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Curved electrodialysis membranes: an innovative approach to enhance ion separation in EDMEM stacks

K. Mohammadi[a], M.R. Malayeri[a,b,*]

[a]*Institute of Thermodynamics and Thermal Engineering (ITW), University of Stuttgart, Pfaffenwaldring 6, D-70550 Stuttgart, Germany, Tel. +49 0 711/6856 9443; Fax: +49 0 711/6856 3503 (K. Mohammadi), Tel. +49 0 711/6856 7656; Fax: +49 0 711/6856 3503; emails: m.malayeri@itw.uni-stuttgart.de; malayeri@shirazu.ac.ir (M.R. Malayeri)*
[b]*School of Chemical, Petroleum and Gas Engineering, Shiraz University, Shiraz, Iran*

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ABSTRACT

The predominant driving force for ion separation in electrodialysis membrane (EDMEM) stacks is the electrical potential gradient. Nevertheless, excessive electrical potential gradient, besides higher energy consumption, could lead to other detrimental impacts such as concentration polarization and fouling. The orientation of electric field lines with respect to the main flow stream would instead be changed to offset the effect of such drawbacks. The present study introduces a potentially innovative technique utilizing curved EDMEM stacks by changing the orientation of electric field lines such that it would result in better stack performance. Mathematical simulations based on the analogy between heat and electrical conduction have been employed which confirmed the effectiveness of curved membrane. A mathematical model has also been developed which characterizes the performance of curved membranes.

Keywords: Electrodialysis; Membrane; Electric field; Ion separation; Ion mobility; Separation performance

1. Introduction

Electrodialysis membrane (EDMEM) is gaining increased attention for separation and purification purposes in various processes. These include, for example, the production of potable water as well as demineralization and ultrapure water treatment required for special industries like semiconductor, chemical, and food industries; and glycerin purification for antifreeze. In addition, most recently, the use of EDMEM stacks are studied for zero liquid discharge treatment of highly concentrated brine in RO plants [1].

For an optimal operation of electrodialysis (EDMEM) stacks, not only the order of magnitude of electrical potential gradient is important for ion separation, but the quality of generated electric field would also play a key role. As the demand for more efficient EDMEM stacks increases for various applications, the investigations which deal with the characteristics of electric field in ion exchange membranes seem crucial for an effective design and operation of such devices.

Increasing the overall electrical potential of an EDMEM stack might accelerate the ion separation.

*Corresponding author.

However, the sole increase in energy would cause the stack to encounter drawbacks such as concentration polarization and formation of deposits on membrane surfaces as typically illustrated in Fig. 1 [2]. For aqueous solutions, in particular, the concentration polarization occurs when the electrolyte concentration decreases on the desalting membrane surface in the boundary layer formed on the surface [3–6]. This would, in turn, deteriorate heat/mass transfer rates, causing severe problems in operating of EDMEM stacks especially for highly concentrated solutions [2].

Low fluid flow rates and poor design of spacers account for the concentration polarization in EDMEM stacks [5,6]. In addition, the uneven distribution of electric potential fields in the EDMEM stacks should be also accounted for such drawbacks. Mohammadi and Malayeri [7] showed that the orientation of electric field lines with respect to the flow direction of both concentrate and dilute streams would profoundly influence the ion separation. They showed that for a given operating condition, the maximum enhancement of ion separation will be achieved when the electric field lines tend to be perpendicular to the main flow streams at each spatial position of stack. Nevertheless, this is possible when the electrodes, which are utilized for generating the required electric potential, are large enough, i.e. when the electrodes have the same wetted area as membranes. However, some commercial EDMEM stacks employ smaller electrodes mainly to avoid excessive heating of membranes [8].

The present study aims at enhancing the performance of EDMEM stacks through structural change of membrane surfaces. To do so, this paper introduces innovative structural stack with "curved membranes" for enhancing the performance of ion separation. This is carried out by reorientation of the electric field lines perpendicularly with respect to the main flow stream to its maximum level. The paper begins with the introduction of objectives then proceeds with the conception and characterization of curved membranes followed by the development of a model which describes the impact of membrane curvature on separation performance.

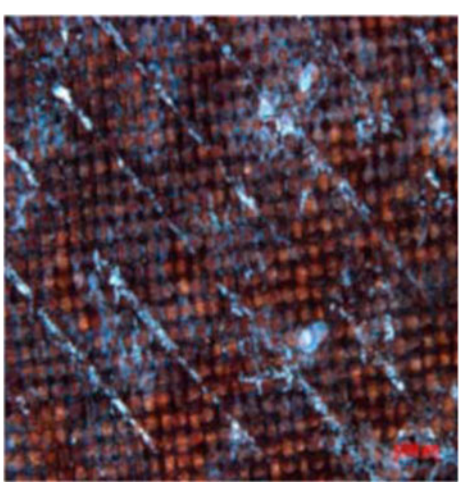

Fig. 1. Scale formation on a EDMEM membrane, adopted from Oren [2].

2. Problem statement

In electrodialysis, ions are transported through semi permeable membranes from one solution to another by exerting an electric potential gradient. The primary product of electrodialysis, i.e. the desalinated solution, is called dilute. In contrast, the byproduct known as concentrate is the other solution which will excessively be saturated in the course of the process. For desalination purposes, the saline water is desalinated to produce fresh water which is here the dilute, and the salty water with increased salinity is the concentrate.

In electrodialysis, generally speaking, the separation process is performed in a configuration called an electrodialysis cell, containing dilute and concentrate compartments. The cells are separated from each other by an anion exchange membrane (AEM) and a cation exchange membrane (CEM). The whole configuration is placed between two electrodes as typically depicted in Fig. 2. E-stream, as schematically shown in Fig. 2, represents the stream which flows near the electrodes. The composition of E-stream may be similar to dilute; however, it could also consist of different compositions to that of dilute. Ions may be transferred from the E-stream into the concentrate or from the dilute into the E-stream. The ion transfer from or into the

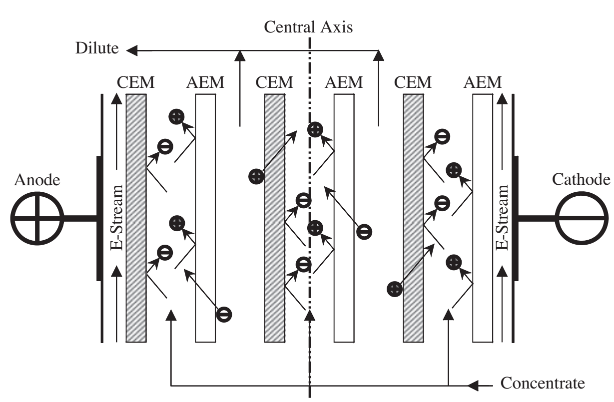

Fig. 2. Schematic of an electrodialysis stack.

E-stream depends on stack configuration and is essential to carry current across the stack and preserve electrically neutral stack solutions.

When an ion is subjected to an electric field $\vec{E}$, it moves due to an electric force in which its magnitude depends on the extent of electric field and the charge density of ion. The direction of this movement depends on the ion charge, i.e. anions move in the opposite direction of cations. Hence, and by referring to Fig. 2, the ion separation in an EDMEM stack depends on the exerted electric field and the characteristic velocity of both dilute and concentrate.

An effective ion separation can be achieved when the electric field lines are perpendicular to velocity field lines in all spatial points. This, nonetheless, is not attainable in many commercial EDMEM stacks due to geometrical limitations. Instead, the electric field lines are not perpendicular to flow field in all spatial points, and hence, the ion separation is not effective as schematically shown in Fig. 3.

Mohammadi and Malayeri [7] showed that the separation performance of an EDMEM stack can be enhanced by decreasing the characteristic perimeter and/or characteristic slope of electric field lines. One possibility to reduce these two characteristic parameters is to increase the size of electrodes used in stack. However, increasing the size of electrodes may result in complexities as augmentation of energy required for generating the indispensable electric field potential and consequently excessive heating of membranes. Accordingly, it is a common practice to place the smaller electrodes concentrically to the membranes (see Fig. 2). Under such circumstances, the characteristic perimeter and slope of electric field lines are not relatively small since the electric field lines are not perpendicular to the main flow streams at all spatial positions. One way to avoid this from happening is to modify the geometrical configuration of EDMEM stack such that the two characteristic parameters could be minimized. For this purpose, one possible option is to change the membrane curvature.

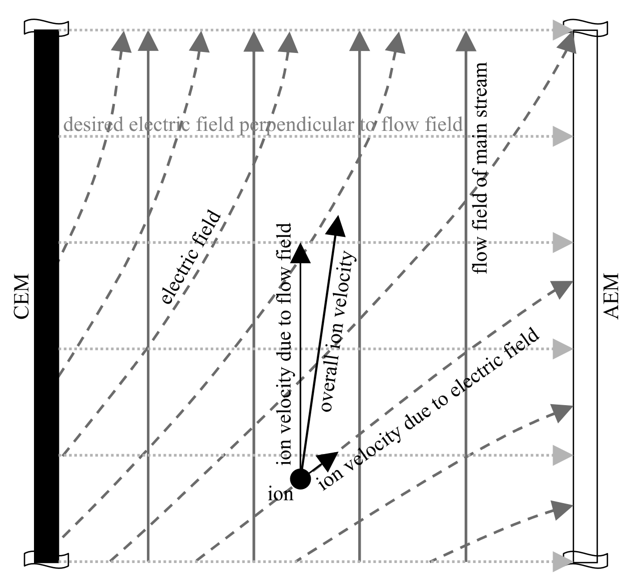

Fig. 3. Orientation of flow and electric fields under real and ideal conditions of an EDMEM stack.

The present common type of membranes used in EDMEM stacks are flat, i.e. the surface of both sides of membrane which is exposed to concentrate or dilute or even E-stream is a vertical flat plate. Instead, curved membranes can be implemented which would redirect the electrical potential fields. These are membranes in which at least one side is not planar but curved, i.e. a surface deviates from being flat. Now the challenge is whether the membrane curvature could influence the orientation of electric field lines. Provided an affirmative impact, it would then be imperative to develop a model that can characterize the impact of membrane curvature on the ion separation performance of EDMEM stacks.

3. Impact of membrane curvature on the orientation of electric field lines

At first, it is important to state that the geometrical curvature of a medium with different physical properties than its surrounding, e.g. permittivity, changes the orientation of electric field. A medium is defined as a material which is able to transmit, or permit, the electric field. For example, the membranes, dilute, and concentrate solutions are different media in an EDMEM stack.

In order to investigate the impact of curvature on the flow direction of electric field, simple two-dimensional (2D) simulations for conduction heat transfer have been performed considering analogy between heat and electrical conductions. To do so, a heat conductive medium with known thermal diffusivity is considered as the surrounding, see Fig. 4. Another medium with a thermal diffusivity of i.e. one-fourth of the surrounding is defined as the thermal barrier. For this thermal barrier different surface curvatures have been considered, namely flat, biconcave, and biconvex surfaces. The thermal barrier is located at the middle of the surrounding medium, and by this manner a 2D stack is built up. A temperature difference as the main driving force for heat conduction is imposed on the stack such as the complete area of the left wall defines the heat sink and one-fourth of the right wall

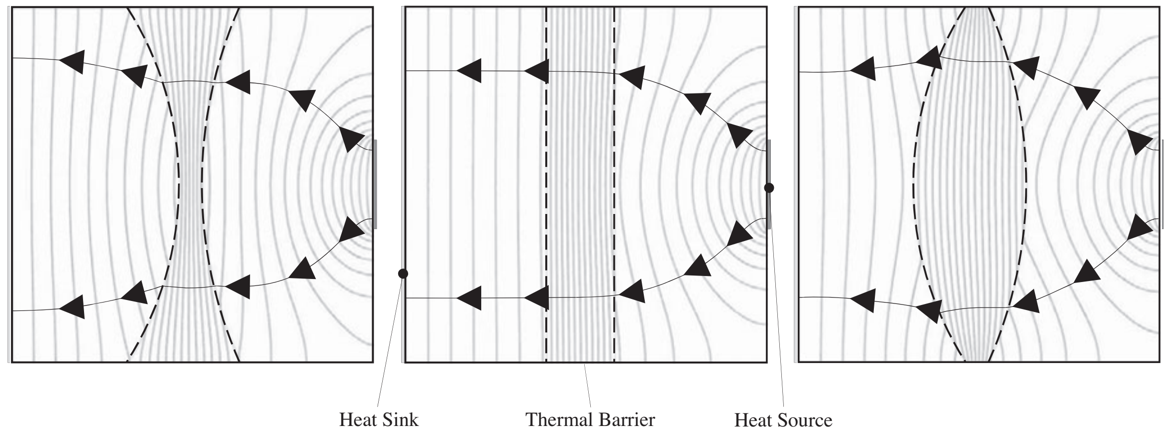

Fig. 4. The change in direction of heat flow using thermal barrier with different curvatures: biconcave (left), flat (middle), and biconvex (right) thermal barrier. The gray curves represent the isothermal lines and the black curves with arrows present schematically the heat flow direction. The border of thermal barrier is also presented by dash lines.

describes the heat source, as it is illustrated in Fig. 4. The heat sink is described by a constant wall temperature while a higher constant wall temperature defines the heat source. All other walls are supposed to be adiabatic.

As for EDMEM stacks, the surrounding medium represents the electrolyte, by analogy, and the thermal barrier corresponds to the membrane. Therefore, assuming a homogeneous surrounding medium accounts for a homogeneous electrolyte. This, however, may not be a plausible assumption since the electrolytes in both sides of a membrane are not similar to each other: the membrane is exposed on one side to dilute and on other side to concentrate. However, for the sake of simplicity, a homogeneous electrolyte in both sides of membrane has been assumed. Moreover, if the electrolyte on one side is different from the other side of the membrane, then impact of the change in physical properties of electrolyte on flow direction of electric field, or heat transfer, has to be taken into account which may cause difficulties in analysis of the final results. More studies are presently underway to address this. The resultant isothermal curves as well as the schematic direction of heat flow for the above-mentioned 2D heat transfer problem using different thermal barriers with different surface curvatures are presented in Fig. 4.

The results presented in Fig. 4 confirm that the heat flow lines diverge using a biconcave thermal barrier while they converge when a biconvex thermal barrier is used. Using the analogy between heat and electrical conduction, and considering the thermal barrier and its surrounding as membrane and dilute or concentrate, respectively, the results confirm that the membrane curvature will change the orientation of electric field lines: biconcave membrane causes divergence of electric field lines while biconvex membrane accounts to convergence. This is analogous to the effect of optical lenses in converging or diverging the light beam. Therefore, a thorough acquaintance with the physics of light transmission and refraction using optical lenses would be beneficial for characterization of the impact membrane curvature on separation performance of EDMEM stacks.

4. Short introduction to optical lenses

An optical lens can be considered as a contrivance which transmits and refracts light. The refraction, as an important phenomenon, essentially takes place on or near the surface of a medium and hence can be considered as a surface phenomenon. It is generally defined as the alteration in direction of a propagating flow, like light or other electromagnetic waves, due to a change in its transmission medium.

Generally speaking, optical lenses are classified according to the curvature of both optical surfaces. When both surfaces are convex, the lens is biconvex, and likewise if they are concave, the lens is biconcave. A plano-convex lens defines a lens which is flat on one side and the other side is convex. Accordingly, a lens with one flat and one concave side is plano-concave lens. A convex–concave or meniscus lens can be considered as a lens which is convex on one side and the other side is concave.

Fig. 5 depicts schematically a biconvex optical lens. When a collimated beam of light traveling parallel to the lens axis passes through a biconvex or

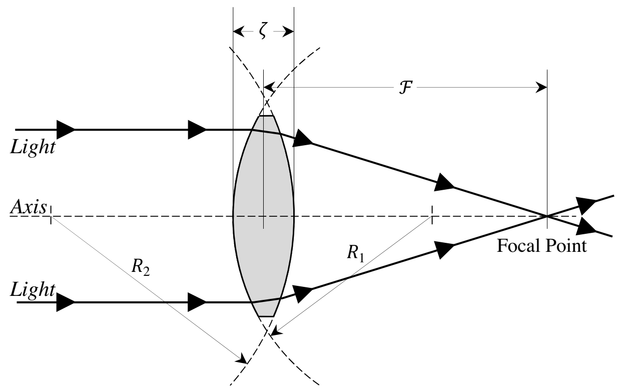

Fig. 5. Definition of focal length $\mathcal{F}$, lens thickness ζ, and radius of curvature R_1 and R_2 for a converging lens.

plano-convex lens, it will be focused to a point, as illustrated in this figure. The intersection point of these light rays is at a certain distance behind the lens. This point is called focal point and the corresponding distance is called focal length. Under such circumstances, the lens is a converging or positive lens. On the contrary, a biconvex or plano-convex lens is a diverging or negative lens. The light rays traveling parallel to the lens axis will be spread when they pass through the lens. However, one can also consider that they appear to be emanating from a particular point in front of the lens. This point is also focal point and the distance from the focal point to the lens is also known as the focal length. However, the focal length of a diverging lens is negative with respect to the focal length of a converging lens. Meniscus or convex–concave lenses can be either positive or negative, subject to the relative surface curvatures of both sides. A positive meniscus lens has a steeper convex surface and therefore is thicker at the center than at the periphery. On the contrary, a negative meniscus lens is thinner at the center than at the periphery since it has a steeper concave surface.

The focal length of a lens in vacuum can be calculated from:

$$\frac{1}{\mathcal{F}} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1-\frac{1}{n}}{R_1 R_2}\zeta\right) \quad (1)$$

where $\mathcal{F}$ is the focal length, n is the refractive index, ζ is the thickness of the lens, and R_1 and R_2 are the radius of curvature of the lens surface closest to and farthest from the light source [9]. These parameters are shown schematically in Fig. 5 for a converging lens.

The signs of the lens' radii of curvature indicate whether the corresponding surfaces are convex or concave. If R_1 is positive then the surface closest to the light source is convex, and if this surface is concave then R_1 is negative. On the contrary, the signs are reversed for the other surface farthest from the light source: for concave surface R_2 is positive, however, for convex surface R_2 is negative. If either radius is infinite, the corresponding surface is flat.

If an object is located at distance $\mathcal{D}$ from the lens, it may appear larger or smaller than its actual size. This enlargement is quantified by magnification number M. $M > 1$ denotes the enlargement while $M < 1$ refers to a reduction in size which is sometimes called minification or de-magnification. When $M = 1$, the object appears in the same size to that of its actual.

$$M = \frac{\mathcal{F}}{\mathcal{F} - \mathcal{D}} \quad (2)$$

One important parameter in optical lenses is the dimensionless number n. Refraction index n describes how light, or any other radiation, propagates through a medium and is defined in optics as the ratio of speed of light in vacuum to the speed of light in the medium, i.e. c/v.

The effect of optical lenses on divergence or convergence of light rays is similar to the effect of curved membranes on redirection of electric field lines (compare Figs. 4 and 5). Therefore, it can be hypothesized that when the electric field passes through a curved membrane, the electric field lines will virtually converge to a focal point located at a distance far from the membrane equal to the focal length. A positive focal length relates to converging curved membrane, however, a negative focal length counts for diverging curved membrane. Consequently, a magnification concept can be postulated for curved membrane: when the source of electric field, e.g. the electrode, is located at a defined distance from the curved membrane, it behaves like a bigger or smaller electrode depending on the corresponding focal length of the curved membrane. These are explained in the next section mathematically in more details.

5. Characterization of curved membranes

In optics, the medium refraction index n depends on the speed of light in the medium. In other words, n indicates how fast light moves in a medium. In conduction heat transfer, thermal diffusivity $\alpha = k/\rho c_p$, in the same sense, describes how fast heat moves in a medium since thermal diffusivity is, in fact, the measure of thermal inertia. By analogy, the inertia of electrical conduction moving in a medium can be described by an equivalent diffusivity equal to $\mathcal{V}\sigma/C$, where $\mathcal{V}$ is the volume, σ is the electrical conductivity, and C is the capacitance of the medium [10]. Since, in an EDMEM stack, the geometrical arrangement of membranes and flow channels are mostly similar to parallel plates (see Fig. 2), therefore, volume $\mathcal{V}$ and capacitance C can be related to capacitor surface area A_C and the capacitor thickness d_C as $\mathcal{V} = A_C d_C$ and $C = \varepsilon A_C/d_C$, where ε is the permittivity [11]. The permittivity of a medium describes how much electric field per unit charge is produced in that medium: the higher the permittivity, the more are the electric field lines. Thus, the corresponding refraction index of a medium for electrical conduction can be defined as the root of the ratio of a reference diffusivity to the medium diffusivity or $\sqrt{(\mathcal{V}\sigma/C)_0/(\mathcal{V}\sigma/C)} \sim \sqrt{(\varepsilon/\varepsilon_0)/(\sigma_0/\sigma)}$, where subscript 0 indicates the reference medium. Nevertheless, this definition is not precise and even sufficient, and indeed, the refraction index has to be described in a more rational form.

The inertia of an electromagnetic radiation moving through a medium can be expressed by the medium permeability μ and permittivity ε. Permeability describes the ability of a medium to support the formation of a magnetic field within itself. Therefore, the refraction index of a medium for an electromagnetic radiation can be formulated as $\sqrt{\mu_r \varepsilon_r}$, where $\mu_r = \mu/\mu_0$ is called relative permeability and is the ratio of the medium permeability to the vacuum permeability. Likewise, relative permittivity $\varepsilon_r = \varepsilon/\varepsilon_0$ is the ratio of the medium permittivity to the vacuum permittivity. Importantly, the relative permeability of most materials and also any material at sufficiently high field strength tends to 1. Hence, in most cases, the corresponding refraction index could be estimated as $\sqrt{\varepsilon_r}$ [12–14]. This is a reasonable conclusion since by considering the wave-like nature of electric field, it can be applied for describing the refraction index of a medium for electric field [15,16]. Thereby, the refraction index n_e concerning the electric field can be described as:

$$n_e = \sqrt{\varepsilon_r} \tag{3}$$

By substituting Eq. (3) in Eq. (1), the corresponding characteristic focal length $\mathcal{F}_e$ for a curved medium, e.g. curved membrane, as for the electrical field can be introduced as:

$$\frac{1}{\mathcal{F}_e} = (n_e - 1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1 - \frac{1}{n_e}}{R_1 R_2}\zeta\right) \tag{4}$$

For very thin shapes, the thickness ζ is significantly smaller than the magnitude of both radii of curvature, hence $\zeta \ll |R_1 R_2|$. This is a practical assumption since in EDMEM stacks the thickness of membranes as well as the distance between two adjacent membranes where the dilute or concentrate solution flows are very thin in comparison to the width and length of stack. Therefore, for very thin mediums, Eq. (4) will yield to the following simpler form:

$$\frac{1}{\mathcal{F}_e} = (n_e - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{5}$$

It is important to mention that for two thin curved media 1 and 2 which are separated by some distance ξ, the focal length for the combined system is given by [9]:

$$\frac{1}{\mathcal{F}_e} = \left(\frac{1}{\mathcal{F}_e}\right)_1 + \left(\frac{1}{\mathcal{F}_e}\right)_2 - \xi\left(\frac{1}{\mathcal{F}_e}\right)_1\left(\frac{1}{\mathcal{F}_e}\right)_2 \tag{6}$$

Eq. (6) facilitates the calculation of the characteristic focal length $\mathcal{F}_e$ for an EDMEM stack with curved membranes. Through Fig. 2, and considering the membranes depicted in this figure as curved membranes, if m denotes the number of AEM membranes, then the number of CEM membranes is also equal to m. Consequently, the number of concentrate (or dilute) compartments is m, while the number of dilute (or concentrate) compartments is $m-1$. Neglecting the effect of E-Stream, and knowing that $m > 1$, $\mathcal{F}_e$ for an EDMEM stack, can be defined as:

$$\begin{aligned}\frac{1}{\mathcal{F}_e} = {} & \left(n_e^{S1} - 1\right)\sum_{j=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\left(\frac{1}{R_{4j}} - \frac{1}{R_{4j+1}}\right) \\ & + \left(n_e^{M1} - 1\right)\sum_{j=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\left(\frac{1}{R_{4j+1}} - \frac{1}{R_{4j+2}}\right) \\ & + \left(n_e^{S2} - 1\right)\sum_{j=0}^{\left\lfloor\frac{m}{2}\right\rfloor-1}\left(\frac{1}{R_{4j+2}} - \frac{1}{R_{4j+3}}\right) \\ & + \left(n_e^{M2} - 1\right)\sum_{j=0}^{\left\lfloor\frac{m}{2}\right\rfloor-1}\left(\frac{1}{R_{4j+3}} - \frac{1}{R_{4j+4}}\right)\end{aligned} \tag{7}$$

The formulation of $\mathcal{F}_e$ in Eq. (7) is obtained for a half of the stack considering its central axis, see Fig. 2. Superscript S1 refers to stream 1, e.g. dilute (or concentrate), while S2 denotes stream 2, e.g. concentrate (or dilute). Importantly, stream 1 is defined as the stream which also flows through the middle compartment where the central axis of the stack is defined. For instance, referring to Fig. 2, stream 1 is the concentrate stream. Moreover, superscript M1 and M2 refers to membrane 1, e.g. AEM (or CEM), and membrane 2, e.g. CEM (or AEM), respectively.

Considering the sign convention used to represent the signs of the medium's radii of curvature, Eq. (7) shows that, when for all media index n_e is greater than 1, then $\mathcal{F}_e < 0$ for a series of negative meniscus curved media, while $\mathcal{F}_e > 0$ for a sequence of positive meniscus curved media. For a special condition where $n_e^{S1} \cong n_e^{S2} \cong n_e^{M1} \cong n_e^{M2} = n_e$, knowing that $R_0 = +\infty$, $\mathcal{F}_e = -|R_{2\mathcal{M}}|/(n_e - 1)$ and $\mathcal{F}_e = +|R_{2\mathcal{M}}|/(n_e - 1)$ for negative and positive meniscus curved media.

Referring to Figs. 2 and 4 and considering Eq. (2), if an electrode with height $\mathcal{H}$ is located at distance $\mathcal{D}$ from the last membrane (in other words, by considering Fig. 2, $\mathcal{D}$ is the thickness of cross-sectional flow area of E-Stream), the apparent height of electrode will be then:

$$Mh = \frac{\mathcal{F}_e}{\mathcal{F}_e - \mathcal{D}} h = \begin{cases} \frac{|\mathcal{F}_e|}{|\mathcal{F}_e| - \mathcal{D}} h & : \quad \text{set of } + \text{ meniscus curved membranes} \\ \frac{|\mathcal{F}_e|}{|\mathcal{F}_e| + \mathcal{D}} h & : \quad \text{set of } - \text{ meniscus curved membranes} \end{cases} \tag{8}$$

Since, in most cases $|\mathcal{F}_e| > \mathcal{D}$, Eq. (8) shows that using a set of positive meniscus curved membranes causes magnification in apparent size of electrode, while a set of negative meniscus curved membranes minifies it. Eq. (8) also states that the apparent size of electrode does not change when all the membranes are flat, since the radius of curvature of a flat membrane is ∞ and therefore $\mathcal{F}_e = +\infty$.

The apparent height of electrode can be attributed to the ion separation performance of an ED stack. Indeed, the reduction in characteristic perimeter and/or characteristic slope of electric field lines, which will be, respectively, denoted by $\overline{\mathcal{P}_E}$ and $\overline{\tan\beta}$, cause enhancement in separation performance. Noteworthy, both the aforesaid characteristic parameters will reduce by increasing the electrode height as shown by Mohammadi and Malayeri [7] thus:

$$\overline{\mathcal{P}_E} = \left(3(1+\phi) - \sqrt{(3+\phi)(1+3\phi)}\right)\frac{\pi\Lambda}{4}\bigg|\phi = \frac{\mathcal{H}}{\Lambda}\sqrt{1 - \left(\frac{h}{\mathcal{H}}\right)^2} \geq 0 \tag{9}$$

$$\overline{\tan\beta} = \tan\left(\frac{\Lambda}{\mathcal{H}}\overline{\beta}\ \tan\ \overline{\beta}\right)\bigg|\ \overline{\beta} = \text{Arctan}\left(\frac{\mathcal{H} - h}{\Lambda}\right) \tag{10}$$

In the above equations, Λ is the distance between two electrodes or the width of stack, $\mathcal{H}$ is the height of stack, and h is the height of electrodes.

The implementation of curved membranes in an EDMEM stack will represent the electrodes with an apparent height Mh. Therefore, instead of using h, the apparent height Mh has to be considered in the corresponding equations for $\overline{\mathcal{P}_E}$ and $\overline{\tan\beta}$. Combining Eqs. (8)–(10) yields:

$$\overline{\mathcal{P}_E} = \left(3(1+\phi) - \sqrt{(3+\phi)(1+3\phi)}\right)\frac{\pi\Lambda}{4}\bigg|\phi = \frac{\mathcal{H}}{\Lambda}\sqrt{1 - \left(\frac{Mh}{\mathcal{H}}\right)^2} \geq 0 \tag{11}$$

$$\overline{\tan\beta} = \tan\left(\frac{\Lambda}{\mathcal{H}}\overline{\beta}\ \tan\ \overline{\beta}\right)\bigg|\ \overline{\beta} = \text{Arctan}\left(\frac{\mathcal{H} - Mh}{\Lambda}\right) \tag{12}$$

Eqs. (8), (11), and (12) show that the implementation of positive meniscus curved membranes will decrease the characteristic perimeter $\overline{\mathcal{P}_E}$ and the characteristic slope $\overline{\tan\beta}$. In this case, enlarging $\mathcal{D}$ or decreasing $\mathcal{F}_e$ which may be accomplished by reducing the radii of curvature of membranes and/or by increasing the relative permittivity ε_r of membranes and electrolytes; see Eq. (3), will amplify the magnification number M, and consequently, the apparent height of electrode which will enhance the separation performance.

The variation of $\overline{\tan\beta}$ and $\overline{\mathcal{P}_E}$; in dimensionless form of $\overline{\mathcal{P}}_E/\mathcal{H}$, as a function of electrode height and focal length; dimensionless forms of $h/\mathcal{H}$ and $|\mathcal{F}_e|/\mathcal{H}$, for a typical value of $\mathcal{D}$; $\mathcal{D}/\Lambda = 0.02$, is represented in Figs. 6–9. In these figures, it is assumed that all media have similar refraction indices, i.e. $n_e^{S1} \cong n_e^{S2} \cong n_e^{M1} \cong n_e^{M2} = n_e$. Therefore, if R_{2m} defines the curvature of the nearest membrane to electrode, then $|\mathcal{F}_e|/\mathcal{H}$ will be equal to $|R_{2m}|/(\mathcal{H}(n_e - 1))$. This means that for a given $\mathcal{H}$, for instance, increasing the value of $|\mathcal{F}_e|/\mathcal{H}$ can be achieved by enlarging the curvature $|R_{2m}|$ and/or by reducing the refraction index n_e. Figs. 6–9 also show that as the focal length increases, $\overline{\tan\beta}$ and $\overline{\mathcal{P}}_E$ will decrease using convex while will increase using concave membranes. Therefore, convex membranes will enhance the ion separation and vice versa. Interestingly, these figures show that the membrane curvature does not significantly change the values of $\overline{\tan\beta}$ and $\overline{\mathcal{P}}_E$ for $|\mathcal{F}_e|/\mathcal{H}$ greater than about 0.25. Moreover, the effect of electrode height h on $\overline{\tan\beta}$ and $\overline{\mathcal{P}_E}$ is more profound to that of the corresponding effect of focal length $|\mathcal{F}_e|$. The impact of focal length $|\mathcal{F}_e|$ on $\overline{\tan\beta}$ and $\overline{\mathcal{P}}_E$, on the other hand, is more significant on large electrode

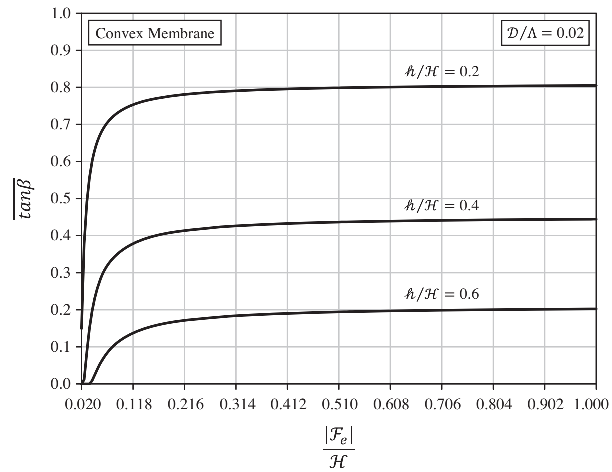

Fig. 6. Impact of focal length $|\mathcal{F}_e|$ on $\overline{\tan\beta}$ in a stack using convex membranes.

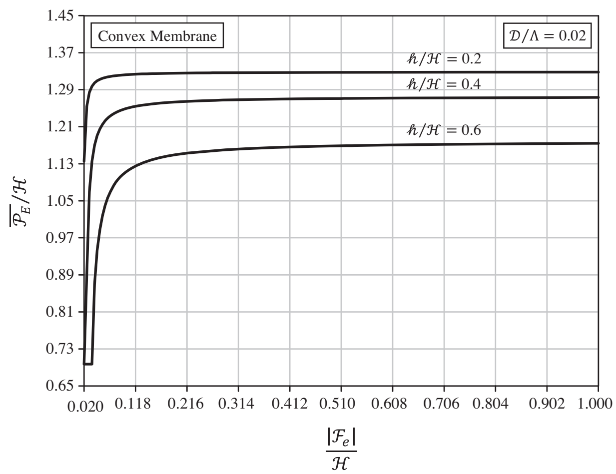

Fig. 7. Impact of focal length $|\mathcal{F}_e|$ on $\overline{\mathcal{P}_E}$ in a stack using convex membranes.

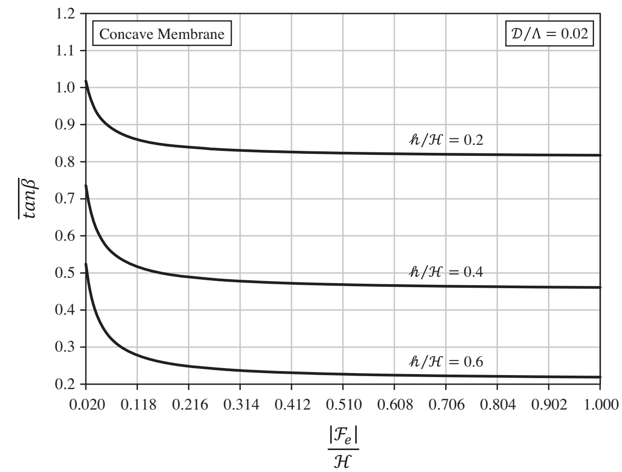

Fig. 8. Impact of focal length $|\mathcal{F}_e|$ on $\overline{\tan\beta}$ in a stack using concave membranes.

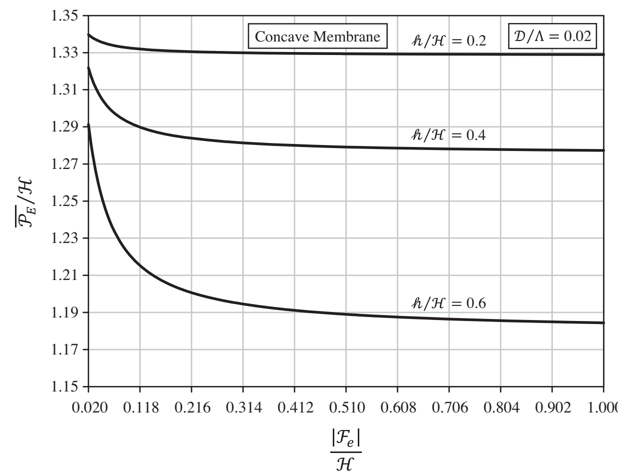

Fig. 9. Impact of focal length $|\mathcal{F}_e|$ on $\overline{\mathcal{P}_E}$ in a stack using concave membranes.

height, e.g. for $h/\mathcal{H} \geq 0.6$. Similar analysis of the performance of flat membranes under various operating conditions and physical properties of the fluids is reported by Mohammadi and Malayeri [7].

6. The importance of n_e in EDMEM stacks with flat membranes

Considering Eq. (8), the change in M with respect to $\mathcal{F}_e$ can be obtained as:

$$\frac{\mathrm{d}M}{\mathrm{d}\mathcal{F}_e} = \begin{cases} \dfrac{-\mathcal{D}}{(|\mathcal{F}_e| - \mathcal{D})^2}\dfrac{\mathcal{F}_e}{|\mathcal{F}_e|} = \dfrac{-\mathcal{D}}{(|\mathcal{F}_e| - \mathcal{D})^2} & : \text{ set of } + \text{ meniscus curved membranes} \\ \dfrac{\mathcal{D}}{(|\mathcal{F}_e| + \mathcal{D})^2}\dfrac{\mathcal{F}_e}{|\mathcal{F}_e|} = \dfrac{-\mathcal{D}}{(|\mathcal{F}_e| + \mathcal{D})^2} & : \text{ set of } - \text{ meniscus curved membranes} \end{cases} \tag{13}$$

Eq. (13) indicates that when $\mathrm{d}M/\mathrm{d}\mathcal{F}_e < 0$, then any decrease in $\mathcal{F}_e$ will augment the magnification number M. This interpretation has some practical implications, especially for ED stacks with flat membrane: when the radii of curvature are kept constant, only if the index n_e increases will $\mathcal{F}_e$ decrease.

The significance of the above conclusion especially for EDMEM stacks with flat membranes can be described if one considers two stacks in which all the geometrical and operational conditions and physical properties except index n_e are equal. For these two stacks, since the membranes are flat, the radius of curvature is ∞, hence $\mathcal{F}_e = \infty$. In order to show that the stack with greater n_e has a better separation performance than the other one with lesser n_e, it is compulsory to apply a mathematical attestation.

Set $\{\mathcal{F}_e\}$ is defined as a well-ordered set with 0 as its least element in which all elements of this set are less than the absolute value $|\mathcal{F}_e|$. The cardinality of $\{\mathcal{F}_e\}$, which will be presented as $\#\{\mathcal{F}_e\}$, shows the size of $\{\mathcal{F}_e\}$. Ratio n_r is now defined as $n_r = (n_e - 1)/(n_e^\circ - 1) > 1$, where superscript "$\circ$" refers to the stack with lower index. Hence $n_r\mathcal{F}_e$ corresponds to the characteristic focal length of the stack with n_e°. Comparison of set $\{\mathcal{F}_e\}$ with new set $\{n_r\mathcal{F}_e\}$ shows that $\#\{n_r\mathcal{F}_e\} > \#\{\mathcal{F}_e\}$. This confirms that even for set $\{\mathcal{F}_e\}$ which has no upper bound, i.e. when $\mathcal{F}_e = \infty$, always $\#\{n_r\mathcal{F}_e\} > \#\{\mathcal{F}_e\}$ which consequently means $n_r\mathcal{F}_e > \mathcal{F}_e$. The practical implication of this mathematical statement is that if the calculated values of separation performance for different ED stacks are equivalent; even if all the stacks have flat membranes where $\mathcal{F}_e = \infty$, then the stack which has the greater index n_e is preferred. Therefore, the new introduced refraction index n_e should be considered as an important parameter in design and judgment between different EDMEM stacks. From economic point of view, the curved membranes allow smaller but electrically stronger electrodes which in turn are less prone to burnt-out and weaker concentration polarization. These would then potentially reduce the cost of operation and troubleshooting in EDMEM stacks. Nonetheless such technical benefits must be addressed through rigorous further theoretical and experimental studies.

7. Concluding remarks

In an ED stack, the effect of surface curvature of membrane either converging, or diverging, the lines of electric field are very similar to the effect of optical lens on light. The present study shows that the performance of an ED stack in terms of ion separation can be improved by applying converging or positive curved membranes. Moreover, an important parameter which has to be considered in design and characterization of ED stacks, whether the implemented membranes are curved or flat, is the refraction index $n_e = \sqrt{\varepsilon_r}$. An ED stack with greater refraction index n_e has a better separation performance than the one with lower n_e.

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Nomenclature

Latin symbols

A_C	—	capacitor surface area (m^2)
c	—	speed of light in vacuum to the speed of light in the medium (m/s)
C	—	capacitance of a medium (F)
c_p	—	specific heat capacity (J/(kg K))
$\mathcal{D}$	—	distance between an object or electrode and the lens or curved medium (m)
d_C	—	capacitor thickness (m)
$\mathcal{F}$	—	focal length (m)
h	—	height of electrodes (m)
$\mathcal{H}$	—	height of stack (m)
k	—	thermal conductivity (W/(m K))
m	—	number of AEM or CEM membranes (–)
R	—	radius of curvature (m)
v	—	speed of light in a medium (m/s)

Greek symbols

α	—	thermal diffusivity (m^2/s)
$\bar{\beta}$	—	corresponds to characteristic orientation angle of electric field lines (rad)
ε	—	permittivity (F/m)
ε_0	—	permittivity of vacuum ($8.854187817 \times 10^{-12}$ F/m)
ζ	—	thickness of lens or curved medium (m)
Λ	—	distance between two electrodes or the width of stack (m)
μ	—	permeability (H/m)
μ_0	—	permeability of vacuum ($1.2566370614 \times 10^{-6}$ H/m)
ξ	—	separation distance between two lenses or curved media (m)
ρ	—	density (kg/m^3)
σ	—	electrical conductivity (S/m)

Mathematical symbols, operators, and special notations

#	—	the cardinality of set which shows the size of the set
$\vert$	—	mathematical symbol which means "such that"
$\vert\ \vert$	—	absolute value
$\lceil\ \rceil$	—	ceiling: $\lceil x \rceil$ means the ceiling of x, i.e. the smallest integer greater than or equal to x
$\lfloor\ \rfloor$	—	floor: $\lfloor x \rfloor$ means the floor of x, i.e. the largest integer less than or equal to x
$\{\ \}$	—	well-ordered set
$\overline{\mathcal{P}_E}$	—	characteristic perimeter of electric field (m)
$\overline{\tan\beta}$	—	characteristic slope of electric field (–)
$\mathcal{V}$	—	volume of a medium (m^3)

Subscripts

0	—	refers to the reference medium or the position at central axis of ED stack

1 — refers to the surface of lens or curved medium closest to the light source or electrode
2 — refers to the surface of lens or curved medium farthest from the light source or electrode
$2m$ — refers to the nearest membrane to electrode
e — refers to the electric field

Superscripts

$\circ$ — refers to a medium or stack which has a lower refraction index
$M1$ — refers to membrane 1, e.g. AEM (or CEM)
$M2$ — refers to membrane 2, e.g. CEM (or AEM)
S1 — refers to stream 1, e.g. dilute (or concentrate), which also flows through the middle compartment where the central axis of the stack is defined
S2 — refers to stream 2, e.g. concentrate (or dilute)

Dimensionless numbers

n — refraction index
n_r — refraction index ratio, $n_r = (n_e - 1)/(n_e^\circ - 1) > 1$
M — magnification number
ε_r — relative permittivity
μ_r — relative permeability
ϕ — characteristic aspect ratio of an ED stack

References

[1] M. Asraf-Snir, J. Gilron, Y. Oren, Gypsum scaling on anion exchange membranes during Donnan exchange, J. Membr. Sci. 455 (2014) 384–391.
[2] Y. Oren, The fate of sparingly soluble salts in electrodialysis employed in ZLD treatment chains, Meeting on Water Technology Research, 11–13 October, Darmstadt, Germany, 2011.
[3] Y. Tanaka, Concentration polarization in ion-exchange membrane electrodialysis: The events arising in an unforced flowing solution in a desalting cell, J. Membr. Sci. 244(1) (2004) 1–16.
[4] H.J. Lee, S.H. Moon, S.P. Tsai, Effects of pulsed electric fields on membrane fouling in electrodialysis of NaCl solution containing humate, Sep. Purif. Technol. 27 (2002) 89–95.
[5] V. Mavrov, W. Pusch, O. Kominek, S. Wheelwright, Concentration polarization and water splitting at electrodialysis membranes, Desalination 91 (1993) 225–252.
[6] K.E. Bouhidel, K. Oulmi, Concentration polarization in electrodialysis: Buffer solutions experimental method, Desalination 132 (2000) 199–204.
[7] K. Mohammadi, M.R. Malayeri, Characterization of ion separation in electrodialysis membranes using ${H_E}^{\beta-}$, a dimensionless number, Desalin. Water Treat. (2014) 1–14, doi: 10.1080/19443994.2014.940213.
[8] PCCell GmbH, *Electrodialysis Cell Unit: ED 64 0 02, Operation & Maintenance Instruction*, Heusweiler, Germany, 2006.
[9] W.J. Smith, Modern Optical Engineering, the Design of Optical Systems, third ed., McGraw-Hill, New York, NY, 2000.
[10] J. Pyrhnen, T. Jokinen, V. Hrabovcov, Design of Rotating Electrical Machines, John Wiley & Sons, Chichester, 2008.
[11] J. Bird, Electrical and Electronic Principles and Technology, fourth ed., Elsevier, Amsterdam, 2010.
[12] D.K. Cheng, Field and Wave Electromagnetics, second ed., Addison-Wesley, New York, NY, 1989.
[13] H. Huang, D.C. Fraser, Magnetic permeability and electrical resistivity mapping with a multifrequency airborne EM system, Explor. Geophys. 29 (1998) 249–253.
[14] X. Ma, A.J. Peytona, Y.Y. Zhao, Eddy current measurements of electrical conductivity and magnetic permeability of porous metals, NDT & E Int. 39 (2006) 562–568.
[15] J.C. Maxwell, A dynamical theory of the electromagnetic field, Philos. Trans. R. Soc. London 155 (1865) 459–512.
[16] J.C. Maxwell, A Treatise on Electricity and Magnetism, vols. 1–2, Clarendon Press, Oxford, 1873.