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Parameter sensitivity and uncertainty analysis of a stormwater runoff model

D. Kim[a], H. Kim[b], G. Pak[c], M. Jung[b], K.J.B. Mallari[b], A.C.C. Arguelles[c], J. Yoon[b,c,*]

[a]*National Institute of Environmental Research, Incheon 404-170, Korea, email: doguyam@hanmail.net*
[b]*Program in Environmental Technology and Policy, Korea University, Sejong 339-700, Korea, emails: tinymons@korea.ac.kr (H. Kim), inver@korea.ac.kr (M. Jung), kristinemallari@korea.ac.kr (K.J.B. Mallari), jyyoon@korea.ac.kr (J. Yoon)*
[c]*Department of Environmental Engineering, Korea University, Sejong 339-700, Korea, emails: gjpak00@korea.ac.kr (G. Pak), cathianne@korea.ac.kr (A.C.C. Arguelles)*

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ABSTRACT

In this study, Generalized Likelihood Uncertainty Estimation (GLUE) methodology was used to perform uncertainty analysis of a stormwater model, which randomly generates parameter sets and identifies behavioral ones with higher likelihood. Latin Hypercube Sampling (LHS) is used to generate parameter sets. The Model for Urban Stormwater Improvement Conceptualization was chosen as an appropriate model for stormwater runoff. Prediction limits are determined by selecting the cutoff threshold for likelihood function. Study area is Goonja Drainage Basin located in the city of Seoul, Korea. From the results, maximum likelihood value for calibration is 0.78 and 0.73 for validation. The *p*-factors for the calibration and validation are 87 and 83%, respectively. The *p*-factor for all storm events is 85%. These are all acceptable values as the results are considered good when 60% or more of the observed data are bracketed by prediction limits. Overall, it was shown that, using GLUE methodology with LHS, the model calibrated well for the basin considered in this study.

Keywords: GLUE; LHS; MUSIC; Sensitivity analysis; Uncertainty analysis

1. Introduction

Urbanization has caused the distortion of natural water cycle because increased impervious areas interrupt the process of infiltration [1]. To resolve this problem, increasing interest has been focused on facilities that are able to reduce the runoff at urban areas such as Best Management Practices (BMPs) and Low Impact Development (LID) [2]. The prediction of effectiveness of BMPs is required in order to make an informed decision on the selection and sizing of BMPs [3]. To achieve this, application of models that are able to assess the BMPs are essential for the quantitative interpretation. However, these models have a lot of uncertainty in parameter estimation. Therefore, it is necessary to perform uncertainty analysis in order to quantify the uncertainty in the results of model application [4].

Of the models that are capable of simulating BMPs, the Model for Urban Stormwater Improvement Conceptualization (MUSIC) [5] is most versatile and has specialty in covering the wide array of BMPs [6]. Like many other models, it is also subjected to parameter uncertainty, and relevant studies were done

*Corresponding author.

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in the literature [7,8]. Dotto et al. [7] used frequentist and Bayesian method, while Dotto et al. [8] used simplified Monte Carlo Makov Chain for uncertainty analysis. Another popular approach in uncertainty analysis is Generalized Likelihood Uncertainty Estimation (GLUE) methodology [9]. Although it is simple in concept and relatively easy to implement [10], no work has been done so far on the uncertainty of MUSIC model using GLUE method.

To fill this gap, in this study, we aimed to perform uncertainty analysis on MUSIC model using GLUE method. In order to increase computational efficiency, Latin Hypercube Sampling (LHS) was used which samples much less number of parameters sets than Monte Carlo simulation yet still representing quite well the statistical estimates of model output [11]. A data-set, collected during four rainfall events, was used for the analysis.

2. Study methods and materials

2.1. Study site

The Goonja Drainage Basin, located in the Seoul metropolitan region, was selected as the test site due to data availability. This basin has an area of 96.4 ha and is located on the left bank downstream of the Joongrang Stream (Fig. 1). Hydrologic and water quality observatories are located at the outlet of the drainage basin. The four runoff data-sets (17 May 2005, 24 August 2005, 22 May 2006, and 28 June 2007) and matching rainfall data at 10 min interval from the Seoul Rainfall Observatory, were used for calibration and validation.

Fig. 1. Aerial view of the Goonja Drainage Basin.

2.2. Model description

MUSIC is an urban runoff model developed by Cooperative Research Centre for Catchment Hydrology in Australia. MUSIC can be employed in modeling LID devices. The model time step may range from a minimum of 6 min to a day while the model can be applicable in a range of different watershed areas (0.01–100 km^2). The model is classified as a semi-distributed model that can be used for both lumped and distributed modeling. In addition, the model has the capacity of both single event and continuous simulation. The runoff component of MUSIC consists of impervious area runoff, pervious area runoff, and baseflow. Runoff from impervious area is generated when rainfall exceeds impervious store capacity. Runoff from pervious area is generated by two methods: infiltration excess and saturation excess. Infiltration excess can occur when rainfall exceeds infiltration, and saturation excess can occur when soil moisture exceeds soil moisture capacity. Finally, baseflow is calculated as a constant ratio of groundwater store.

2.3. Sensitivity analysis

Prior to uncertainty analysis of runoff-related parameters of MUSIC, sensitivity analysis was first performed to understand the sensitivity of the parameters using Condition Number (CN) [12]. The runoff component in MUSIC consists of 12 parameters. These parameters are distributed into four categories such as impervious area, pervious area, groundwater, and routing properties. Default values of each parameter were set up from MUSIC Version 5.0 User Manual [5] where these values as used in Brisbane, Australia are given. Each default parameter value is increased and decreased by 20%, and peak flows corresponding to these perturbed parameters are then used to calculate CN as:

$$\mathrm{CN} = \frac{k}{Q_{\mathrm{peak}}} \quad \frac{\Delta Q_{\mathrm{peak}}}{\Delta k} \tag{1}$$

where k is a default value of a parameter, and Q_{peak} is peak flow corresponding to the default value of a parameter. The larger the CN, the more sensitive the parameter is. The sign means either proportional, (if positive) or inversely proportional (if negative) relationship.

2.4. Uncertainty analysis

Matott et al. [13] reviewed and compared 26 programs for uncertainty analysis according to three categories: assessment method, frequency of use in previous studies, and availability to public. This study presented that GLUE is most widely used and has been applied in numerous literature for uncertainty analysis. It is also easily accessible to users. Therefore, GLUE was selected in this study for the method of uncertainty analysis. The method has been applied for the parameter assessment of various hydrology and water quality models such as IHDM (Institute of Hydrology Distributed Model), SWAT (Soil Water Assessment Tool), and HSPF (Hydrologic Simulation Program Fortran).

Uncertainly analysis through GLUE is performed as follows using GLUEWIN [14], a code that implements GLUE methodology:

(1) Find a defined distributional pattern or the maximum and minimum values of an input parameter.
(2) Construct set of parameters by sampling processes such as Random Sampling and Latin Hypercube Sampling (LHS).
(3) Calculate likelihood function value using observed and simulated data corresponding to constructed set of parameters.
(4) Select the cutoff threshold for behavioral simulations.
(5) Calculate rescaled likelihood weights using the behavioral parameter sets and form a cumulative distribution for the model output.
(6) Suggest confidence interval using the cumulative distribution of the model output.

Before the application of GLUE method, it is very important to appropriately select configuration parameters, likelihood function, and the cutoff threshold because these have a considerable effect in the uncertainty analysis. Generation of parameter sets are done using LHS method which was first developed by McKay et al. [15]. The biggest advantage of LHS method is that it can generate a small number of parameter sets whose effect is the same as those represented by a large number of parameter sets. The LHS method is done by extracting one unique sample from each sector after a parameter range is divided into n sectors. Each parameter was assumed to be of uniform distribution because there was no known statistical distribution about selected parameters for uncertainty analysis which are impervious area, rainfall threshold, K and θ. The method proposed by Melching [11] was used in determining the number of parameter sets which calculated the mean and standard deviation of average flow, peak flow, and runoff volume for the simulation results. Uhlenbrook and Sieber [16] also used this method for generating parameter sets.

3. Results and discussion

3.1. Sensitivity analysis

Table 1 shows default parameter values and the result of sensitivity analysis expressed in terms of CN. The CN values in the table represents average CN for four rainfall events mentioned earlier in Section 2.

As can be seen in Table 1, impervious area properties (impervious area ratio and rainfall threshold) and

Table 1
Parameters used in MUSIC model and corresponding condition numbers

Category	Parameter	Unit	Default setting	Condition number
Impervious area properties	Impervious area	%	50	0.99
	Rainfall threshold	mm	1	−0.12
Pervious area properties	Soil storage capacity	mm	120	0
	Initial storage	% of soil storage capacity	30	0
	Field capacity	mm	80	0
	Infiltration capacity coefficient-a	mm	200	0
	Infiltration capacity Exponential-b	–	1	0
Groundwater properties	Initial depth	mm	10	0
	Daily recharge rate	%	25	0
	Daily baseflow rate	%	5	0
Routing properties	K	min	30	−0.35
	θ	–	0.25	0.25

routing properties (K and θ) were found to have a keen effect on peak flow. Among these four parameters, impervious area was found to be the most sensitive parameter controlling peak flow as indicated by the highest CN. The CN of rainfall threshold has a negative value because increasing rainfall threshold in impervious area decreases the simulated runoff. The CN values of the parameters related to pervious area properties were all zeros, showing insensitivity of these parameters on peak runoff. In addition, the CN values of groundwater properties related to base flow were also all zeros. These are thought to be because pervious runoff and base flow are significantly smaller than impervious runoff in this heavily urbanized drainage basin. On the contrary, routing properties were found to be sensitive on peak flow as they control the timing of peak flow. Based on the results of sensitivity analysis, uncertainly analysis was performed on the most sensitive parameters which were found to be impervious area ratio, rainfall threshold, K, and θ.

3.2. Uncertainty analysis

3.2.1. Parameter sampling

For uncertainty analysis, different number of parameter sets (integer multiples of four, which is the

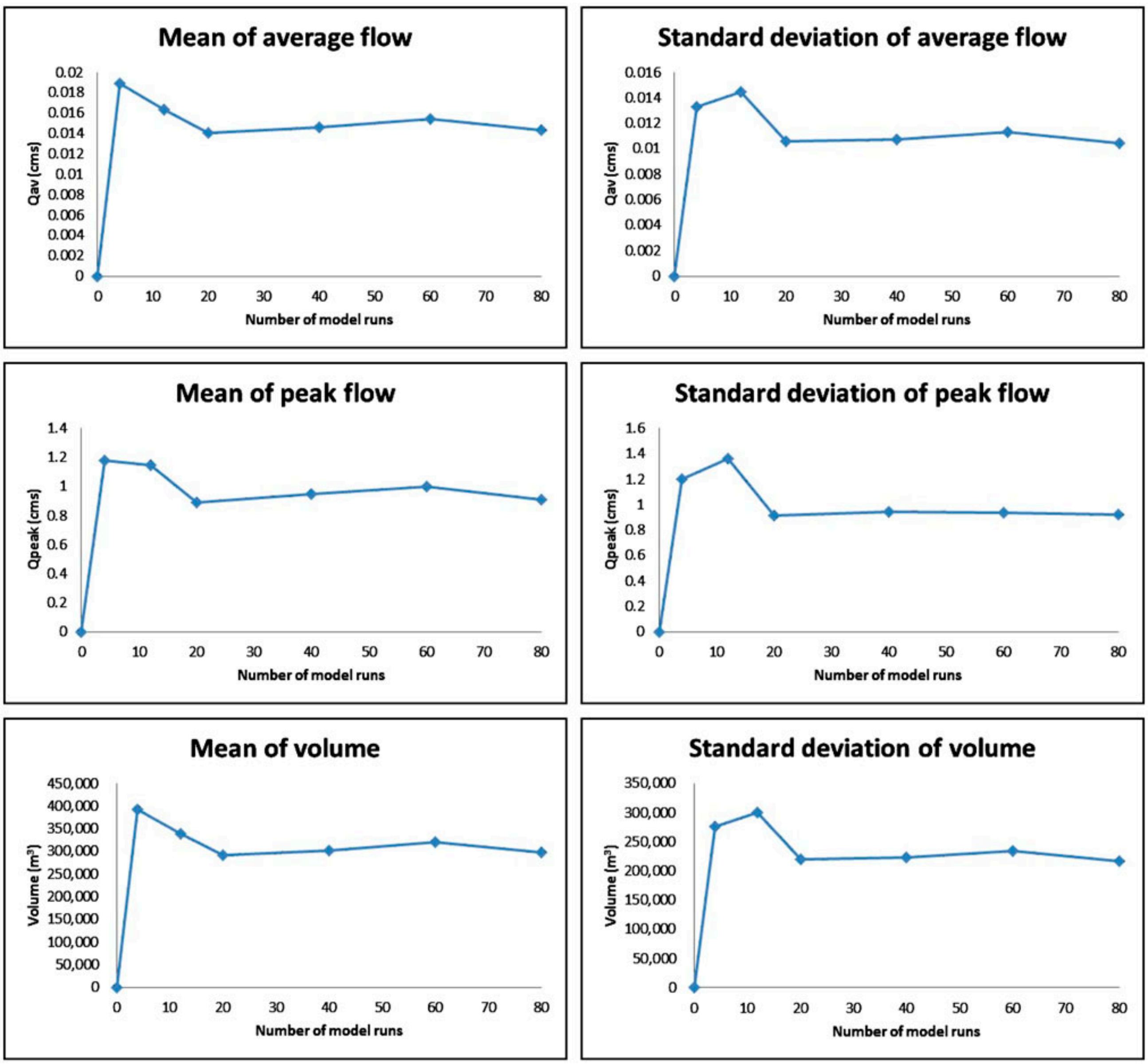

Fig. 2. Mean and standard deviation of average flow, peak flow, and volume.

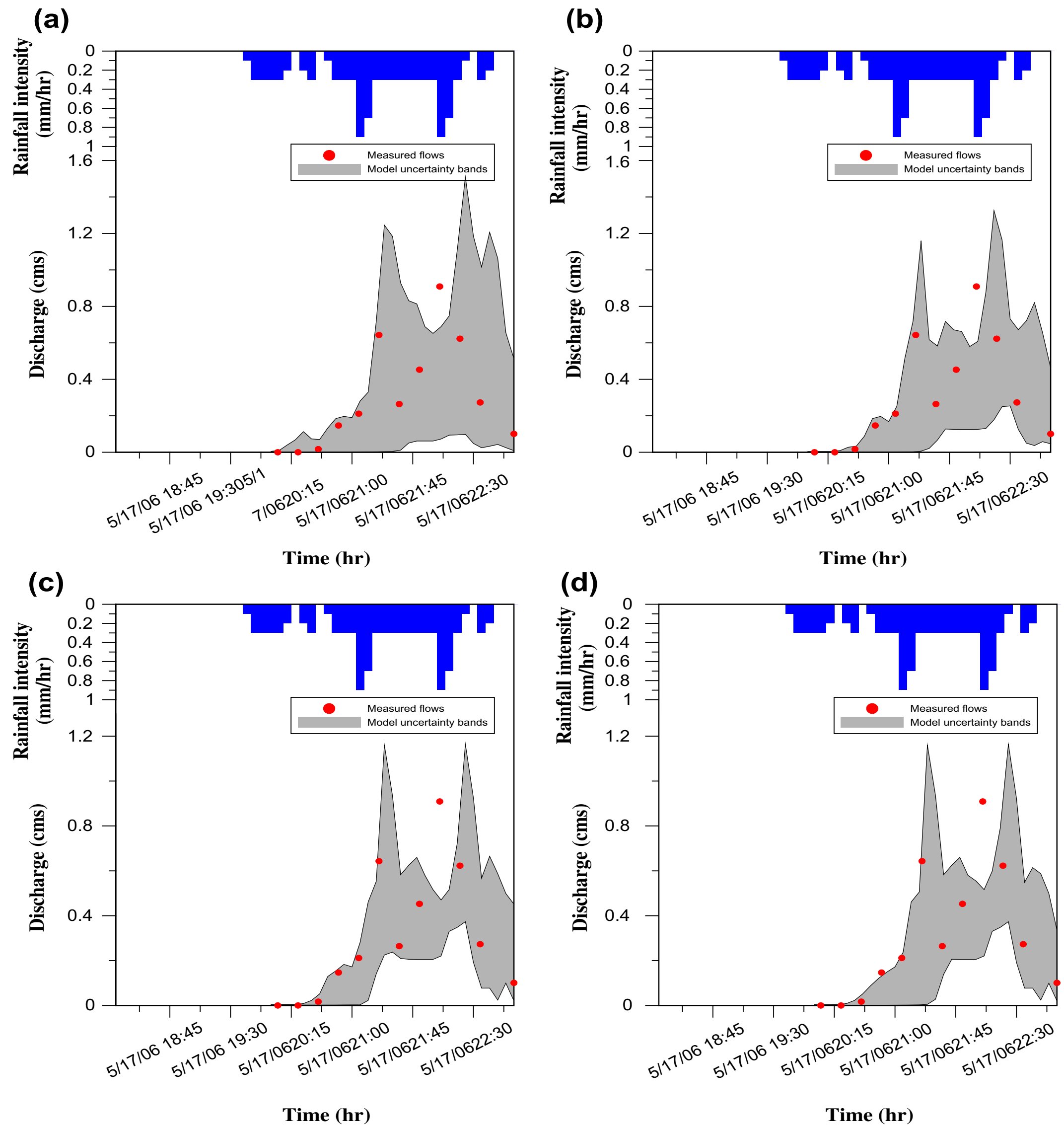

Fig. 3. The 90% prediction limits for different threshold values. (a) 30 parameter sets—0.168 (threshold); (b) 20 parameter sets—0.26; (c) 15 parameter sets—0.317; (d) 10 parameter sets—0.361.

Table 2
p-factor values for different thresholds used for likelihood function

Threshold value	0.168	0.26	0.317	0.361
Behavioral parameter sets	30	20	15	10
p-factor (%)	90	90	90	80

number of parameters analyzed) were generated using LHS for the four selected sensitive parameters. The model was then run using these parameter sets for 22 May 2006 rainfall event, and statistics regarding flow such as average flow, peak flow, and volume were plotted against the number of model runs. As can be seen from Fig. 2, the values of the mean and standard deviation of average flow, peak flow, and runoff volume converged to a more or less constant value at about 20 model runs.

Accordingly, the 90% prediction limits of the simulation results based on 20 or more parameter sets were calculated and compared to determine how much observed data were contained in the 90% confidence intervals. From the result, it was identified that, for 40 parameter sets, the range considerably widened and also the number of the observed data in the range were increased (*p*-factor = 78%). There was little difference, however, with the 40 and 60 parameter sets, and

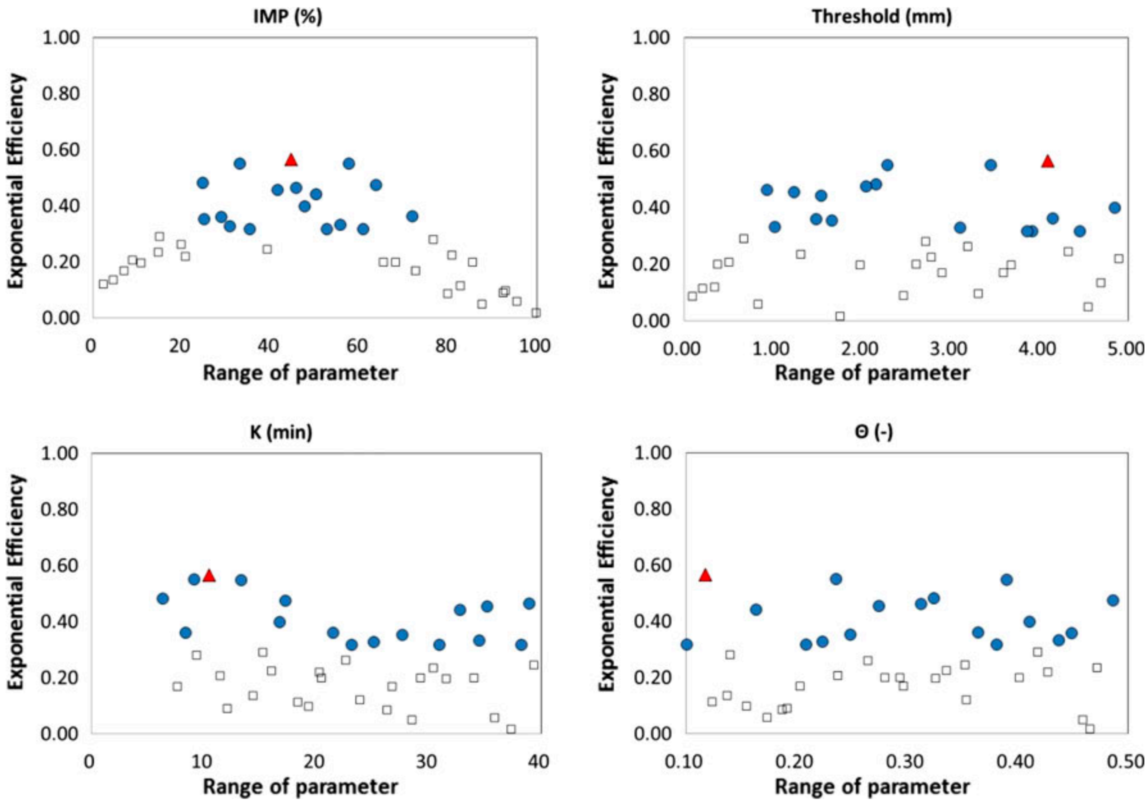

Fig. 4. Dotty plots of parameters with likelihood function value ranging 0.32–0.56 (17 May 2005 rainfall event).

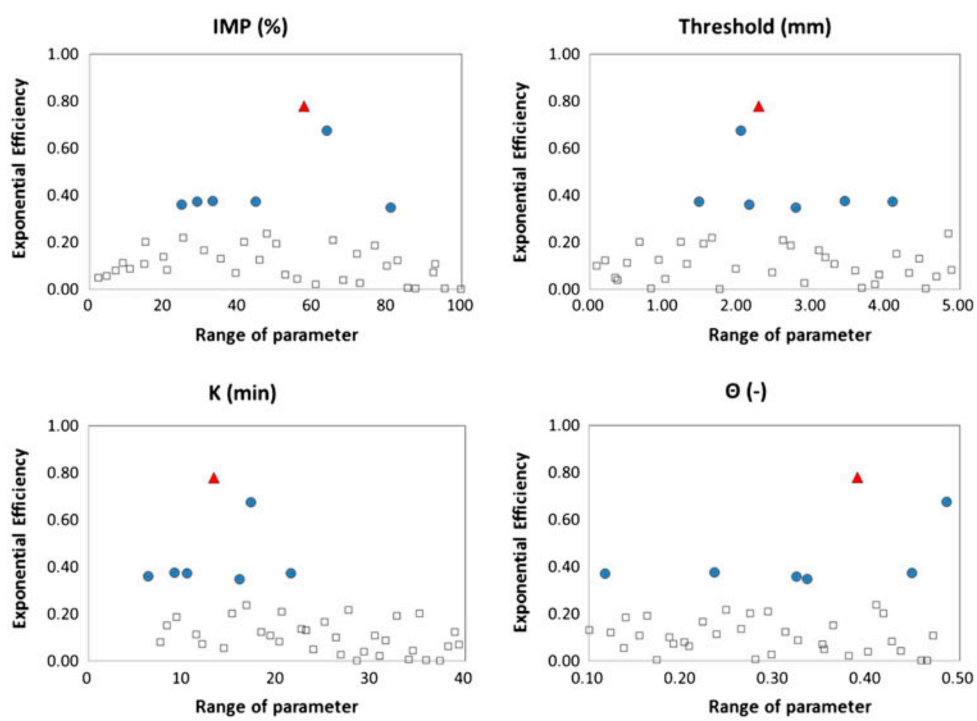

Fig. 5. Dotty plots of calibration with likelihood function value ranging 0.35–0.78 (24 August 2005).

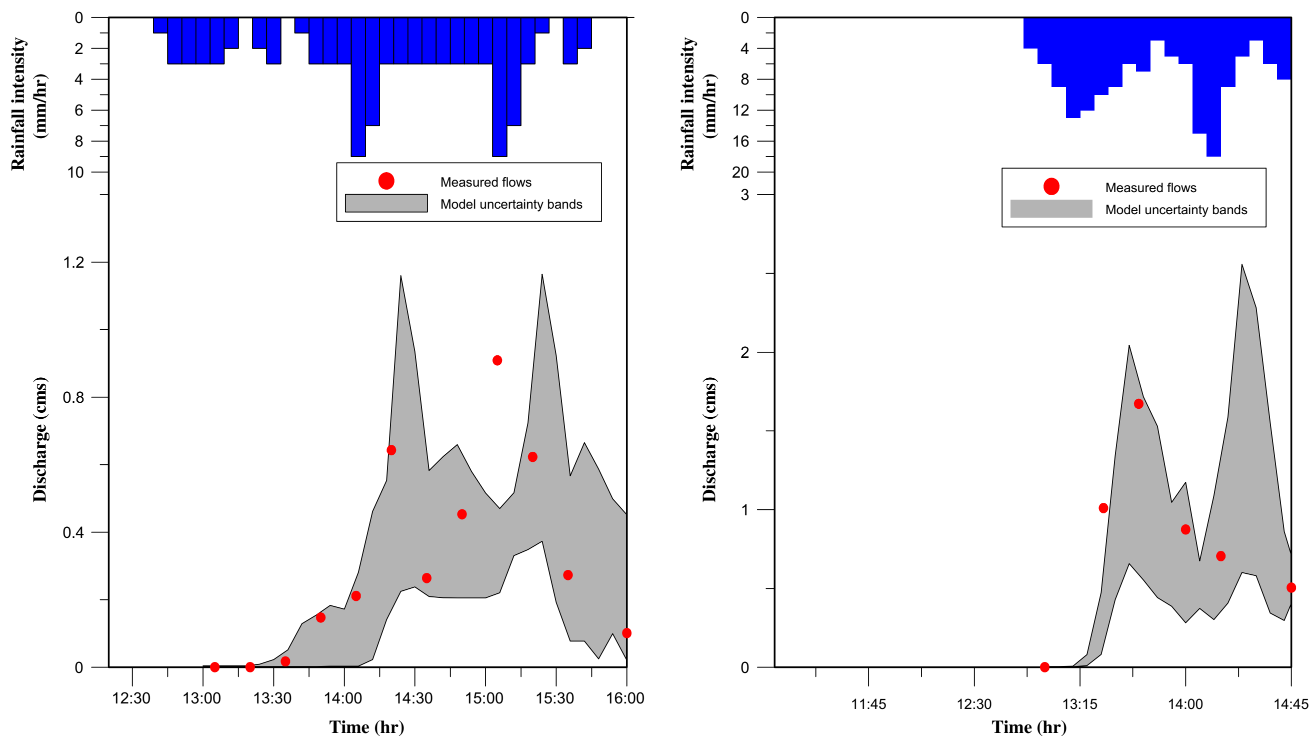

Fig. 6. 90% Prediction limits of calibration on 17 May 2005 (left) and 24 August 2005 (right).

it was found that the number of the observed data in the range were the same. Therefore, it was decided that the suitable number of parameter sets using LHS is 40.

3.2.2. Likelihood function and cutoff threshold

Likelihood function used in the uncertainty analysis is exponential efficiency which can separate easily behavioral from non-behavioral parameter sets [17]. The cutoff threshold values for likelihood function that lead to the selection of 30, 20, 15, and 10 behavioral parameter sets out of 40 parameter sets are tried, and the output corresponding to behavioral parameter sets was analyzed for 90% prediction limits. Fig. 3 shows confidence interval determined for the simulation results of 17 May 2005 rainfall event.

The *p*-factor was used to quantify model prediction uncertainty, which is the percentage of observation data bracketed by the prediction limits. Table 2 shows the *p*-factor for each threshold values used for likelihood function. From this result, the threshold was determined as 0.317 because this value was found to contain 90% (maximum) of the observed data within the prediction limits with the narrowest possible width. This way of selecting the threshold value is based on Blasone et al.'s work [18].

3.2.3. Calibration

Uncertainty analysis was done first for model calibration using behavioral parameter sets. Of the four rainfall events available, earlier two events were used for model calibration. Results were first given in the form of "dotty plot," a scatter plot of parameter value against a likelihood function value. Fig. 4 shows the dotty plot of the 17 May 2005 rainfall event with likelihood function value ranging from 0.32 to 0.56. On the dotty plots, the values over the 0.317 threshold are shown in blue circles, and the highest value is shown by a red triangle. The range of behavioral parameters sets is 25–72 for impervious area, 0.93–4.85 for rainfall threshold, 6–39 for a routing *K*, and 0.1–0.49 for routing θ.

As for the 24 August 2005 rainfall event, likelihood function value ranges between 0.35 and 0.78, and seven parameter sets are selected as behavioral parameter sets out of 40 parameter sets (Fig. 5). The range of behavioral parameters are is 25–81 for impervious area, 1.48–4.09 for rainfall threshold, and 6–21 and 0.12–0.49 for routing properties *K* and θ, respectively.

Summarizing the results of calibration, the likelihood function of the behavioral parameter sets ranges from 0.32 to 0.78, and the corresponding range is 25–81 for impervious area, 0.93–4.85 for rainfall threshold, and 6–39 and 0.12–0.49 for routing parameters *K* and θ,

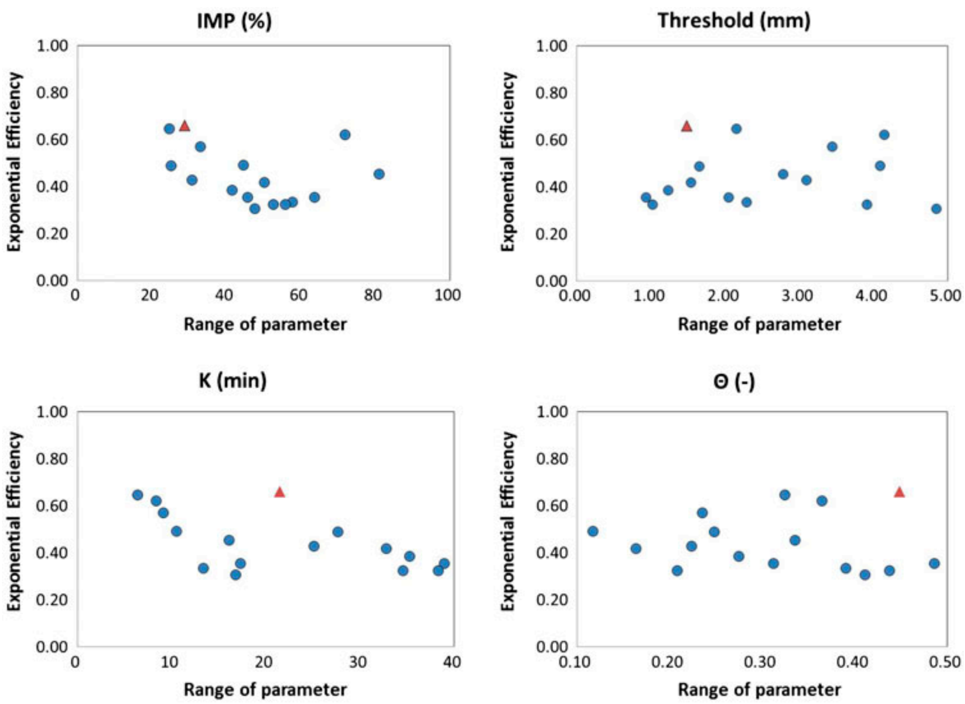

Fig. 7. Dotty plots of parameters with likelihood function value ranging 0.31–0.66 (22 May 2006 rainfall event).

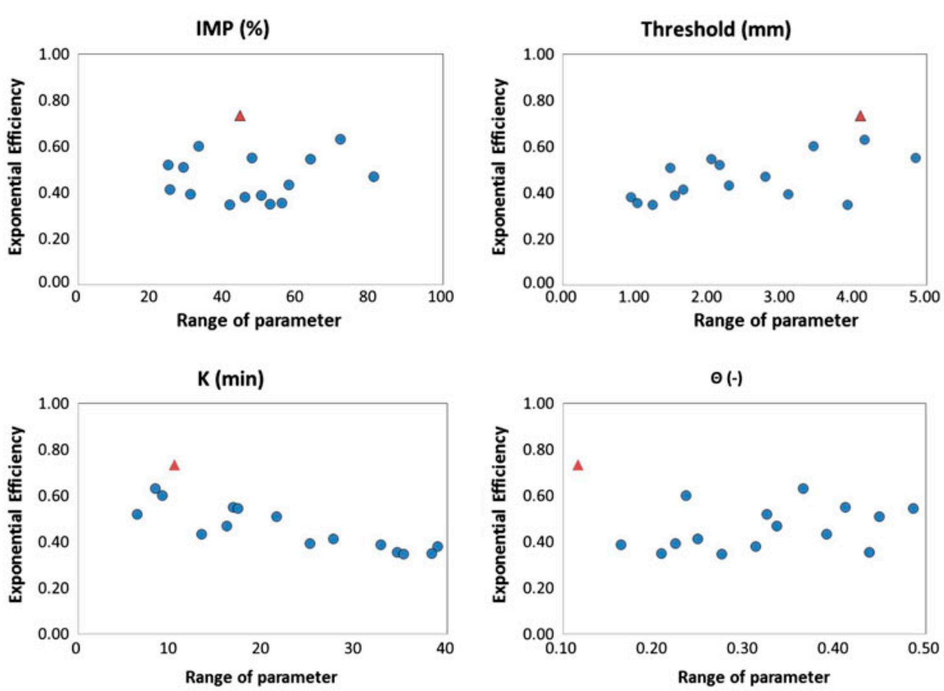

Fig. 8. Dotty plots of parameters with likelihood function value ranging 0.34–0.73 (28 June 2007 rainfall event).

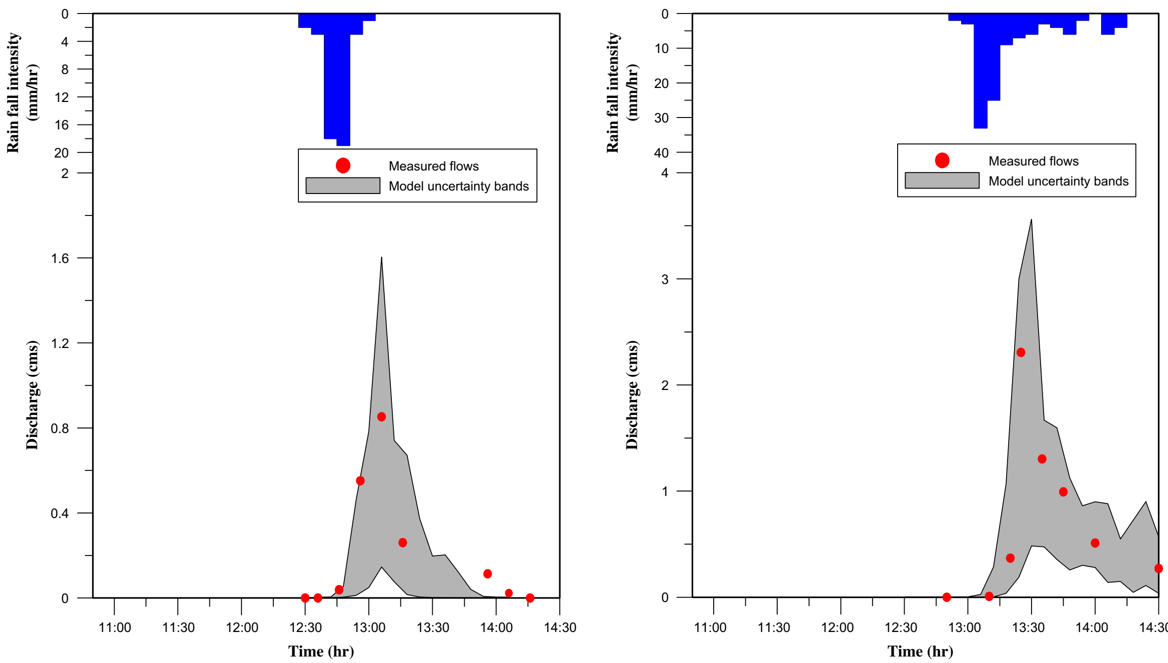

Fig. 9. 90% Prediction limits of validation on 22 May 2006 (left) and 28 June 2007 (right).

Table 3
Posterior distribution of behavioral parameters and optimal parameter value for calibration

	Impervious area (%)				Rainfall threshold (mm)			
Date	Opt.	Min	Max	L_{max}	Opt.	Min	Max	L_{max}
17 May 2005	45	25	72	0.56	4.09	0.93	4.85	0.56
24 August 2005	58	25	81	0.78	2.29	1.48	4.09	0.78
	K (min)				θ			
17 May 2005	10	6	39	0.56	0.12	0.1	0.49	0.56
24 August 2005	13	6	21	0.78	0.39	0.12	0.49	0.78

respectively. Table 3 presents the range of parameters and optimum values obtained from maximum likelihood, representing the posterior distributions of each parameter for calibration.

Prediction limits of time series is given by 90% confidence interval. Fig. 6 shows the 90% prediction limits for the calibration period. The p-factor can be used to judge the prediction uncertainty. Setegn et al. [19] suggest that the p-factor is considered good when 60% or more of the observed data are bracketed by the prediction limits. For the whole calibration period, the p-factor for the two rainfall events is 87% (Table 4), which is considered an acceptable value. Therefore, the 90% prediction limits provided by GLUE is considered satisfactory.

3.2.4. Validation

The 16 behavioral parameter sets selected during the calibration are used for the validation period. According to the results of validation, the range of likelihood function value is 0.31–0.73. Dotty plots for validation are shown in Figs. 7 and 8 for the other two events during the validation period.

Table 4
p-factor values of calibration

Date	p-factor (%)
17 May 2005	90
24 August 2005	80
Total	87

Table 5
Posterior distribution of behavioral parameters and optimal parameter value for validation

	Impervious area (%)				Rainfall threshold (mm)			
Date	Opt.	Min	Max	L_{max}	Opt.	Min	Max	L_{max}
22 May 2006	29	25	81	0.66	1.48	0.93	4.85	0.66
28 June 2007	45	25	81	0.73	4.09	0.93	4.85	0.73
	K (min)				θ			
22 May 2006	21	6	39	0.66	0.45	0.1	0.49	0.66
28 June 2007	10	6	39	0.73	0.12	0.1	0.49	0.73

Table 6
p-factor values of validation

Date	*p*-factor (%)
22 May 2006	67
28 June 2007	100
Total	83

From the dotty plots of the impervious area, the distributions are focused on the 20–60 range. Meanwhile, rainfall threshold results showed a wider distribution. Routing parameters (K and θ) are widely distributed similar to the rainfall threshold parameter. Table 5 presents the range of parameters and optimum values obtained from maximum likelihood, representing the posterior distributions of each parameter for the validation.

Fig. 9 shows the 90% prediction limits of validation results. For the whole validation period, total observed data are 12 and those bracketed by the 90% prediction limits are 10. Therefore, *p*-factor value is 83% (Table 6) which is quite good and satisfactory.

The values of maximum likelihood function are 0.78 for calibration and 0.73 for validation. The range of optimized value is 29–58 for impervious area, 1.48–4.09 for rainfall threshold, and 10–21 and 0.12–4.09 for routing parameters K and θ, respectively (Tables 3 and 5). The values of likelihood function tend to be similar for both calibration and validation. Maximum likelihood function value is high in the range of 0.56–0.78, therefore verifying the applicability of the MUSIC model.

4. Conclusions

In this study, uncertainty analysis of runoff-related parameters of MUSIC was performed using GLUE for four rainfall events. Using the results of the sensitivity analysis, four sensitive parameters were chosen and used for uncertainty analysis. LHS method was used for the generation of 40 parameter sets. GLUE method typically requires a large number of parameter sets but, in this study, uncertainty analysis was performed using significantly less number of parameter sets using the LHS method. Exponential efficiency that is commonly adopted in GLUE is used as a likelihood function to evaluate the model performance. The choice of the cutoff threshold for behavioral parameter sets was determined as 0.317 because this value was found to contain most observed data within 90% prediction limits with the narrowest possible width.

From the results of model calibration, the range of parameter for behavioral sets was found to be 25–81 for impervious area, 0.93–4.85 for rainfall threshold, 6–39 for routing property K, and 0.1–0.49 for routing property θ. The range of likelihood function for behavioral parameter sets during calibration is 0.32–0.78. The *p*-factor value using the results of the 90% prediction limits for calibration is 87%, an acceptable value since the results are considered good when 60% or more of the observed data are bracketed by prediction limits. The likelihood function value for the validation, using 16 parameter sets selected from calibration, lies in the 0.31–0.73 range which is similar to the calibration results. In addition, 83% *p*-factor value confirms the adequacy of prediction uncertainty for validation. Overall, maximum likelihood value for calibration is 0.78 and 0.73 for validation. The results prove that the MUSIC model is appropriate for estimating runoff for the drainage basin considered in this study.

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