on MHD mixed convection are examined. The following ranges of the main parameters are considered: Ri = -0.5, -0.1, 0.0, 0.1, 1.0, and 10.0; Pr = 1.0, Ec = -0.2, -0.1, 0.0, 0.1, and 0.2 and magnetic interaction parameter Mn = 0.0, 1.0, 2.0, and 3.0. The combined effects of Ri, Ec, and Mn on the momentum and heat transfer are analyzed.

The Richardson number, Ri represents a measure of the effect of the buoyancy in comparison with that of the inertia of the external forced or free stream flow on the heat and fluid flow. Outside the mixed convection region, either the pure forced convection or the free convection analysis can be used to describe accurately the flow or the temperature field. Forced convection is the dominant mode of transport when  $\text{Ri} \rightarrow 0$ , whereas free convection is the dominant mode when  $\text{Ri} \rightarrow \infty$ . Buoyancy forces can enhance the surface heat transfer rate when they assist the forced convection [24]. The Eckert number, Ec represents the relative importance of viscous dissipation to thermal

$\xi = \frac{4}{r_0} \left(\frac{vx}{u_\infty}\right)^{1/2}$	Chen and Mucoglu [1]		Chang [10]		Present results	
	$f''(\xi,0)$	$- heta'(\xi,0)$	$f''(\xi,0)$	$- heta'(\xi,0)$	$f''(\xi,0)$	$- heta'(\xi,0)$
0.0	1.3282	0.5854	1.3280	0.5852	1.3281	0.5856
1.0	1.9172	0.8669	1.9133	0.8658	1.9134	0.8659
2.0	2.3981	1.0986	2.3900	1.0940	2.3922	1.0958
3.0	2.8270	1.3021	2.8159	1.2982	2.8198	1.2988
4.0	3.2235	1.4921	3.2187	1.4925	3.2212	1.4918

Comparison of the local skin friction and heat transfer rates with Pr = 0.7, Ec = 0.0, Mn = 0.0, and Ri = 0.0

Table 2

Table 1

Comparision of the local skin friction and local heat transfer rates with Pr = 0.7, Ec = 0.1, Mn = 0.0, and Ri = 0.0

$\xi = \frac{4}{(vx)} \int_{1/2}^{1/2} dx$	Datta et al. [11]		Present results	
$r_0(u_{\infty})$	$\overline{C_f \operatorname{Re}_x^{1/2}}$	$Nu \operatorname{Re}_{x}^{-1/2}$	$\overline{C_f \operatorname{Re}_x^{1/2}}$	$Nu \operatorname{Re}_{x}^{-1/2}$
0.0	0.6511	0.2523	0.6537	0.2538
0.5	0.8202	0.3023	0.8241	0.3042
1.0	0.9614	0.3543	0.9673	0.3555
1.5	1.1012	0.4123	1.1044	0.4153
2.0	1.2321	0.4601	1.2409	0.4658
2.5	1.3410	0.5076	1.3483	0.5103
3.0	1.4432	0.5520	1.4497	0.5544

$\overline{\xi} = \frac{4}{r_0} \left(\frac{vx}{u_\infty}\right)^{1/2}$	Ri	Chen and Mucoglu [1]		Takhar et al. [6]		Singh et al. [14]		Present results	
		$f''(\xi,0)$	$- heta'(\xi,0)$	$f''(\xi,0)$	$- heta'(\xi,0)$	$f''(\xi,0)$	$- heta'(\xi,0)$	$f''(\xi,0)$	$- heta'(\xi,0)$
0	0	1.3282	0.5854	1.3281	0.5854	1.3281	0.5854	1.3281	0.5856
0	1	4.9666	0.8221	4.9663	0.8219	4.9665	0.8221	4.9664	0.8216
0	2	7.7126	0.9305	7.7119	0.9302	7.7124	0.9303	7.7122	0.9305
1	0	1.9172	0.8669	1.9167	0.8666	1.9168	0.8668	1.9134	0.8659
1	1	5.2584	1.0621	5.2578	1.0617	5.2580	1.0618	5.2581	1.0619
1	2	7.8871	1.1690	7.8863	1.1685	7.8870	1.1692	7.8868	1.1688

(a)

3.5

3.0

2.5

Table 3 Comparison of the local skin friction and heat transfer rates with Pr = 0.7, Ec = 0.0, and Mn = 0.0

diffusion. Viscous dissipation plays a role like an energy source and therefore it affects the temperature profile [25].

Fig. 2 shows the dimensionless velocity (a) and temperature (b) profiles inside the boundary layer for

different values of the magnetic parameter Mn. The increasing of the magnetic parameter Mn increases the momentum and the temperature boundary layer thickness (i.e. decreases velocity and temperature gradients at the wall). Both the local skin friction and

> Pr=1.0 Ri=0.1

Ec=0.0

Mn=0.0

Mn=1.0

- Mn=2.0 - Mn=3.0



f"(ξ, 0.0) 2.0 1.5 1.0 0.5 0.0 0.0 0.5 1.0 1.5 2.0 2.5 . 3.0 ξ (b) 0.60 Mn=0.0 Pr=1.0 ----- Mn=1.0 Ri=0.1 0.55 Mn=2.0  $E_{c=0}$ Mn=3.0 0.50 -0'(ξ, 0.0) 0.45 0.40 0.35 0.30 0.0 0.5 1.0 1.5 2.0 2.5 3.0

Fig. 2. Dimensionless velocity (a) and temperature (b) profiles for different Mn at Ri = 0.1, Mn = 0.0, Ec = 0.1, Pr = 1.0, and  $\xi$  = 1.0.

Fig. 3. Effect of Mn on the local skin friction (a) and local heat transfer (b) parameters against  $\xi$  at Ri = 0.1, Ec = 0.1, and Pr = 1.0.

ξ

the local heat transfer parameters decrease with the magnetic parameter Mn (Fig. 3).

The effect of the mixed convection parameter Ri on the velocity and temperature profiles for aiding and opposing flow case are shown in Fig. 4. Increasing the Ri (-0.5 to 10) decreases the thermal and momentum boundary layer thickness (i.e. increases the velocity and temperature gradients at the wall as shown in Fig. 5).

For a positive value of Ec, since the temperature of the wall is greater than that of the free stream, there will be a heat transfer from the wall to the fluid. The viscous dissipation will cause to a heat generation inside the fluid, which results in the increase of the temperature distribution in the flow region. This is due to the fact that heat energy is stored in the fluid due to frictional heating. Due to the increased bulk fluid temperature, the temperature gradient will decrease. For a negative value of Ec, since  $T_w < T_{\infty}$ , the viscous dissipation will increase the temperature distribution in the flow region more. Finally, this leads to an increased temperature gradient, which will result in increased heat transfer values. The effect of the viscous dissipation parameter Ec on the dimensionless velocity and temperature profiles is given in Fig. 6. The momentum boundary layer decrease and thermal boundary layers increase with Ec [24].

Fig. 7 shows the variation of local skin friction and the local heat transfer parameter, with the Eckert number at Ri = 1.0, Pr = 1.0, and Mn = 0.0. As well known, viscous dissipation behaves like a heat generation source inside the fluid. As expected, increasing Ec increases local skin friction and decreasing local heat transfer parameters.

Fig. 4. Dimensionless velocity (a) and temperature (b) profiles for different Ri at Mn = 1.0, Ec = 0.0, Pr = 1.0, and  $\xi = 1.0$ .

Fig. 5. Effects of Ri on the local skin friction (a) and local heat transfer (b) parameters against  $\xi$  at Mn = 1.0, Ec = 0.0, and Pr = 1.0.







Fig. 6. Dimensionless velocity (a) and temperature (b) profiles for different Ec at Mn = 1.0, Ri = 1.0, Pr = 1.0, and  $\xi = 1.0$ .

## 5. Conclusions

In this article, we have studied numerically the effects of viscous dissipation and magnetic parameters on a steady MHD mixed convective flows about a vertical slender cylinder. A transformed set of nonsimilar equations have been solved using the Keller box scheme. From the present numerical investigation, the following conclusions can be drawn:

- (1) An increase in the magnetic and viscous dissipation parameters decrease the local skin friction and local heat transfer parameters.
- (2) Increasing the buoyancy parameter (both aiding and opposing case) increases the local skin friction and local heat transfer parameters.



Fig. 7. Effects of Ec on the local skin friction (a) and local heat transfer (b) parameters against  $\xi$  at Mn = 1.0, Ri = 1.0, and Pr = 1.0.

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## Nomenclature

$c_p$	specific heat of the convective fluid
Gr	Grashof number
Ec	Eckert number

- *f* dimensionless stream function
- Ha Hartman number
- Mn magnetic parameter
- Pr Prandtl number
- Re Reynolds number

- Ri Richardson number
- T temperature
- *u*, *v* velocities in *x* and *r* directions, respectively
- *x*, *r* coordinates in axial and radial directions, respectively

#### Greek symbols

- $\eta$  similarity variable
- $\xi$  dimensionless streamwise coordinate
- $B_0$  magnetic flux density
- $\sigma$  electrical conductivity of the fluid
- $\rho$  fluid density
- $\mu$  dynamic viscosity
- v kinematic viscosity
- $\theta$  dimensionless temperature profile in Eq. (5)

## Subscripts

- w wall
- $\infty$  free stream

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