



## Multi-site modeling and prediction of annual and monthly precipitation in the watershed of Cheliff (Algeria)

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### ABSTRACT

The quantity of precipitation as well as its distribution varies in time, space, and even in small areas. This temporal and/or spatial variability is due to the support of several quite complex climatic and physiographic factors. The description and the prediction of this variability is a fundamental requirement in a wide variety of human activities, as well as in the elaboration and the design of hydraulic projects. The objective of this work is to study the applicability of Kalman filter (KF) technique for the modeling and prediction of annual or monthly rainfall amounts in the Cheliff watershed, as well as the assessment of the prediction error. The major advantage of KF is to provide with the prediction error covariance an indicator of the filter accuracy. In addition, its algorithm works in the temporal domain with a recursive nature and has an optimal estimator in the least squares concept. Another aspect of its optimality is the incorporation of all the available information on the system, measurements and errors in an adaptive operator, which is reset each time as a new measurement is available. For the implementation of this filter, time series of monthly and annual rainfall data registered over a period of 51 years (1959–2009) in 39 precipitation stations are studied in the Cheliff watershed and the obtained results are quite satisfactory.

*Keywords:* Kalman filter; Multi-site prediction; Precipitation; Cheliff watershed; Algeria

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### 1. Introduction

In a semiarid country like Algeria, water is an element of survival which is directly or indirectly related to any economic and social development.

Unfortunately, water resources are facing great challenges due to the lack and scarcity of rainfall as well as its geographical variability. It is important to note that among the 100 billion cubic meters received in the form of rainfall per year, on the northern part of Algeria, only 4.8 billion are captured in operational

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dams in 2008 [1]. In 2012, according to the National Agency of Dams and Transfers (ANBT), a filling rate of 76% was recorded in the operational 65 dams in Algeria, which corresponds to a stored amount of 5.5 billion cubic meters. This amount is inevitably decreased if different types of losses are included in addition to the increase in the demand for agricultural, industrial, and domestic purposes. According to recent studies [2–4], the demand for water has reached 5 billion cubic meters per year, with a supply of 170 (m<sup>3</sup>/capita/d), instead of a minimum standard of 250 (m<sup>3</sup>/capita/d), which is already considered as a deficit. This deficit is becoming more and more problematic from one year to another and thus an urgent and effective solution is necessary to ensure a regular water supply for all users.

The new Algerian policy focuses on the integrated management of water resources [5]. Therefore, two types of actions are to be taken into consideration. The first is related to the economy of these resources while the second concerns the optimal management of water resources, which relies mainly on successful models for prediction.

Such models in hydrology are numerous and diverse; the literature offers a wide range of work, but in this study, our attention is focused on a particular type, the state-space models. The latter include the distinction between the observed (signal) and hidden (internal state) variables, and they generally consist of an equation of state, showing the dynamics of the hidden variable and a measurement equation describing the way how observations are generated by hidden variables and residuals [6,7].

The objective of this study is the Kalman filter (KF) application. The KF combines two independent estimations to get a better weighted estimation. The first one is an estimate (prediction) based on a prior knowledge and the second is an estimate based on new information (measurement) arrival.

The term “filter” is borrowed from the electronic language and it means the extraction of a signal from a noisy environment, and the filtering operation is defined as the operation in estimating the state of a dynamic system from partial and noisy observations. In other words, it means any mathematical operation that uses past data or measurements of a dynamic system to obtain better estimations in the past (interpolation), in the present (filtering), or in the future (forecast).

By definition, the forecast is the likely future behavior of a process. The trust assigned to it depends on the nature of the studied process and the quality of the model adequacy it represents [8,9].

In hydrology, these processes are complex, nonlinear depending on time and a certain number of

uncertain parameters, [10]. As a consequence, the formulations based on simplifying assumptions of linearity and time invariance can cause substantial differences between the observed and predicted values [11]. In the context of a real-time forecasting [9], it is important to distinguish between the two modes with which a model can work. In simulation mode, one can use the outputs of the model as new input in the calculation of new outputs; however, it is excluded to use the observed output as input to the model. In adaptive mode, the current output of the model can be based on earlier observed output of the system [12]. In the real-time forecasting, it is necessary to work in the adaptive mode, which involves the use of a model with a retroactive structure that makes the model output at the present time connected to the previous observed output as in the case of the KF [13].

The KF is a quite effective tool, since its introduction in the sixties of the last century Kalman [14] has not ceased to intrigue scholars, and until now its results in all areas are so satisfactory that it has become a very popular tool [15–17]; in the field of data assimilation can be found [18–21]. The spectrum of its application has been extended to include other areas, one can cite [22–24] among others, particularly in the hydrological estimation and prediction. In this respect, a quite interesting reference [10] included several articles that were presented at the Chapman Conference on Applications of the Theory and Technique of KF in Hydrology, Hydraulics and Water Resources, The Geophysical American Union. There are several applications such as modeling and prediction of flows, studies of rainfall-runoff system, flow hydraulics and other transport processes in rivers, studies related to the quality of water, groundwater problems, and other areas of water resources and geophysics.

Among recent publications on the KF one can refer to [25–29]. Harrison and Stevens [30] has mentioned that the KF is the most general approach to statistical estimation and prediction, and all prediction methods are special cases of it.

In this paper, we consider the discrete KF, which presents difference equations rather than differential equations, because the predictions in hydrology are made in discrete time; the objective is to study its applicability for the modeling and multi-site prediction of precipitation, as well as the assessment of the prediction error. The expected result is an online optimal prediction model that would not be fixed in space or in time; in addition, it would automatically adapt itself to the changes in meteorological conditions in the watershed under study.

## 2. Materials and methods

### 2.1. Study area

The Cheliff-Zahrez watershed covers an area of 56,227 km<sup>2</sup>, bounded to the north by the Mediterranean Sea, to the west by the Oran-Chott Chergui region, to the south by the desert, and to the east by the region of the Algiers-Chott Hodna (Fig. 1). The study area is bordered by two main chains: the Tellian Atlas to the north and the Saharian Atlas to the south. It lies between the longitude of 3° 50' East and 0° 08' West, corresponding to the upper valley of Chellif watershed between the longitude of 2° 82' East and 1° 58' West and the latitude of 33° 53' South and 36° 14' North.

Landform, the effects of crests, as well as the influence of the site are major factors in the structuring of rainfall fields and their orientation in space.

### 2.2. Data

The data used in this study are the annual and monthly rainfall registrations recorded by the National

Agency for Water Resources (ANRH) in the Cheliff watershed. These data are recorded in 39 rainfall stations; four of them are located on operational dams. Their distribution in the Cheliff watershed is shown in Fig. 1. These data constitute 39 time series with a common observation period of 51 years lying from 1959 to 2009.

### 2.3. Discrete KF

The KF is a statistical approach of data assimilation, whose principle is to correct the model trajectory by combining observations with the information provided by the model, so as to minimize the error between the true state and the filtered one. This method uses a prediction that is based on a deterministic model and a registration that relies on innovation (the difference between the measured and predicted output) [14]. One of the most important steps in the application of the KF technique is the formulation of the state and the measurement equations according to the structure of a state space model. In the present case, these can be formulated as follows:

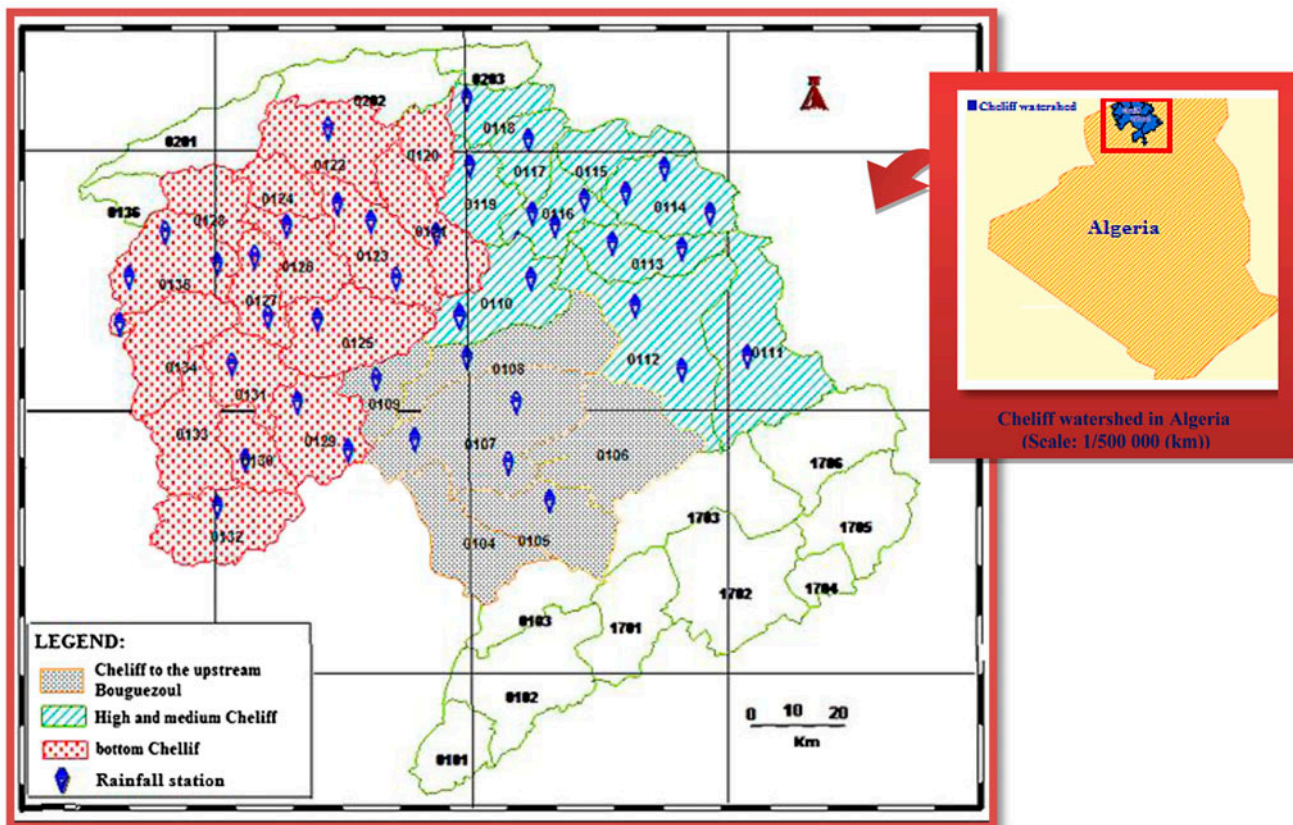


Fig. 1. Location of Cheliff watershed and distribution of rainfall stations.

State equation:

$$X_k = \phi_{k/k-1}X_{k-1} + W_{k-1} \quad (1)$$

Measurement equation:

$$Z_k = H_k X_k + W_k \quad (2)$$

where

$X_k, X_{k-1}$	$(n \times 1)$ vectors of the state of the system at time $t_k$ and $t_{k-1}$ , respectively
$\phi_{k/k-1}$	$(n \times n)$ matrix of state transition permitting the passage of $X_{k-1}$ up to $X_k$
$W_{k-1}$	$(n \times 1)$ noise vector of the system at time $t_k$ , assumed to be white Gaussian noise with a known covariance structure
$Z_k$	$(m \times 1)$ measurement vector at time $t_k$
$H_k$	$(m \times n)$ measurement matrix at time $t_k$
$V_k$	$(m \times 1)$ noise vector of measurement at time $t_k$ , assumed to be a white noise with a known covariance structure and uncorrelated with $W_{k-1}$

Once the formulation of the model is achieved, certain initializations such as the initial state vector and the associated error covariance matrix, the system and measurement noise covariance matrices, as well as the state transition matrix and the measurement matrix, are required to start the calculations. The latter are made according to the following five matrix relations:

Update of the state vector:

$$\hat{X}_{k/k} = \hat{X}_{k/k-1} + K_k (Z_k - H_k \hat{X}_{k/k-1}) \quad (3)$$

Gain estimation:

$$K_k = P_{k/k-1} + H_k^T (H_k P_{k/k-1} H_k^T + R_k)^{-1} \quad (4)$$

Correction of the matrix of covariance of estimation error:

$$P_{k/k} = (1 - K_k H_k) P_{k/k-1} \quad (5)$$

Prediction of the state vector:

$$\hat{X}_{k+1/k} = \phi_{k+1/k} \hat{X}_{k/k} \quad (6)$$

Prediction of the covariance of the prediction error:

$$p_{k+1/k} = \phi_{k+1/k} p_{k/k} \phi_{k+1/k}^T + Q_k \quad (7)$$

where  $Q_k$  and  $R_k$  are the system noise covariance and the measurement noise matrices, respectively, such as:

$$E[W_k W_j^T] = \begin{cases} Q_k, & j = k \\ 0, & j \neq k \end{cases} \quad (8)$$

$$E[V_k V_j^T] = \begin{cases} R_k, & j = k \\ 0, & j \neq k \end{cases} \quad (9)$$

### 3. Application and discussion

To develop and apply the approach of the KF model to the modeling and multi-site prediction of rainfall in Cheliff watershed, the observations recorded at 39 precipitation stations, for both monthly and annual scales, observed over a period of 51 years (1959–2009) were considered. Hence, the system state variable is a vector giving the observed precipitations simultaneously at the 39 rainfall stations under consideration.

#### 3.1. Annual predictions

The estimation of initial values mentioned above is essential for starting calculations. To this end, the initial state vector  $X_{k/k-1}$  ( $k = 1$ ) consists of the average annual precipitation observed at the 39 stations ( $X_1, 2 \dots 39$ ); the associated error covariance  $P_{k/k-1}$  is a matrix with the value of 1,000 on the main diagonal and zero elsewhere; this has the advantage of giving more flexibility to the algorithm in order to fit the sensitive values in a relatively short time. This choice will lead to an increase in both of the covariance matrix  $P_{k/k-1}$  and the filter gain, allowing as such the filter to weigh more heavily the new coming measure. For the initial system noise covariance ( $Q$ ), a matrix with the value of 100 on the main diagonal and zero elsewhere was adopted. Since we hope the measures to be less noisy than the dynamic of the system, we have taken a measurement noise matrix ( $R$ ) with the value of 50 on the main diagonal and zero elsewhere. Concerning the estimation of the initial state-transition matrix ( $\phi_{k/k}$ ), the inter-correlations between the observations of the 39 rainfall stations have been considered. Harrison and Stevens [30] showed that the initial value of such a matrix does not substantially affect the results of the KF. Regarding the measurement matrix ( $H_k$ ), we opt for the unit matrix ( $39 \times 39$ ) as long as all rainfall stations provide observations.



The common observation period for the 39 rainfall stations is 51 years; the first 30 years are used to estimate the parameters of the model, while the last 21 years are used for its validation. The consecutive execution of KF equations over the observation period constitutes the application of the KF approach to the multi-site modeling and prediction of annual rainfall for the above mentioned rainfall stations.

Minimization of the prediction error covariance matrix trace is a convergence and optimality criterion of the filter. As shown in Fig. 2, the trace of this matrix starts with high values at the beginning of the calculations, and then decreases rapidly to converge to a stable positive value very close to zero. This convergence confirms the adequacy of the adjusted model to the studied process and means that the algorithm of calculation is effective, and hence, it provides optimal predictions.

Indeed, in this first phase, measurement is more credible than the estimation provided by the model because the calculations are unavoidably influenced by the initial conditions which are quite subjective, but in the progress of calculations, the latter are quickly discarded and the filter gives more importance to the measurement which contributes continuously to the improvement of the model parameters estimation.

So initially, when the model parameters are only rough estimates, the measure represents any objective information and the role of the gain matrix is precisely to ensure that the measure is heavily weighted in the estimation of the state parameters. In this case, the KF gain matrix takes important values and the result is an automatically bad estimation, which explains the relative big errors in the first iterations. However, with

the progress of the calculations when the confidence assigned to the accuracy of the parameters of the model begins to rise, the values of the gain matrix begin to gradually decrease to a value asymptotically close to zero (Fig. 3), which means decrease in the influence of the measurement in the update of the estimation of the model parameters and the associated errors.

The results also show that the predictions obtained for the 39 stations over the 51 years follow closely the historical observations, and differences between predictions and observations are minimal except for the first iterations. These differences are given in terms of percentage of relative error.

Fig. 4 is an example; it shows the predictions obtained at Teniet-el-had station with the percentage of associated relative errors. Examination of this figure shows how much closely the observations and predictions follow each other; the degree of this concordance is given in terms of relative error in percentage.

Another point of view, Fig. 5, consists of two figures and reveals the observations and predictions obtained simultaneously in 39 rainfall stations with the relative prediction error in percent. The top one corresponds to the beginning (1960), while the bottom one corresponds to the end (2009) of the calculations. On the first figure, one can see the large gap between observations and their corresponding estimates (predictions); this is due to the fact that at the beginning of calculations, the model parameters are not yet well established to give good estimations. The model which is initialized with subjective values must then rely more on the measurement (as the only objective available information) in order to learn from the provided information; this is

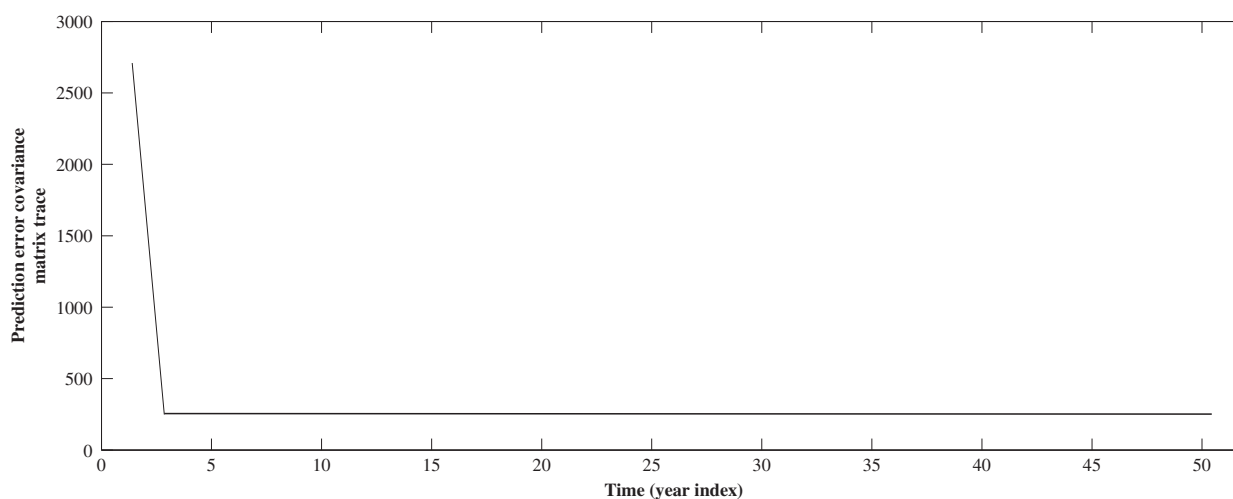


Fig. 2. Annual prediction error covariance matrix trace.

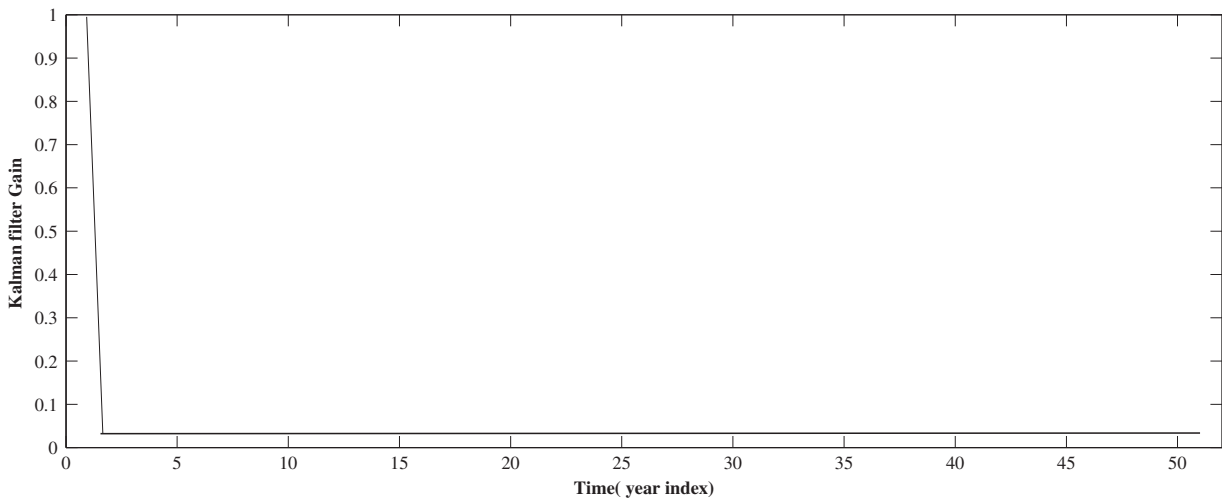


Fig. 3. Annual KF gain.

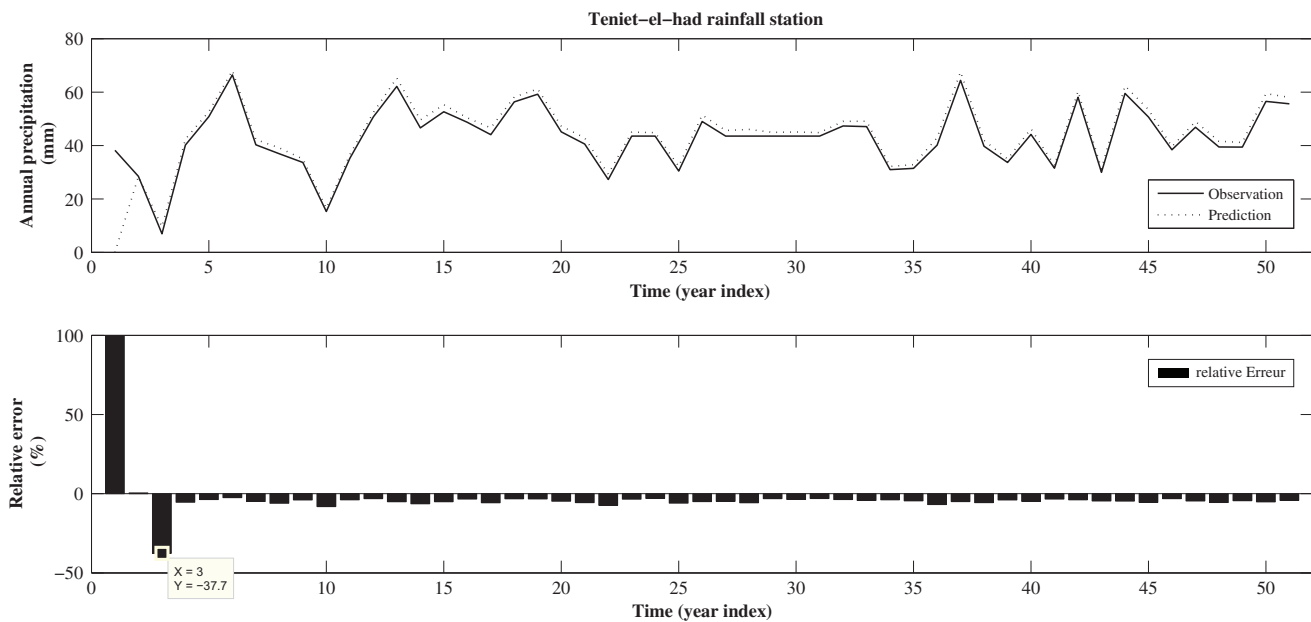


Fig. 4. Annual predictions obtained at Teniet-el-had rainfall station from 1959 to 2009.

essential for the KF to adapt itself automatically as soon as a new piece of information becomes available. Thereafter, over the next iterations as shown in the figure below, the result is a minimum distance between observations and their corresponding predictions. In terms of percentage of the relative error, this difference which begins with a maximum of  $-37.7\%$  at the station 18 (Teniet-el-had), for example, is reduced to the value of  $-4.3\%$  by the end of the calculations (2009).

Table 1 gives some average statistical characteristics for observations and predictions obtained from 39

rainfall stations during the study period. It is a summary of information that makes possible to compare the obtained predictions to the recorded observations; this is done in terms of some statistical parameters of central location (mean) and dispersion around this central position (standard deviation). Each one of these parameters is an average which is calculated for both temporal and spatial dimensions.

Regarding the temporal dimension, it is shown that the average prediction value is about 22.95 mm against an average observation value of 22.37 mm,

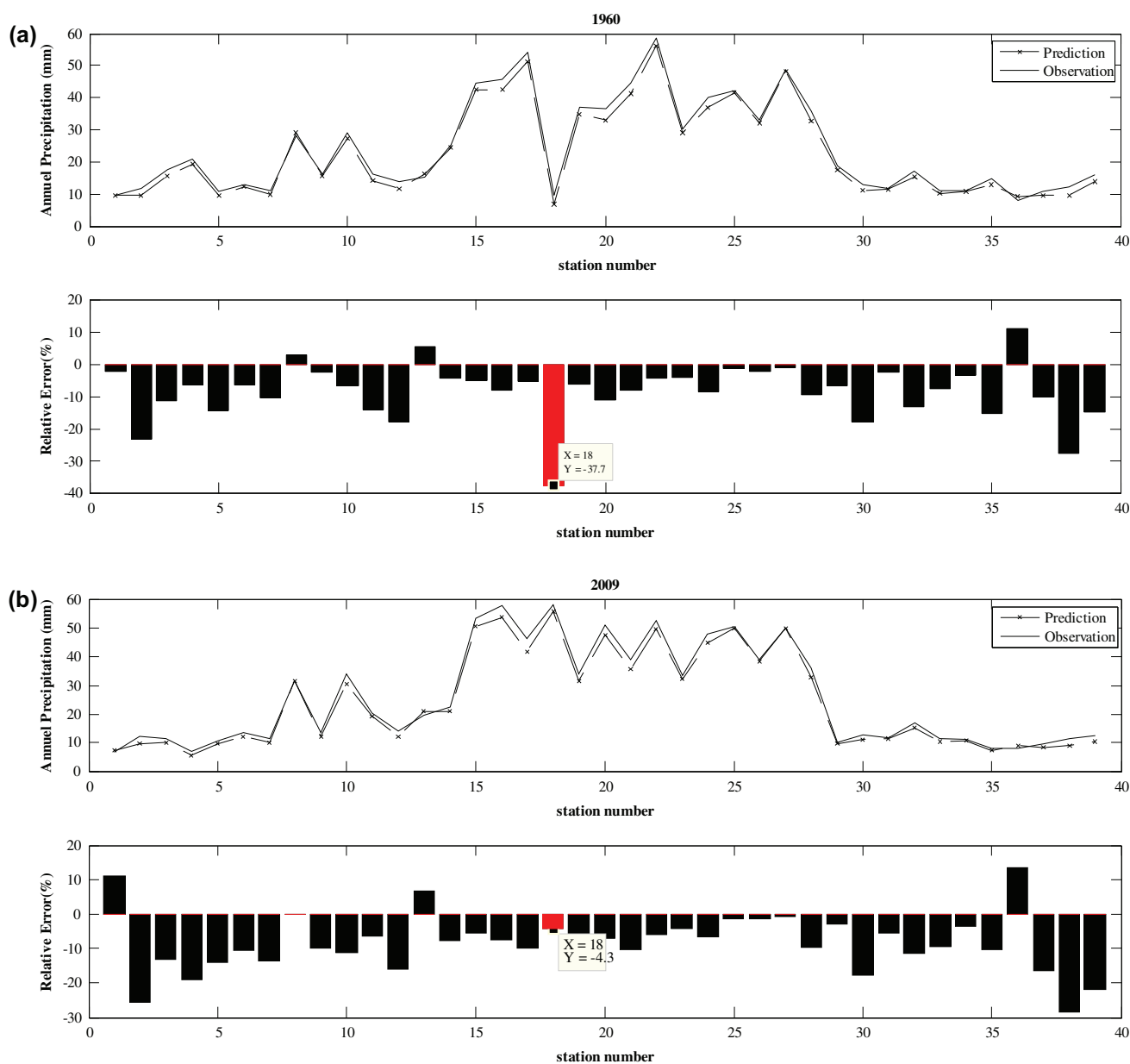


Fig. 5. Multi-site annual rainfall predictions from 1959 to 2009: (a) the beginning (1960) and (b) the end (2009).

Table 1

Some average statistical characteristics of annual observations and predictions obtained at the 39 rainfall stations (1959–2009)

	Observation		Prediction		Relative error (%)	
	Mean (mm)	St. dev. (mm)	Mean (mm)	St. dev. (mm)	Mean	St. dev.
Temporal	22.37	18.16	22.95	18.6	-4.22	-4.04
Spatial	22.6	21.46	24.24	22.36	-7.28	-11.23

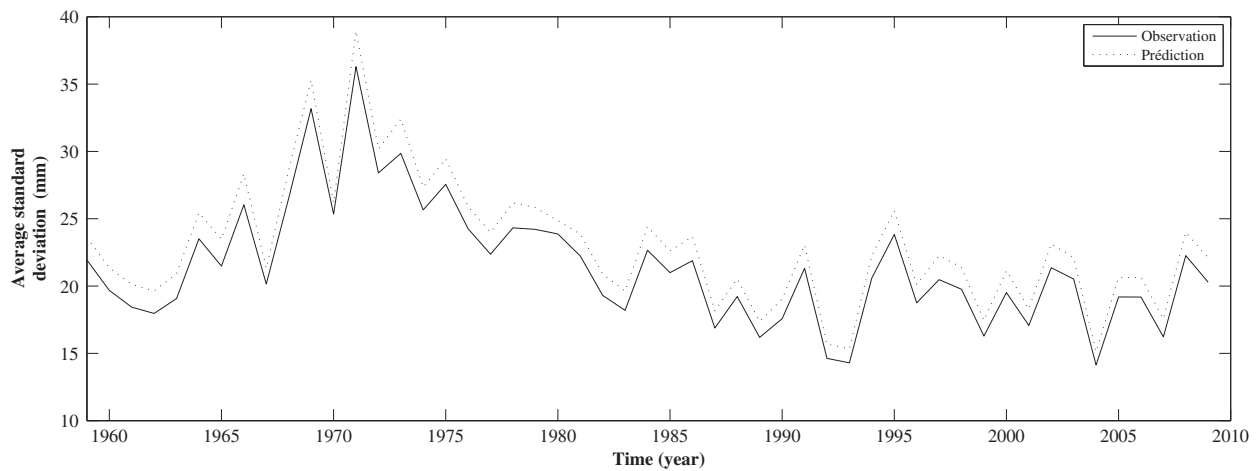


Fig. 6. Average standard deviation for annual observations and predictions at the 39 rainfall stations (1959–2009).

with a relative error of  $-4.22\%$ . On the other hand, the average standard deviation is  $18.6$  mm against a value of  $18.16$  mm for observations, with a relative error of  $-4.04\%$ . For the spatial dimension, an average prediction value of  $24.24$  mm is shown against the value of  $22.6$  mm for the observations average, with a relative error of  $-7.28\%$ , and an average standard deviation value of  $22.36$  mm for predictions against a value of  $21.46$  mm for the observations, with a relative error of  $-11.23\%$ . Fig. 6 depicts the temporal behavior of the average standard deviation for the annual observations and predictions calculated for the 39 stations from 1959 to 2009. It is obvious that values are

constantly higher for predictions, which express a possible tendency of KF to overestimation.

### 3.2. Monthly predictions

The application of KF was also carried out on monthly rainfall of 39 rainfall stations. Those data are observed from September 1959 to August 2009 and the whole observation period is about 612 months. As mentioned above, the first 240 months (20 years) are used for model estimation, whereas the rest of data is used for its validation.

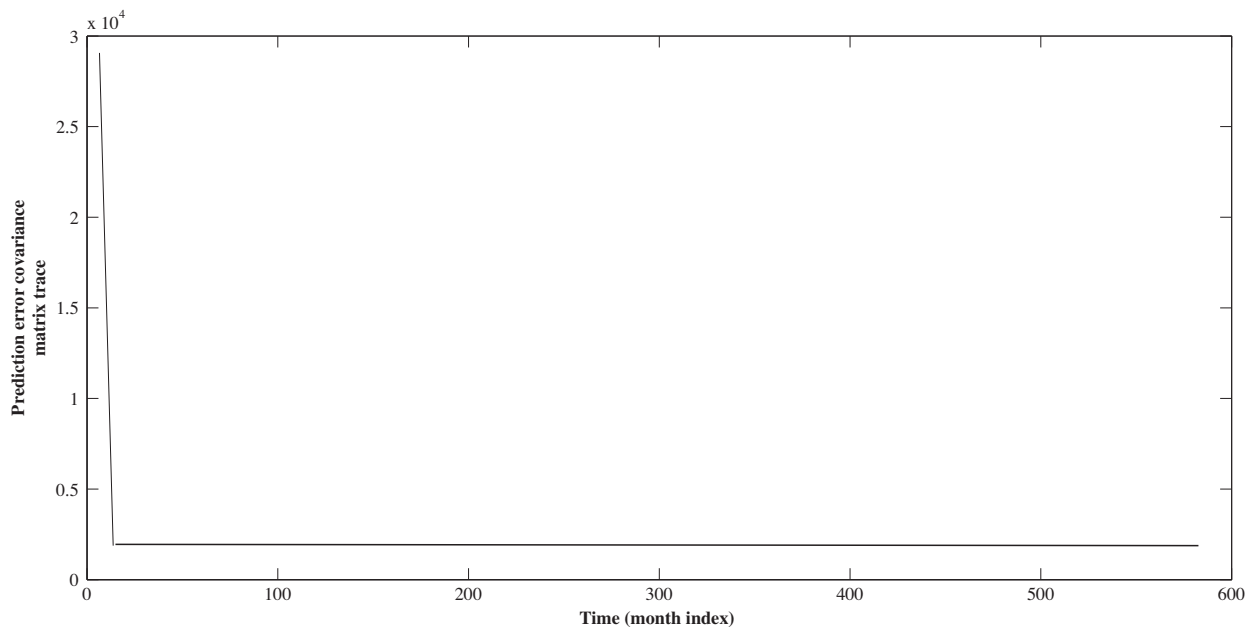


Fig. 7. Monthly prediction error covariance matrix trace.



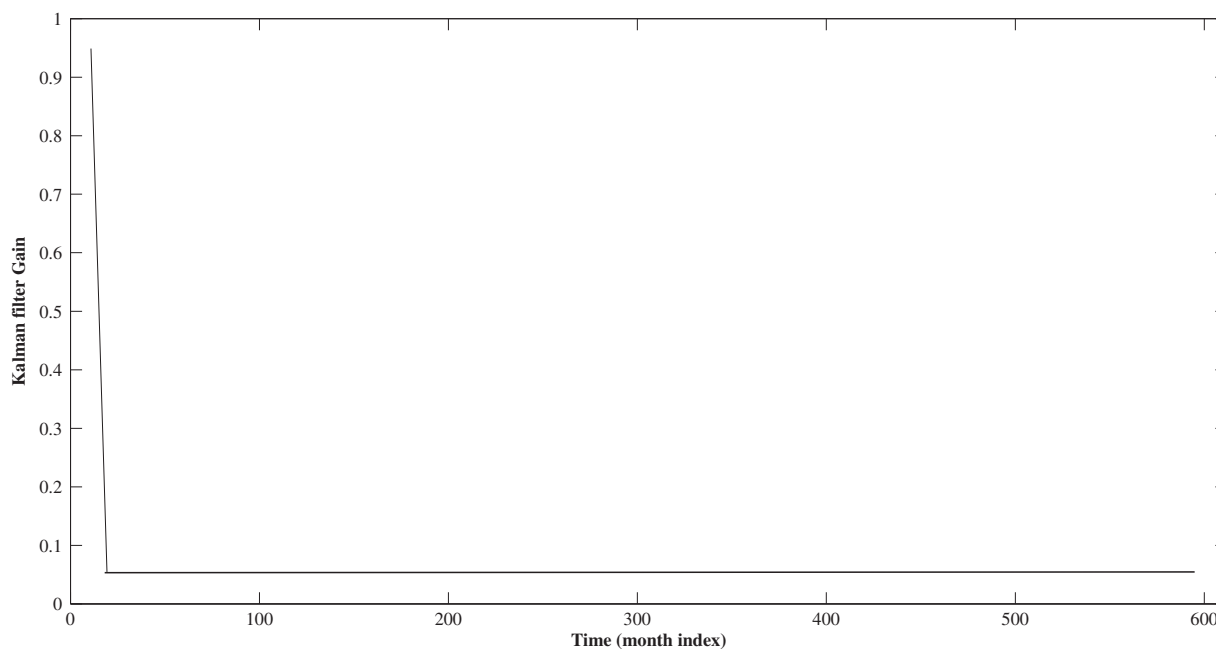


Fig. 8. Monthly KF gain.

The optimality of the obtained results is illustrated by Fig. 7; it is expressed by the shape of the curve, which describes the decreasing behavior of the major diagonal elements of the covariance matrix. To be noticed here, those major diagonal elements are nothing else than the prediction error variances. It is shown how those major diagonal elements take important values in the beginning of

calculations, and how after some iteration till the end of calculations, it continues to decrease in a regular way to reach values close to zero, but still stays positive.

The gain matrix as illustrated in Fig. 8 that presents a similar behavior to that of the prediction error covariance, which is indicative of the KF optimality; this means that the model parameters have reached

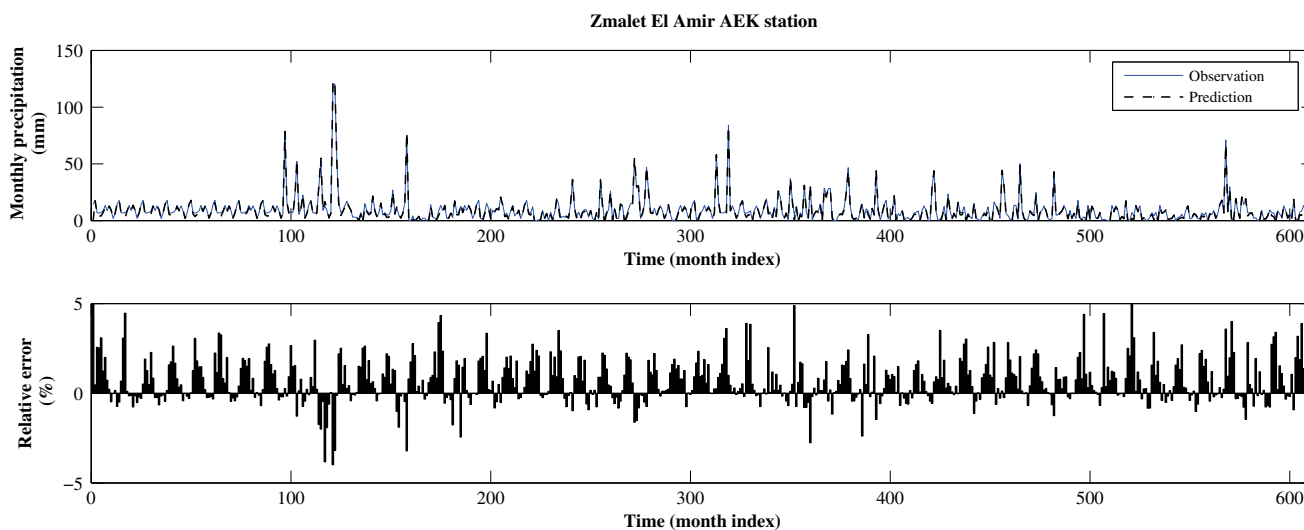


Fig. 9. Monthly predictions obtained at Zmalet El Amir AEK station (September 1959–August 2009).

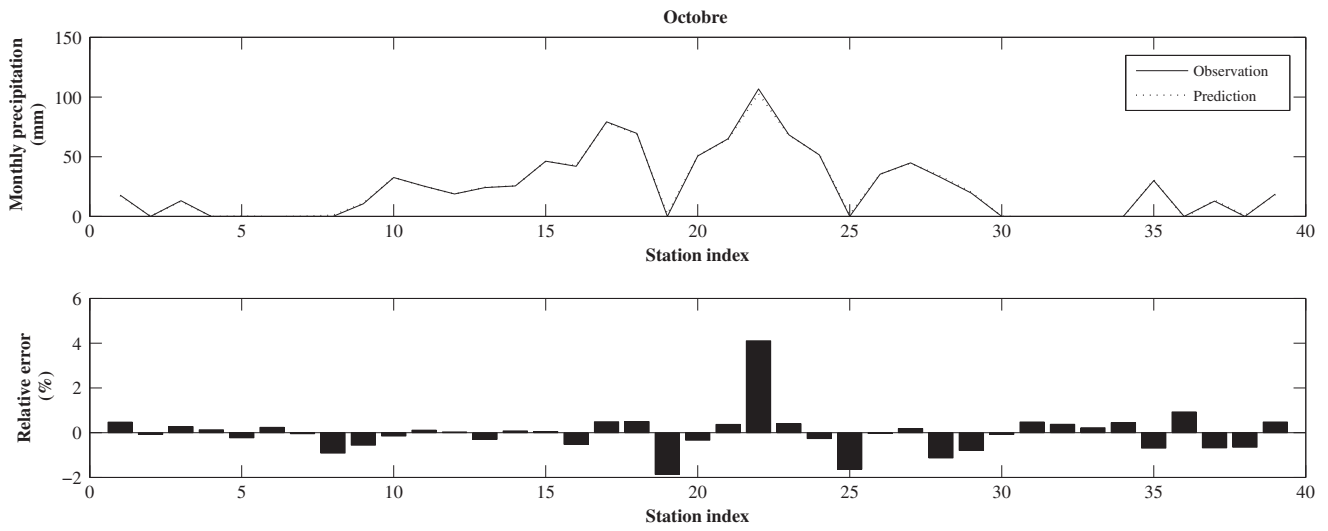


Fig. 10. Multi-site monthly rainfall predictions for October.

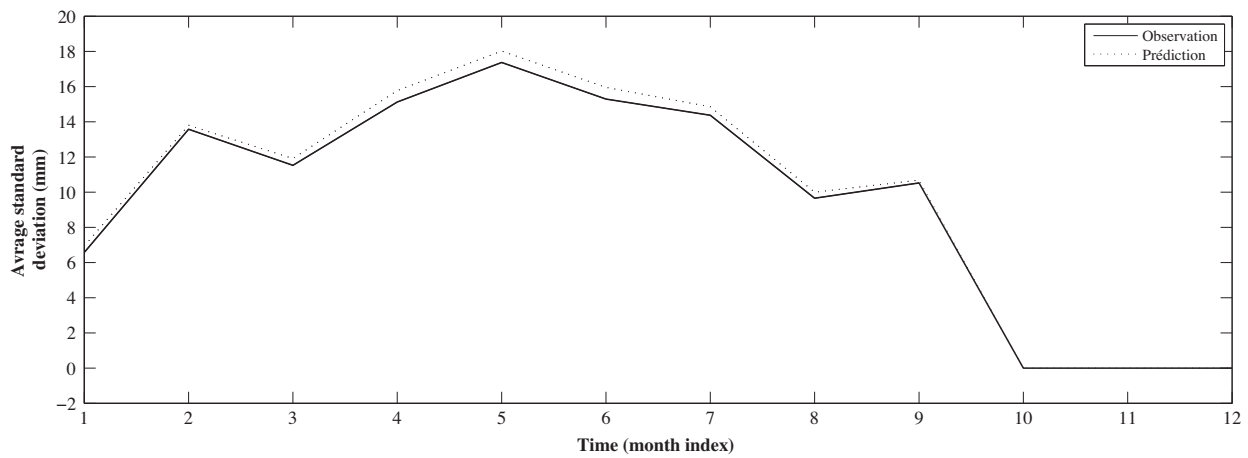


Fig. 11. Average standard deviation for monthly observations and predictions at the 39 rainfall stations (September 1959–August 2009).

Table 2

Some average statistical characteristics of monthly observations and predictions obtained at the 39 rainfall stations (September 1959–August 2009)

	Observation		Prediction		Relative error (%)	
	Mean (mm)	St. dev. (mm)	Mean (mm)	St. dev. (mm)	Mean	St. dev.
Temporal	14.62	12.90	15.22	13.03	-3.37	-1.41
Spatial	22.51	10.50	23.62	10.81	-4.95	-2.30

their optimal values during the estimation period (first 240 months) and presents an additional proof for the model suitability.

Fig. 9 illustrates an example of the multi-site monthly predictions obtained over the 612 months going from September 1959 to October 2009. It shows

how the observations and predictions at Zmalet el Amir AEK station follow each other very closely; the difference between them is given in terms of relative error in percentage, and it is obvious that this later varies only between  $-5$  and  $+5\%$ .

The quality of the obtained predictions can also be appreciated from a spatial point of view; as an example, the monthly observations and predictions obtained at the 39 stations during the month of October are illustrated in Fig. 10. A visual inspection of such figure shows that the differences between the observations and the corresponding predictions are so minimal that one cannot even distinguish between the two curves. This difference, translated in terms of relative error in percentage varies between 4 and 2%, and confirms once again the performance of the filter for the monthly scale.

The eventual tendency of overestimation of the model developed throughout this work is also observed for the monthly scale. Fig. 11 and Table 2 confirm this eventuality. It is shown that the average values of mean and standard deviation are higher for predictions relative to observations; this is observed both for temporal and spatial points of view.

Definitively, the obtained results show that the multi-site predictions obtained by KF closely follow the recorded observations; the average relative error in absolute value is about 7.28% and 4.95%, respectively, for annual and monthly scales. These prediction errors are minima; they are in all cases well below 10% which is highly acceptable. This constitutes a proof of KF efficiency in the modeling and the multi-site prediction of rainfall at Cheliff watershed, even in the presence of seasonality and in spite of the geographical variability and the disparity of rainfall stations.

#### 4. Conclusion

Throughout this work, a KF model is developed for multi-site rainfall predictions. For this purpose, monthly and annual rainfall time series have been investigated. These data are observed during a common observation period of 51 years (1959–2009) at 39 precipitation stations, in Cheliff watershed in northern Algeria.

The obtained predictions are very close to the observed values at the aforementioned stations over the study period. This is observed for the temporal as well as the spatial dimension, and indicates that the multi-site KF is practically an effective tool for rainfall modeling and prediction in Cheliff watershed.

The performance of the developed model was highlighted by calculating the percentage of the relative error of multi-site predictions; this average

percentage over the entire observation period is significantly less than 10%, which is quite acceptable.

A possible model tendency for overestimation was also highlighted by means and standard deviations, which are more important for the predictions than for the observations.

The developed model shows the great advantage of taking into account not only the stochastic nature of rainfall, but also their temporal and spatial variations.

In the end, one can say that, KF is a technique based on the concept of least squares, which has a very important property as the sequential optimization, which means that the model is updated in an adaptive fitting way as soon as a new system output becomes available. One advantage of this technique is that the stationary is not prerequisite, as it is the case in most of models in hydrology; this allows for changes in the model parameters and the variances, which is a way of accounting for the non-linearity of the concerned hydrological system response. Another advantage of the technique is that the application is made in the temporal domain. This characteristic plays an important role in the real-time forecasting of time series in hydrology. In addition, the KF algorithm can be started with minimum available objective information, and then a self-learning is automatically launched when new data arrive. These features make the KF one of the most appropriate tools, and therefore, the most used, particularly in situations where everything changes. Moreover, the results are optimal.

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