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A novel method to estimate the specific gravity and refractive index of seawater

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ABSTRACT

Seawater is described by a number of physical and chemical parameters that are useful in measurement and analysis of materiel effects. The material dissolved in seawater will not only affect its specific gravity, but also its optical properties, or rather, the degree to which light is refracted as it passes through the sample of water. The specific gravity and refractive index of seawater are related directly to salinity and temperature. In this work, an attempt has been made to develop simple predictive tools to estimate specific gravity and refractive index of seawater as a function of salinity and temperature. Estimations are found to be in excellent agreement with reported data in the literature with average absolute deviation being less than 0.2%.The predictive tool developed in this study can be of immense practical value for engineers to have a quick estimate on the specific gravity and refractive index of seawater without opting for any experimental trials. In particular, process and water treatment engineers would find the proposed method to be user-friendly with transparent calculations involving no complex expressions.

Keywords: Predictive tool; Seawater; Salinity; Refractive index; Specific gravity anomaly

1. Introduction

Oceans and seas act as a large repository for all wastes, solid and dissolved, from the continents and the atmosphere, and all of these salts are by-products of the weathering of crustal rocks and are continuously being added into the ocean [1–5].

Salinity is a characteristic of seawater and is defined as the ratio of the weight of the dissolved salts to the weight of the sample of water. The salinity varies somewhat from area to area [6–11].

Surface seawater is relatively fresh for considerable distances off the mouths of large rivers, while the highest surface salinities are found in semi-isolated areas in low latitudes (such as the Red Sea) where excessive evaporation takes place [5]. Salt water is also found in land-locked lakes and seas; in fact, all natural waters are salty to a greater or lesser extent. The properties of interest in this discussion are primarily those associated with salt in solution,

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wherein it ionizes and provides a vehicle for the transport of electric charge.

The material dissolved in water will not only affect its specific gravity, but also its optical properties, or rather, the degree to which light is refracted as it passes through the sample of water. The more material dissolved in a parcel of water (the higher its salinity) the greater the degree of refraction. The refractive index of seawater can exploit this phenomenon as an advantage by using it to measure the salinity of the sample [3–7].

A small change in density will correlate with a rather large change in a sample's salinity. It is therefore helpful to present small changes in specific gravity in a more readable way.

Since all specific gravity values will begin with the number one and will only show changes in the hundredth or thousandth decimal, the value for specific gravity is commonly presented as a specific gravity anomaly which can be calculated as follows:

$$\text{Specific gravity anomaly } (\mathrm{SG_A}) = (\text{Specific gravity} - 1.0) \times 10^5 \tag{1}$$

Thus, a sample with a specific gravity of 1.025 will have a specific gravity anomaly of:

$$\text{Specific gravity anomaly } (\mathrm{SG_A}) = (1.025 - 1.0) \times 10^5 = 2{,}500 \tag{2}$$

In view of the above, in this work an attempt has been made to develop accurate and simple correlations to estimate refractive index and specific gravity anomaly as a function of salinity and temperature.

This paper discusses the formulation of such a predictive tool in a systematic manner to show the simplicity of the model and usefulness of such a tool. The proposed method leads to well-behaved (i.e. smooth and non-oscillatory) equations enabling accurate and non-oscillatory predictions and this is the distinct advantage of the proposed method.

2. Development of correlation

The primary purpose of the present study is to accurately estimate specific gravity (SG) and refractive index (RI) of seawater as a function of total dissolved solid (TDS) and temperature (T) using Vandermonde matrix [12–14]. Vandermonde matrix id used to interpolate among experimentally obtained data points.

$$\mathrm{RI} = f(\mathrm{TDS}, T) \tag{3}$$

$$\mathrm{SG} = f(\mathrm{TDS}, T) \tag{4}$$

The required data [15] to develop this correlation include salinity and temperature (K). The following methodology has been applied to develop this predictive tool [16,17] using Matlab [18].

Firstly specific gravity and refractive indexes of seawater data [15] are correlated as a function of salinity, then calculated coefficients for these

Table 1
Tuned coefficients

Coefficient	Values for salinity in Eqs. (6–9)	Values for refractive index in Eqs. (11)–(14)
A_1	$-1.32364692719 \times 10^5$	$3.04185089235 \times 10^4$
B_1	$1.23507638737 \times 10^3$	$-3.32174425485 \times 10^2$
C_1	-3.74149992895	1.35004509865
D_1	$3.63834248184 \times 10^{-3}$	$-1.81639354990 \times 10^{-3}$
A_2	$7.96883129939 \times 10^3$	$-1.06383575763 \times 10^4$
B_2	$-8.04835938493 \times 10^{-1}$	$1.12407179180 \times 10^2$
C_2	$2.74059026321 \times 10^{-1}$	$-3.94607814988 \times 10^{-1}$
D_2	$-3.11609157690 \times 10^{-4}$	$4.61107682989 \times 10^{-4}$
A_3	$-4.13444127989 \times 10^{-2}$	$5.33161746339 \times 10^2$
B_3	4.29346598849	-5.58010386854
C_3	$-1.48537207520 \times 10^{-2}$	$1.94479528410 \times 10^{-2}$
D_3	$1.71179978257 \times 10^{-5}$	$-2.25732455561 \times 10^{-5}$
A_4	5.75987790050	-7.54853816695
B_4	$-5.98321497746 \times 10^{-2}$	$7.88619753037 \times 10^{-2}$
C_4	$2.07039642448 \times 10^{-4}$	$-2.74357732293 \times 10^{-4}$
D_4	$-2.38626415367 \times 10^{-7}$	$3.17866166738 \times 10^{-7}$

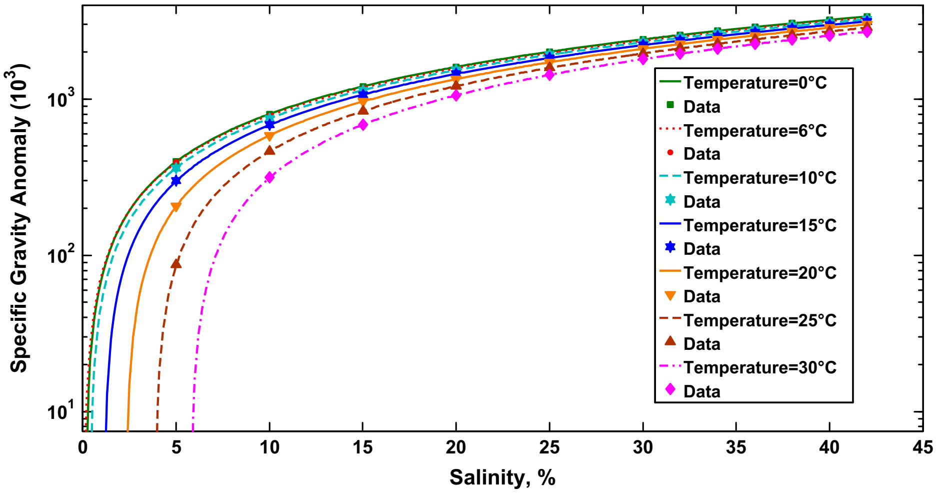

Fig. 1. The results of predictive tool for the estimation of seawater specific gravity anomaly in comparison with data [15].

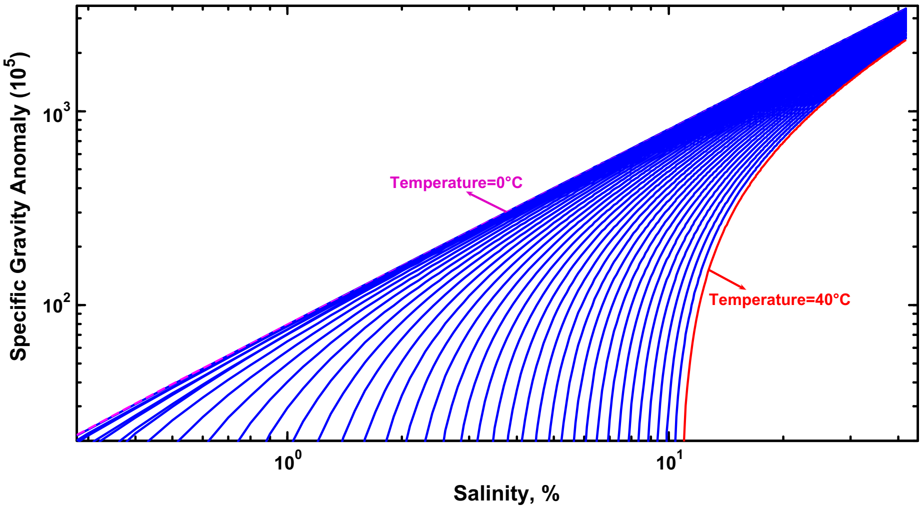

Fig. 2. The smoothness of results of predictive tool for the estimation of specific gravity anomaly.

equations are correlated as a function of temperature. The derived equations are applied to calculate new coefficients for Eq. (5) to calculate specific gravity and refractive index of seawater as a function of salinity and temperature. Table 1 shows the tuned coefficients for Eqs. (6)–(9) for predicting specific gravity and refractive indexes of seawater. In brief, the following steps are repeated to tune the correlation's coefficients using Matlab[18] for specific gravity of seawater:

(1) Correlate seawater specific gravity data as a function of salinity for a given temperature value.
(2) Repeat step 1 for other temperature data.
(3) Correlate corresponding polynomial coefficients, which were obtained for specific gravity data versus temperature for a given salinity value. $a = f(T)$, $b = f(T)$, $c = f(T)$, $d = f(T)$ [see Eqs. (6)–(9)].

Eq. (5) represents the proposed governing equation in which four coefficients are used to correlate seawater specific gravity anomaly data as a function of salinity and temperature where the relevant coefficients have been reported in Table 1.

$$(SG_A) = a + bS + cS^2 + dS^3 \tag{5}$$

where

$$a = A_1 + B_1T + C_1T^2 + D_1T^3 \tag{6}$$

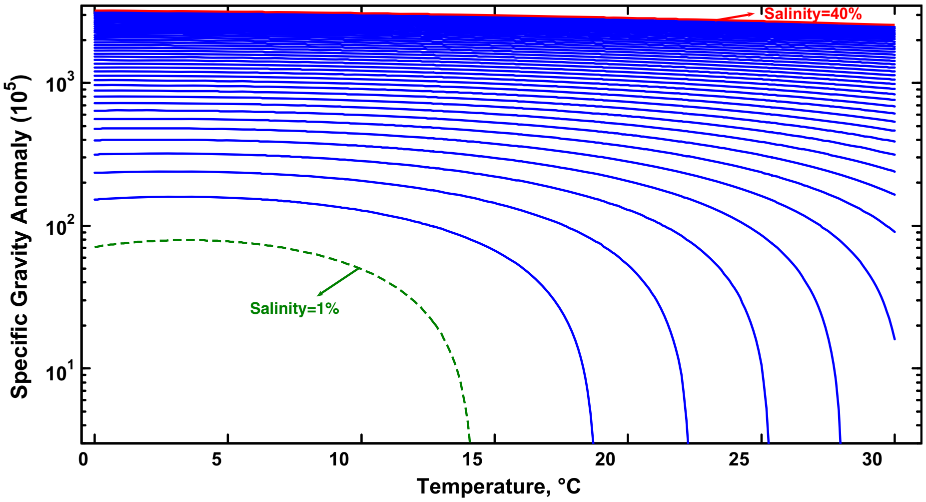

Fig. 3. The smoothness of the results of predictive tool for the estimation of specific gravity anomaly in another view poit.

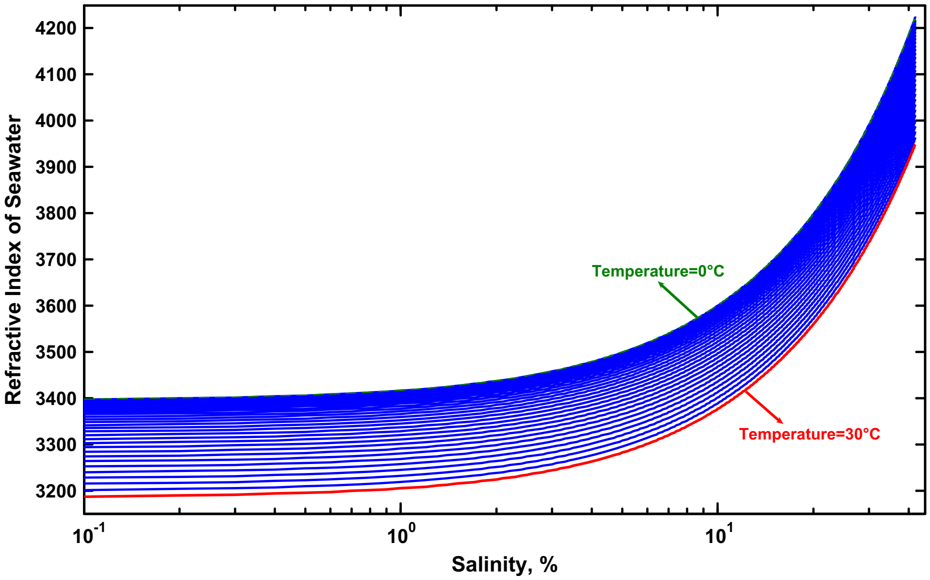

Fig. 4. The results of predictive tool for the estimation of seawater refractive index.

$$b = A_2 + B_2T + C_2T^2 + D_2T^3 \quad (7)$$

$$c = A_3 + B_3T + C_3T^2 + D_3T^3 \quad (8)$$

$$d = A_4 + B_4T + C_4T^2 + D_4T^3 \quad (9)$$

These optimum tuned coefficients help to cover temperature up to 40°C and salinity up to 40%. The optimum tuned coefficients given in Table 1 can be further retuned quickly according to proposed approach if new data become available in the future.

The above procedure is applied to develop a predictive tool to estimate refractive index of seawater as a function of salinity and temperature.

Eq. (10) represents the proposed governing equation in which four coefficients are used to correlate seawater refractive index as a function of salinity and temperature where the relevant coefficients have been reported in Table 1.

Table 2
Comparison of calculated values with typical data for specific gravity anomaly

Temperature (°C)	Salinity (%)	Specific gravity anomaly, reported data [15]	Calculated specific gravity anomaly values	Absolute deviation percent
0	10	801	802.4	0.17
0	40	3,216	3216.4	0.01
4	5	402	397.1	1.2
4	10	799	797.3	0.12
4	20	1,589	1,591	0.12
4	30	2,384	2383.5	0.02
4	40	3,179	3178.1	0.03
8	5	381	379.7	0.34
8	10	772	772.2	0.03
8	20	1,551	1555.2	0.03
8	30	2,340	2337.5	0.11
8	40	3,122	3121.9	0.003
15	0	−87	−87.6	0.70
15	10	685	682.9	0.31
15	20	1,450	1449.3	0.05
15	30	2,215	2215.2	0.01
15	40	2,985	2984.4	0.02
20	0	−177	−176.4	0.34
20	10	586	585.7	0.05
20	20	1,342	1342.2	0.01
20	30	2098	2098.2	0.01
20	40	2,860	2858.8	0.04
25	0	−293	−292.3	0.24
25	10	462	462.2	0.04
25	20	1,210	1,211	0.08
25	30	1960	1959.4	0.03
25	40	2,714	2713.3	0.02
30	0	−433	−432.6	0.09
30	10	315	314	0.31
30	20	1,057	1057.3	0.03
30	30	1801	1801.2	0.01
30	40	2,550	2550.2	0.01
Average absolute deviation	0.14%			

$$(R_{\text{index}}) = a + bS + cS^2 + dS^3 \tag{10}$$

where

$$a = A_1 + B_1T + C_1T^2 + D_1T^3 \tag{11}$$

$$b = A_2 + B_2T + C_2T^2 + D_2T^3 \tag{12}$$

$$c = A_3 + B_3T + C_3T^2 + D_3T^3 \tag{13}$$

$$d = A_4 + B_4T + C_4T^2 + D_4T^3 \tag{14}$$

In this work, our efforts directed at formulating a correlation which can be expected to assist engineers for rapid calculation of specific gravity and refractive indexes of seawater as a function of salinity and temperature. The proposed novel tool is simple and unique expression which is non-existent in the literature. Furthermore, the selected function to develop the tool leads to well-behaved (i.e. smooth and non-oscillatory) equations enabling reliable and more accurate predictions.

Table 3
Comparison of calculated values with typical data for seawater refractive index

Temperature (°C)	Salinity (%)	Refractive index reported data [15]	Calculated refractive index, values	Absolute deviation percent
0	5	3,500	3498.6	0.04
0	10	3,600	3599.8	0.01
0	20	3,795	3797.2	0.06
0	30	3,991	3990.8	0.01
0	40	4,185	4184.6	0.01
5	5	3,485	3487.7	0.08
5	10	3,585	3586.9	0.05
5	20	3,780	3,779	0.03
5	30	3,966	3967.4	0.04
5	40	4,157	4157.9	0.02
10	5	3,465	3466.7	0.05
10	10	3,565	3563.5	0.04
10	20	3,750	3751.4	0.04
10	30	3,935	3936.1	0.03
10	40	4,124	4,123	0.02
15	5	3,435	3435.3	0.01
15	10	3,530	3,530	0
15	20	3,715	3714.8	0.01
15	30	3,898	3897.3	0.018
15	40	4,080	4080.1	0.01
20	5	3,395	3394.6	0.02
20	10	3,485	3487.2	0.06
20	20	3,670	3670.2	0.01
20	30	3,851	3,851	0
20	35	3,940	3,941	0.02
20	40	4,031	4030.9	0.01
25	5	3,345	3343.4	0.02
25	10	3,435	3435.8	0.02
25	20	3,620	3618.1	0.05
25	30	3,798	3797.6	0.01
25	35	3,886	3886.6	0.02
25	40	3,976	3975.3	0.02
Average absolute deviation	0.03%			

3. Results

Fig. 1 shows the proposed method results to estimate specific gravity anomaly as a function of salinity and temperature in comparison with data [15]. Figs. 2 and 3 show the smooth performance of predictive tool in the prediction of specific gravity of seawater as a function of salinity and temperature in two different viewpoints. Fig. 4 shows the proposed method smooth results to estimate refractive index of seawater as a function of salinity and temperature. Tables 2 and 3 illustrate the accuracy of proposed correlation to predict seawater specific gravity and refractive index in comparison with reported data [15]. The deviation of correlation in terms of average absolute deviation is less than 0.2%

It is expected that our efforts in formulating a simple tool will pave the way for arriving at an accurate prediction of specific gravity and refractive indexes of seawater as a function of salinity and temperature which can be used by practitioners and engineers for monitoring the key parameters periodically. The tool developed in this study can be useful for experts and engineers to have a quick check on specific gravity and refractive indexes of seawater at various conditions without opting for any experimental trials. In particular, process engineers and water treatment practitioner would find the approach to be user-friendly with transparent calculations involving no complex expressions.

Example: Calculate the refractive index of seawater with 10% salinity at 10˚C:

Solutions: Using new developed correlation (Eqs. (10)–(14)):

$a = 3367.33436621$
$b = 20.19784772712$
$c = -0.066804085967$
$d = 0.0008.52669875077$
Refractive index = 3563.48510

4. Conclusions

In this work, simple-to-use equations, which are easier than existing approaches, less complicated, with fewer computations, and suitable for engineers is presented here to estimate specific gravity and refractive indexes of seawater as a function of salinity and temperature. Unlike complex mathematical approaches, the proposed correlation is simple-to-use and would be of immense help for engineers especially those dealing with seawater desalination and processing. Additionally, the level of mathematical formulations associated with the estimation of specific gravity and refractive indexes of seawater can be easily handled by an engineer or practitioner without any in-depth mathematical abilities. The proposed method has clear numerical background, wherein the relevant coefficients can be retuned quickly if more data become available in the future.

Nomenclature

A — coefficient
B — psychrometric constant
C — coefficient
D — coefficient
R_{index} — refractive index
S — salinity, g/kg
SG_{A} — specific gravity anomaly
T — temperature, K

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