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Analysis of the filtration curve and the effect of temperature on silt density index (SDI)

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ABSTRACT

Filtration curves of samples obtained at three different seawater pretreatment plants were analyzed in order to identify the blocking mechanism during silt density index (SDI) measurements of seawater and filtered seawater. This was done in two ways. Firstly, by direct comparison of the measured filtration curves with the calculated curves based on different constant-pressure dead-end filtration models, the coefficient of determination R^2 was analyzed. Secondly, by changing the sample water temperature the measured SDI was compared to the calculated SDI in each model. The R² of both standard blocking models and intermediate blocking models show higher values than those of the other models in the case of seawater filtration curves, indicating that these fit the measured filtration curves. The R^2 of the cake filtration model is clearly smaller than those of other models. The measured SDI value of the same filtered seawater sample increases with temperature rise, this tendency coincides with the results calculated by all the filtration models. More precise analysis through R^2 comparison of each model shows that the standard blocking model and the complete blocking model, which indicate relatively strong dependence of water temperature on SDI, best coincide with the measured results. The temperature dependency on SDI of the cake filtration model shows poor correlation with test results. Consequently, blocking coefficient (k_s) of the standard blocking model is the most appropriate fouling indicator. The indicator $k_{\rm s}$ can be the consistent fouling index, widely adaptable from seawater to filtered seawater after pretreatment. It is also noted that the standard blocking model can be applied for the quantitative evaluation of the effect of water temperature on SDI.

Keywords: SDI; Seawater; Filtration curve; Standard blocking; Temperature

1. Introduction

In seawater reverse osmosis (SWRO) desalination plants it is essential to control the fouling potential through various pretreatment technologies, maintaining a high and stable performance of the reverse osmosis membrane. As a characteristic indicator for fouling potential, silt density index (SDI), as defined in the Standard D4189 of American Society for Testing and Materials (ASTM), is widely used due to its simple measuring method [1]. However, this index is only an empirical parameter and is not defined

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based on filtration theory. Therefore, it cannot be ensured that SDI has the linearity with the concentration of particles in the sample water. In the case of relatively high particle concentration, such as raw seawater, the application of SDI is limited due to the SDI's low sensitivity of particle concentration. Standard D4189 states that plugging rate percent ($\% P_{30}$ in Eq. (4)) should not exceed 75% at 207 kPa feed pressure. If this occurs, a shorter time such as 5 or 10 min should be used instead of usual 15 min elapsed flow time. Note, $\% P_{30} = 75\%$ corresponds to SDI₁₅ = 5 which raw seawater exceeds in many cases, therefore it may not be appropriate to use SDI₁₅ for the index of raw seawater.

On the other hand, SDI_{15} is commonly used for the fouling index of filtered seawater after pretreatment as the SDI_{15} is close to 4 which, in many cases, meets membrane manufacturer's requirements. If a consistent indicator from raw seawater through filtered seawater after pretreatment is available, it is easy to conduct the evaluation of the pretreatment performance, such as the removal efficiency of particles in water. In that case, it is highly expected that the consistent indicator has linearity with particle concentration.

The modified fouling index (MFI) was derived as the indicator having linearity with particle concentration [2]. The parameter MFI specifically corresponds to the resistance of the cake layer formed onto a $0.45 \,\mu\text{m}$ membrane surface during filtration under the constant-pressure dead-end conditions. Therefore, it is regarded that the MFI has linearity with particle concentration in the test water. Moreover, MFI is also advantageous over SDI in that it takes into account the whole filtration curve, whereas SDI is only based on an initial and a final measurement. However, there is little literature published on the MFI of actual seawater making it difficult to determine if MFI can be applied to the fouling index of raw seawater.

Recently, Wei et al. analyzed the filtration curves of actual seawater and compared the specific linearity of four filtration models, finding that standard blocking, rather than cake filtration, is the dominant fouling mechanism [3]. This report concluded that the standard blocking coefficient (k_s) is the most appropriate fouling index for actual seawater. In addition, analysis of filtered seawater from pretreatment processes was carried out in the same manner and showed that standard blocking is also the dominant mechanism for filtered seawater.

The main purpose of this paper is to determine the most consistent fouling indicator from raw seawater through filtered seawater after pretreatment and analyze its relationship to the SDI. Taking into consideration the results of the previous study [3], we first directly compared the measured filtration curves of both seawater and filtered seawater with calculated curves based on different constantpressure dead-end filtration models and tried to determine which blocking mechanism is dominant by quantitative analysis of the coefficient of determination R^2 . Secondly, we derived the equations which express the relationship between the blocking coefficient of four filtration models and the SDI.

According to these relationships, temperature dependency of SDI was expected. The SDI was measured by changing the temperature of the test samples which were obtained after pretreatment, and these results were then compared with the calculated SDI of each model. The best fit models were specified quantitatively by comparing R^2 .

The dependency of water temperature on SDI was reported by the previous study however, water samples and the comparison of test results with the prediction models was limited to artificial suspension and cake filtration models [4,5].

2. Theory and background

2.1. Filtration models

The empirical equation presented by Hermans and Bredee expresses that infinitesimal change to the dV of the filtration resistance dt/dV is proportional to $(1/J)^n$ as the proportional constant *K* (blocking coefficient) [6]. Hermia revealed the physical meaning of the four filtration models that were derived from Eq. (1) [7]. The four models: complete blocking (n = 2), standard blocking (n = 1.5), intermediate blocking (n = 1), and cake filtration (n = 0) are currently used:

$$\frac{\mathrm{d}^2 t}{\mathrm{d}V^2} = K \left(\frac{\mathrm{d}t}{\mathrm{d}V}\right)^n = K \left(\frac{1}{J}\right)^n \tag{1}$$

In these equations, t is the filtration time, V is the cumulative permeate volume per membrane area, and, J is the flux.

Table 1 shows the basic equations of the four models obtained by the integration of Eq. (1) and the definition of *K* for each model. The blocking coefficients for all four filtration models (K_b , k_s or K_s , K_i , K_c) is expressed as a function of the properties of the test water (η , C_b , C_p , n_p , ε_p , α_v), the characteristics of membrane (A_M , R_M , N_T , V_m), and the operating conditions (ΔP , T_m), considering the blocking mechanism of each model. The blocking coefficients of these filtration models are defined on the assumption that all

Basic equation of each fi	ltration 1	model and the derived equations for the	parameters of SDI calculation	
Model	и	Basic equations	Blocking coefficient	Derived equations for calculation of SDI _{Tm} Δt_1 , Δt_2 (min)
Complete blocking	2	(a) $K_b V/J_0 = 1 - \exp(-K_b t)$	(i) $K_{\rm b} = (n_{\rm p}/N_{\rm T}) f_0 ~({\rm s}^{-1})$	$\Delta t_1 = rac{-1}{60k_0} \ln \Bigl(1 - rac{k_b V_s}{b_0} \Bigr)$
		(a') $V = rac{f_0}{K_b} \{1 - \exp(-K_b t)\}$		$\Delta t_2 = rac{-1}{60 K_{ m b}} \ln \left\{ 1 - rac{K_{ m b}V_{ m b}}{h} \exp(60 K_{ m b} T_{ m m}) ight\}$
Standard blocking	1.5	(b) $J = J_0 - K_b V$ (c) $t/V = k_s t + 1/J_0$	(j) $k_{\rm s} = C_{\rm p} / \{ V_{\rm m} (1 - \varepsilon_{\rm p}) \}$ (m ⁻¹)	$\Delta t_1 = rac{V_s}{60_s} \left(rac{1}{1-V_F} \right)$
		(c') $V = rac{t}{(k_s t + 1/f_0)}$	(k) $K_{\rm s} = 2 k_{\rm s} J_0^{1/2}$ ((s m) ^{-1/2})	$\Delta t_2 = \frac{(60T_{\rm m}k_{\rm s})^2 + (120T_{\rm m}k_{\rm s})I_0) + (1/I_0)^2}{2(10T_{\rm m}k_{\rm s})(10T_{\rm m}k_{\rm s})I_0} \cdot \frac{1}{2(10T_{\rm m}k_{\rm s})}$
Intermediate blocking	1	(d) $J^{1/2} = J_0^{1/2} - k_s J_0^{1/2} V$ (e) $K_i V = \ln(1 + K_i J_0 t)$	(1) $K_i = n_{ m p}/N_{ m T} ~({ m m}^{-1})$	$\Delta t_1 = \frac{1/(J_0 V_s) - 60I_m K_s^2 - (K_s/J_0)}{\cos t} 60$
		(e') $V=rac{1}{K_i}\ln(1+K_i]_0t)$		$\Delta t_2 = rac{60 K_i f_0}{(1+60 T_{ m m} K_i f_0) \{ \exp(K_i V_{ m s}) - 1 \}}$
Cake filtration	0	(f) $1/J = 1/J_0 + K_i t$ (g) $t/V = K_c V/2 + 1/J_0$	(m) $K_{\rm c} = \eta lpha_{\rm v} / \Delta P ({ m sm}^{-2})$	$\Delta t_1 = rac{1}{60} \Big(rac{K_c V_2^2}{2} + rac{V_3}{l_0} \Big)$
		(g') $V = \frac{-1/f_0 + \{(1/f_0)^2 + 2K_ct\}^{1/2}}{K_c}$	$MFI = \frac{1}{2A_{M}^{2} \cdot 10^{6}} = \frac{\eta_{M}^{2A}}{2\Delta P \cdot A_{M}^{2} \cdot 10^{6}} (s/L^{2})$	$\Delta t_2 = rac{K_{ m s}V_{ m s}^2 + \left\{ 4V_{ m s}^2 {\left(rac{1}{b_{ m b}} ight)}^2 + 480K_{ m c}V_{ m s}^2T_{ m m} ight\}^{1/2}$
		(h) $1/J = 1/J_0 + K_c V$		- 120

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Table 1

particles are retained by the membranes. Therefore, these coefficients are proportional to the particle concentration in filtered water [4–9]. Details of the derivation procedure of Eqs. (i)–(m) in Table 1 are described in the previous studies [7,8].

In general K_s is the standard blocking coefficient in Eq. (1). However, normalized k_s , excluding J_0 (initial flux), is used instead of K_s in order to avoid the temperature dependence in this work, as defined in Table 1. The coefficient $K_c/2$ of Eq. (g) in Table 1 was defined as MFI by Alhadidi et al. which expresses the fouling index of test water [4]. The unit of (s/L^2) is commonly used for MFI and, consequently, Eq. (2) is derived by converting the unit from K_c (sm^{-2}) to MFI (s/L^2) :

MFI =
$$\frac{K_{\rm c}}{2A_{\rm M}^2 \cdot 10^6} = \frac{\eta I}{2\Delta P \cdot A_{\rm M}^2 \cdot 10^6}$$
 (s/L²) (2)

I in Eq. (2) indicates the fouling potential index. *I* expresses the product of the specific resistance of the cake accumulated on the membrane α (m/kg) and the particle concentration in test water $C_{\rm b}$ (kg/m³), as defined in Eq. (3):

$$I = \alpha_{\rm v} = \alpha C_{\rm b} \quad ({\rm m}^{-2}) \tag{3}$$

2.2. Calculation of SDI based on filtration model

2.2.1. Determination of SDI

The calculation of SDI is as follows:

$$\mathrm{SDI}_{\mathrm{Tm}} = \frac{\% P_{30}}{T_{\mathrm{m}}} = \left(1 - \frac{\Delta t_1}{\Delta t_2}\right) \times \frac{100}{T_{\mathrm{m}}} \tag{4}$$

where Δt_1 is the time required to collect the first 500 ml permeate and Δt_2 is the time required to collect the second 500 ml after total elapsed flow time $T_{\rm m}$ (usually 15 min). Water is passed through a 0.45 µm membrane filter at a constant applied pressure of 207 kPa (30 psi).

2.2.2. Calculation of Δt_1 and Δt_2 based on filtration model

From the aforementioned determinations of Δt_1 , Δt_2 and each model filtration curve (*V* vs. *t*), Δt_1 and Δt_2 are expressed as a function of *K*, J_0 , $V_{s'}$, and T_m as shown in Table 1. By inputting Δt_1 and Δt_2 into Eq. (4), the corresponding SDI_{Tm} of each filtration model can be calculated. An example of the derivation process of Δt_1 and Δt_2 based on a complete blocking model is shown in Appendix 1. In the other filtration model cases Δt_1 and Δt_2 are derived in the same way.

Additionally, J_0 is calculated by Eq. (5) from its definition using membrane resistance R_{M} , applied pressure ΔP and viscosity of test water η :

$$J_0 = \frac{\Delta P}{\eta R_{\rm M}} \quad (\rm{ms}^{-1}) \tag{5}$$

2.3. Parameters for calculation

2.3.1. Viscosity

Seawater viscosity is calculated using the following predictive equation [10]:

$$\eta = \eta_{\rm w} \left(1 + A \left(\frac{S}{1000} \right) + B \left(\frac{S}{1000} \right)^2 \right) \quad (P_{\rm a}s) \tag{6}$$

$$\eta_{\rm w} = 4.2844 \times 10^{-5} + \left(0.157(\theta + 64.993)^2 - 91.296 \right)^{-1}$$

$$A = 1.541 + 1.998 \times 10^{-2}\theta - 9.52 \times 10^{-5}\theta^2$$

$$B = 7.974 - 7.561 \times 10^{-2}\theta + 4.724 \times 10^{-4}\theta^2$$

$$0 < \theta < 180 (^{\circ}{\rm C}) \quad 0 < S < 150 ({\rm g/kg}) \quad S: {\rm Salinity}$$

2.3.2. Filtration area and filtration resistance of membrane

According to standard ASTM D4189, a membrane with a 47 mm diameter is normally used. Taking into account dead space due to the O-ring when the



Fig. 1. Schematic diagram of filtration system for the measurements of filtration curve and SDI.

membrane is setup to its holder, the effective surface area of the membrane is equal to $13.8 \times 10^{-4} \text{ m}^2$ [4]. Using measured J_0 , obtained from distilled water, the membrane resistance $R_{\rm M}$ is determined from Eq. (5).

3. Materials and methods

3.1. Materials

The test water samples were obtained from three different plants. Site *A* and site *B* are located along the coast of the Seto Inland Sea and the coast of Nagasaki prefecture in Japan, respectively [11], site *C* is located in the Arabian Gulf coast. Both plant A and B are full-scale plants equipped with two-stage sand filter and one-stage dual media filter as a pretreatment process. Plant C is a pilot-scale test unit equipped with multistage sand filters. No coagulants are used at the either plant A or C, whereas FeCl₃ is added as a coagulant at plant B in the pretreatment section.

Two kinds of microfiltration (MF) membrane with nominal pore size $0.45 \mu m$, Millipore HAWP04700, and Advantec A045A47A were used. The material of the membranes is a mixture of cellulose acetate and cellulose nitrate as specified by standard ASTM D4189.

3.2. Experimental setup and methods

The apparatus similar to the previous study was assembled as shown in Fig. 1 [3]. The filtration tests were carried out according to the procedure standardized in ASTM D4189. Using compressed air, the sample water to be filtered is pressurized in a stainless steel water vessel up to 207 kPa.



Fig. 2. Typical comparative example of the measured filtration curve and the calculated filtration curves by each filtration model (raw seawater at site *A*).

The residual air in the pipe is exhausted from the gap between the filter holder and the membrane by loosening the fastening screws of the holder prior to starting the filtration. The permeate from the filter holder is collected in a tank on the electronic balance connected to the computer and the filtration curve is obtained by recording the permeate weight at defined intervals.

In some tests of the SDI measurement only, either a fully automatic SDI meter (SDI-2200, Mabat Chemical Systems Ltd) or a semiautomatic SDI meter (simple SDI, Applied Membranes Inc.) was used. When performing temperature variation test, the water temperature was adjusted to a set point by a heater and kept constant throughout the test.

4. Results and discussion

4.1. Fouling mechanism analysis of raw seawater and filtered seawater after pretreatment

4.1.1. Raw seawater

Fig. 2 shows a typical filtration curve of raw seawater. The calculated curves of each filtration model are also described in Fig. 2. The parameters (K, J_0) used for the model calculation were optimized to minimize the residual sum of squares (RSS) in Eq. (7) by least-squares method:

$$RSS = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$
(7)

where $\hat{y}_i = f(K, J_0, x_i)$, (x_i, y_i) : data point (t_i, V_i) , \hat{y}_i : calculated value of V_i by model (function *f*) corresponding to x_i , *n*: number of data points.

By comparing the measured filtration and the calculated curves in Fig. 2, it seems that the calculated filtration curves, by both standard and intermediate blocking, are relatively close to the measured filtration curve, whereas alternatively, the calculated curve by cake filtration does not fit the measured one.

The coefficient of determination R^2 is calculated by Eq. (8) in order to quantitatively determine the goodness of fit between the calculated curve by models. A higher R^2 signifies a good fit between the model calculation curve and the measured one.

$$R^{2} = 1 - \frac{\sum_{i=1}^{n} (y_{i} - \widehat{y}_{i})^{2}}{\sum_{i=1}^{n} (y_{i} - \overline{y}_{i})^{2}} = 1 - \frac{\text{residual sum of squares}}{\text{total sum of squares}}$$
(8)



Fig. 3. Comparison of the goodness of fit between the measured filtration curves and calculated filtration curves by the coefficient of determination R^2 (raw seawater at site *A* and site *C*).



Fig. 4. Typical comparative example of measured filtration curve and calculated filtration curves by each filtration model (filtered seawater after pretreatment at site *C*).

where \bar{y} : average of y_i , other symbols: same as in Eq. (7).

As described in Fig. 2, the R^2 of the standard blocking and the intermediate blocking is larger than others, indicating a better fit of these two models. The R^2 of the cake filtration model is clearly smaller than others, showing that the cake filtration cannot explain the measured filtration curve.

Fig. 3 shows the average values of R^2 which were calculated using all data obtained at site *A* and site *C*. The R^2 of the standard blocking and the intermediate blocking are larger than that of cake filtration at both sites. The total numbers of water samples (n_s) were 6 and 22, respectively.

From the above results, it is concluded that the fouling mechanism of the MF membrane during SDI

measurement of seawater is dominated by the standard blocking or intermediate blocking rather than cake filtration. Wei et al. noted that the reason a cake layer is not formed during SDI measurement is due to the very low suspended particle concentration in raw seawater [3]. Other studies also confirm that the seawater filtration mechanism is dominated by standard blocking [3,9].

4.1.2. Filtered seawater after pretreatment

Fig. 4 shows a typical example of the measured filtration curve at site *C* and the calculated curves of each model. The parameters (*K*, J_0) used for the model calculation were optimized as in the case of seawater in the previous section.

In appearance, it would seem that the measured filtration curve is very close to any curves by each model calculation. To make a quantitative comparison, as in the case of raw seawater, the average of R^2 at site *A*, *B*, and *C* were calculated as shown in Fig. 5. The n_s was 6, 6, and 14 at site *A*, *B*, and *C*, respectively.

There is no significant difference in the R^2 between the four models. Extremely low particle concentration in the filtered seawater may cause very small changes to the filtration resistance. This is presumably the reason why the measured filtration curve coincides with those calculated by all the models.

Previous studies reported that the standard blocking model dominates the fouling mechanism for the filtered seawater as well as seawater [3]. A longer filtration could possibly bring a clearer difference between the four models.



Fig. 5. Comparison of the goodness of fit between the measured filtration curves and calculated filtration curves by the coefficient of determination R^2 (filtered seawater after pretreatment at site *A*, *B*, and *C*).



Fig. 6. Results of SDI measurements at different sample temperature (symbols tied with the line represent the same sample).

4.2. Effect of temperature on SDI measurement and its analysis based on filtration models

4.2.1. SDI measurements at different temperature

Fig. 6 presents the SDI of the filtered seawater after pretreatment which were measured under the condition of temperature variation at site *A* and *B*. Again, Fig. 6 also shows the measured SDI cited from the previous study by Kunisada [9]. From these test results, it is evident that measured SDI of the same sample increases with the water temperature, irrespective of the membrane type (Millipore/Advantec) and methods of measurement (ASTM/full-automatic/ semiautomatic).

4.2.2. Analysis of the temperature effect on SDI based on filtration models

In order to clarify the water temperature dependency of measured SDI, theoretical analysis by filtration models was carried out as follows.

4.2.2.1. Evaluation of temperature dependency of SDI based on filtration models. Fig. 7 demonstrates the theoretically calculated SDI lines of each model as a function of water temperature. In this calculation, the blocking coefficient *K* of each model at the reference temperature 20 °C is determined to match the SDI of 1, 2, 3, 4, 5 as shown in Table 2.

The values of SDI at a given temperature are calculated by considering the temperature dependency of J_0 and K. The detailed procedure for calculation is shown in Table 3.

It can be seen in Fig. 7 that all calculated SDI values increase with temperature rise. However, the sensitivity of the temperature to SDI differs depending on each fouling model. The sensitivity of the complete blocking model is the highest, followed in order by standard blocking, intermediate blocking, and cake filtration. In addition, it is confirmed that the calculated lines by cake filtration are consistent with those in the literature under the same model parameters ($R_{\rm M} = 1.29 \times 10^{10}$ (m⁻¹), S = 0 (g/kg), others are same as Table 2) [4].

4.2.2.2. Comparison between measured SDI and calculated SDI of each model. Fig. 8 represents a typical example of a comparison of measured SDI with calculated SDI of each mode. The parameter (*K*) used for the model calculation is optimized to minimize the RSS of that noted in Eq. (7), by least-square method.



Fig. 7. Theoretical evaluation of the temperature effect on SDI by each filtration model.

Table 2					
Parameters of the model	calculation	used	in	Fig.	7

SDL- at 20°C	Complete blocking	Standard blocking	Intermediate blocking	Cake filtration		
5D115 at 20 C	$K_{\rm b} ({\rm s}^{-1})$	$k_{\rm s} \ ({\rm m}^{-1})$	$K_i (m^{-1})$	$K_{\rm c}~({\rm sm}^{-2})$	MFI (s/L ²)	
1	$1.80 imes 10^{-4}$	6.79×10^{-3}	1.42×10^{-2}	1.12×10^{0}	2.91×10^{-1}	
2	$3.94 imes 10^{-4}$	1.56×10^{-2}	3.44×10^{-2}	3.05×10^{0}	$8.00 imes 10^{-1}$	
3	6.56×10^{-4}	2.78×10^{-2}	6.57×10^{-2}	6.84×10^{0}	$1.80 imes 10^0$	
4	9.96×10^{-4}	4.61×10^{-2}	$1.20 imes 10^{-1}$	1.61×10^1	$4.24 imes 10^0$	
5	$1.48 imes 10^{-3}$	7.80×10^{-2}	2.41×10^{-1}	5.33×10^{1}	$1.40 imes 10^1$	
Common	$R_{\rm M} = 1.39 \times 10^{10} \text{ (m}^{-1}), S = 35 \text{ (g/kg)}, A_{\rm M} = 13.8 \times 10^{-4} \text{ (m}^2), V_{\rm s} = 0.362 \text{ (m)}, T_{\rm m} = 15 \text{ (min)}, \Delta P = 207 \times 10^3 \text{ (Pa)}$ The values of blocking coefficient are those at 20°C					

Table 3 Calculation procedure of SDI at a given temperature

Step	Description
1	Calculation of J_0 at the reference temperature (e.g. 20°C) using ΔP , θ , S , R_M , Eqs. (5) and (6) Calculation of K at the reference temperature
2	J_0 (step 1) and <i>K</i> (assumed) $\rightarrow \Delta t_1$, Δt_2 (Table 1) \rightarrow SDI (Eq. (4)) until it matches a given SDI at the reference temperature by changing the assumed <i>K</i>
3	Calculation of J_0 and K at a given temperature J_0 is calculated in the same way as step 1. K is calculated using Eqs. (i)–(m) in Table 1 under the condition of the same property values* at the reference temperature. (* n_p/N_T for complete blocking, $C_p/V_m(1 - \varepsilon_p)$ for standard blocking, n_p/N_T for intermediate blocking, α_v for cake filtration)
4	Calculation of SDI at a given temperature J_0 and K (step 3) $\rightarrow \Delta t_1$, Δt_2 (Table 1) \rightarrow SDI (Eq. (4))



Fig. 8. Typical comparative example of measured SDI and calculated SDI of each model at different temperature (filtered seawater after pretreatment at site *A*).

In Eq. (7), $\hat{y_i}$ and y_i means as follows.

 $\hat{y}_i = g(K, J_0, x_i), (x_i, y_i)$: data point (θ_i , SDI_i), \hat{y}_i : calculated value of SDI by model (function *g*) corresponding to x_i .

As previously described, the values of $R_{\rm M}$, measured by the permeation of distilled water, were used for this calculation. The values of $R_{\rm M}$ are 1.39×10^{10} (m⁻¹) for Millipore and 1.48×10^{10} (m⁻¹) for Advantech, respectively. Water viscosity was calculated by inputting the salinity at each site into Eq. (6). The salinities for site *A*, *B*, and *C* were 28, 35, and 45 (g/kg), respectively. In the literature cited [9], the salinity is assumed to be 35 (g/kg) taking into consideration the average salinity of seawater surrounding Japan. The calculation parameters of $R_{\rm M}$ and salinity are used in the model calculations hereafter.

By comparing the measured SDI and the calculated SDI in Fig. 8, it seems that the calculated SDI lines of both the standard blocking and the complete blocking are relatively close to the measured SDI line, whereas the SDI line by cake filtration is far from the measured one due to low temperature dependency.



Fig. 9. Comparison of the goodness of fit between measured SDI and calculated SDI in different sample temperatures by the coefficient of determination R^2 (filtered seawater after pretreatment).



Fig. 10. Comparison between measured SDI and calculated SDI (solid lines represent the calculated lines by standard blocking model).

In order to quantitatively determine the goodness of fit between the calculated SDI by models and the measured SDI, R^2 is also calculated by Eq. (8). As shown in Fig. 8, the R^2 of standard blocking model demonstrates the highest value (0.9843), whereas the cake filtration model exhibits the lowest value (0.7966).

For further quantitative comparison, all the SDI data obtained from site *A* and *B*, together with the data cited from the literature, were compared with the corresponding calculated SDI, and R^2 was calculated as shown in Fig. 9.

According to Fig. 9, the average R^2 of the standard blocking and the complete blocking model are relatively large, indicating the calculated SDI is close to the measured SDI. The R^2 of the intermediate blocking model is rather high, while the cake filtration model shows the lowest R^2 .

Fig. 10 shows the calculated SDI of the standard blocking as a function of water temperature with measured SDI of site *A* and *B* and those also cited from identified literature. From the results of this figure, temperature dependency of SDI can almost be explained by standard blocking model.

4.2.2.3. SDI value at reference temperature. According to standard ASTM D4189, the water temperature must remain constant (\pm 1°C) throughout the SDI test. However, through our investigation, SDI is found to be affected not only by temperature variation during the test but also by the test temperature itself. Consequently, we believe that it is reasonable to convert the measured SDI value to the value at the reference temperature (for example 20°C) by the calculation using the standard blocking model or to measure the SDI at the reference temperature. Concerning the latter, the test conditions must be examined elsewhere as the change of water quality might occur during temperature adjustment.

4.3. Relationship between k_s and SDI, the applicability of k_s

From the analysis of the filtration curve and the temperature dependency on SDI, standard blocking coefficient k_s could be the most appropriate fouling index from raw seawater to filtered seawater after pre-treatment. The reasons for this assumption are as follows:

- (a) It appears that the fouling mechanism of the MF membrane for seawater and filtered seawater is dominated by standard blocking.
- (b) Assuming that the membrane retains all particles, the blocking coefficient k_s is defined as a parameter proportional to the particle volume (C_p) in the test water as described in Table 1. Therefore, k_s could be used as the fouling index having good linearity with the particle concentration.
- (c) Blocking coefficient k_s is very useful for practical use as it can be easily calculated from the measured data obtained by plotting t/V vs. t. The k_s corresponds to the gradient of the line of t/V vs. t as described by Eq. (c) in Table 1. In addition, the advantage of the k_s over SDI is its reproducibility and accuracy, taking into account the evolution of membrane fouling throughout a filtration test, whereas SDI is only based on an initial and a final measurement.



Fig. 11. Relationship between k_s and SDI (filtered seawater after pretreatment at site *C*).

- (d) The k_s is temperature independent in theory, based on the assumption of C_p , V_m , and ε_p are constant values, as indicated in Table 1.
- (e) The relationship between $k_{\rm s}$ and SDI can be obtained theoretically based on standard blocking model.

In particular, the items of (b) and (c) bring very effective advantages for the actual use of k_s which is valid in a wide range, from raw seawater to filtered seawater after pretreatment.

As for item (e), it becomes possible to obtain the relationship between k_s and SDI more clearly by standard blocking model. Examples that demonstrate the relationship between measured SDI₁₅ and measured k_s at site *C* are shown in Fig. 11. In this figure, the measured SDI₁₅ values are converted to those at 20°C from measured temperature by standard blocking model in the same manner as shown in Table 3.

Fig. 11 also shows SDI₁₅ calculated by standard blocking model as a function of $k_{\rm s}$. The value 0.047 (m⁻¹) of $k_{\rm s}$ corresponds to 4 of SDI at 20°C as shown in Fig. 11. The calculation parameters of $R_{\rm M}$ and *S* are 1.39×10^{10} (m⁻¹) and 45 (g/kg), respectively.

From the calculated line in this figure, SDI₁₅ (<4) shows a relatively good linearity with the k_s of the low range, whereas SDI₁₅ (>4) does not increase in proportion to k_s in the high range. The data and the empirical equation by Wei et al. are also shown in Fig. 11, indicating a good linearity of SDI₁₅ (<5) with k_s [3]. From our study, the linearity appears to be lost in the relatively lower range of k_s .

In Fig. 11, taking into account the difference in the definition of k_s , we plotted as k_s the values of one half of the original values by Wei et al. [3].

5. Conclusions

During SDI measurement, the filtration curves of seawater and filtered seawater after granular media filtration were obtained at three sites and directly compared with the calculated filtration curves of four models: complete blocking, standard blocking, intermediate blocking, and cake filtration. The experimental values of SDI obtained by changing the water temperature were also compared with the calculated values based on the four models and the temperature dependency of SDI was analyzed. The experimental results were compared with calculated values by four models using the coefficient of determination R^2 .

As a result, the following was concluded:

(1) For seawater, the calculated filtration curves, of both standard blocking and intermediate blocking models, fit the measured filtration curve relatively well, whereas the calculated filtration curve by cake filtration model does not coincide with the measured curve.

- (2) For the filtered seawater after pretreatment, the calculated filtration curves of all models could explain the measured filtration curve presumably due to both extremely low particle concentration in the filtered seawater and relatively short filtration time.
- (3) Measured SDI values of the same water increased with the water temperature. All four filtration models explain this tendency. Especially the calculation results of the standard blocking model and complete blocking model, which have relatively stronger temperature dependencies and could explain the measured curve very well.
- (4) We identified the relationship between k_s and SDI based on the standard blocking model theoretically, this relationship explains the range of k_s having linearity with the value of SDI₁₅ (<4).

On the basis of aforementioned test results and the analysis of filtration curve and the temperature dependency of SDI, we consider that standard blocking coefficient k_s is the most consistent fouling index. The value of k_s is defined as a parameter proportional to the particle concentration in test water. Therefore, k_s can be utilized for a wide range of water, such as raw seawater through filtered seawater. The performance of seawater pretreatment, e.g. removal efficiency of particles, can be easily evaluated using k_s .

The blocking coefficients of filtration models are promising as fouling indices which other researchers have also pointed out [2–5,8,9]. However, these coefficients could be affected by factors such as particle distribution, membrane properties, and applied pressure [12]. Further research on these factors is needed.

Nomenclature

$A_{\rm M}$	—	membrane area (m ²)
$C_{\rm b}$	—	particles concentration in test water
		(kg/m^3)
$C_{\rm p}$	_	particles volume in test water (m^3/m^3)
I		fouling potential index (m ⁻²)
J		flux (ms ⁻¹)
Jo		initial flux (ms ⁻¹)
Κ		coefficient of blocking in Eq. (1) $(m^{n-2} s^{1-n})$
$K_{\rm b},K_{\rm s},$	—	coefficients of complete blocking (s^{-1}) ,
K_i, K_c		standard blocking ((ms) ^{-1/2}), intermediate
		blocking (m^{-1}) , and cake filtration (sm^{-2})
$k_{\rm s}$		normalized coefficient of standard blocking
		model (m ⁻¹)
N_{T}		number of pores per unit membrane area
		(m^{-2})

n _p	_	number of particles in test water (m ⁻³)
n	_	exponent in Eq. (1) (–) or number of data
		points in Eqs. (7) and (8) (–)
n _s	_	number of samples (–)
ΔP	_	applied pressure (Pa)
R^2	_	the coefficient of determination (–)
R _M	_	membrane resistance (m ⁻¹)
S	_	salinity (g/kg)
$T_{\rm m}$	_	elapsed filtration time (usually 15 min)
		after the start of collecting permeate (min)
Δt_1	_	initial time required to collect 500 mL of
		permeate for SDI measurement (min)
Δt_2	_	time required to collect 500 mL of
		permeate after time $T_{\rm m}$ for SDI
		measurement (min)
t	—	filtration time (s)
V	_	cumulative permeate volume per
		membrane area (m)
$V_{\rm s}$	_	corrected permeate volume per membrane
		area (i.e. $\tilde{V}_{s} = 5 \times 10^{-4} \text{ (m}^{3})/\tilde{A}_{M} \text{ (m}^{2})$) (m)
$V_{\rm m}$	_	pore volume per unit membrane area
		(m^3/m^2)
θ	—	temperature of water (°C)
α	—	specific cake resistance (m/kg)
$\alpha_{\rm v}$	—	specific cake resistance based on permeate
		(m^{-2})
ε _p	—	porosity of the particles layer captured in
		pore of membrane (–)
η	—	water viscosity (P _a s)
$%P_{30}$	—	plugging rate percent at 207 kPa (30 psi)
		feed pressure (%)
SDI	—	silt density index (%/min)
MFI	_	modified fouling index (s/L^2)

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Appendix 1

An example of Δt_1 and Δt_2 derivation process based on complete blocking model

The equation representing the filtration curve is derived from Eq. (a) in Table 1 as follows:

$$V = \frac{J_0}{K_b} (1 - \exp(-K_b t)) \quad (m)$$
 (A1)

or

$$t = \frac{-1}{K_b} \ln \left(1 - \frac{K_b V}{J_0} \right) \quad (s)$$
(A2)

From the definition of Δt_1 and Eq. (A2), Δt_1 is expressed by Eq. (A3):

$$\Delta t_1 = \frac{-1}{60K_b} \ln\left(1 - \frac{K_b V_s}{J_0}\right) \quad (\text{min}) \tag{A3}$$

where $V_{\rm s}$ is collected permeate volume (usually 500 ml) per membrane area in initial and after $T_{\rm m}$ for SDI measurement:

$$V_{\rm s} = \frac{5 \times 10^{-4}}{A_{\rm M}} \quad ({\rm m}) \tag{A4}$$

Then, from Eq. (A1), cumulative permeate volume per membrane area at filtration time at $60T_m$ and $60(T_m + \Delta t_2)$ are represented by the following:

$$V_{60T_{\rm m}} = \frac{J_0}{K_{\rm b}} (1 - \exp(-K_{\rm b} \cdot 60T_{\rm m})) \quad ({\rm m})$$
(A5)

$$V_{60(T_{\rm m}+\Delta t_2)} = \frac{J_0}{K_{\rm b}} \{1 - \exp(-K_{\rm b} \cdot 60(T_{\rm m} + \Delta t_2))\} \quad ({\rm m}) \quad ({\rm A6})$$

From the definition of V_{s} , the following is obtained:

$$V_{60(T_{\rm m}+\Delta t_2)} - V_{60T_{\rm m}} = V_{\rm s} \quad ({\rm m}) \tag{A7}$$

By substituting Eqs. (A5) and (A6) to Eq. (A7),
$$\Delta t_2$$
 is expressed by Eq. (A8):

$$\Delta t_2 = \frac{-1}{60K_b} \ln \left\{ 1 - \frac{K_b V_s}{J_0} \exp(60K_b T_m) \right\}$$
 (min) (A8)

 $\mathrm{SDI}_{\mathrm{Tm}}$ is calculated by substituting Eqs. (A3) and (A8) to Eq. (4).