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Optimization model for water distribution network considering minimization of total replacement cost and stabilization of flow velocity in pipelines

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ABSTRACT

This study proposes a method using genetic algorithms (GA) to optimize selection of appropriate pipe diameter during pipeline replacement planning for water distribution networks. Mathematical programming problems were first formulated to minimize cost of replacement while considering hydraulic constraints such as flow velocity for each pipe and water pressure at each node. In addition to the economic perspective, stability of flow velocity in pipes was considered as another objective function of the multipurpose programming problem. After this, a GA model combined with hydraulic pipe network analysis was created: the HGA model. Finally, a case study was conducted to show the validity of the proposed model. Results reveal that this multipurpose HGA model is useful for optimization of pipeline replacement planning.

Keywords: Water distribution network; Replacement planning; Optimization; Multipurpose; Genetic algorithm

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1. Introduction

Japan's water supply, with an overall coverage of 97%, is one of the most important lifelines for the nation's daily activity. However, as a result of Japan's tremendous economic growth, most of these water supply facilities were installed very rapidly during the 1970s, and they now require large-scale replacement. When it comes to planning replacement of the water supply facilities, water supply pipelines, which occupy around 70% of total water supply assets, are the most important factor to consider.

However, the need for a water supply has changed from simple consideration of matters of quantity to matters of quality [1]. In order to maintain fairness of water supply and stability of water pressure, the issue of "pipe flow velocity" has been raised for serious consideration. Pipe flow velocity depends not only on the demand for water, but also on the region; particularly when the flow velocity is small, this may cause turbidity. During redevelopment or renewal of pipelines, choosing the correct pipe diameter will not only help in cutting the costs of downsizing, but also will help to reduce regional disparities in pipe flow velocity, which will in turn work to reduce and equalize residual chlorine concentrations, and contribute to the formation of a pipeline system capable of supplying fresh water.

In this paper, we utilize a hybrid genetic algorithm model (HGA model) that aims to support optimization of the pipeline system while taking into consideration economic efficiency and flow velocity during renewal of the water distribution network. One characteristic of this

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model is the application of a genetic algorithm (GA) [2–4] to the combination optimization problem for changing and selecting the pipeline diameter, while also considering whether stability of pipe flow velocity and effective water head at each node meet hydraulic constraints through testing and evaluation based on a pipe network analysis method.

This paper is organized into five sections: Section 2 describes the mathematic formulation of an optimal water distribution network model; Section 3 outlines the HGA model and its extension for multipurpose optimization (the multipurpose HGA model); Section 4 explains the results and the utility of this idea by examining a case study; and finally, Section 5 presents some conclusions of this research.

2. Mathematical formulation for an optimal water supply network programming model

2.1. Problem of minimizing pipeline construction cost

In general, the cost of pipeline construction depends on pipeline diameter D, and it is calculated by pipeline installation cost C multiplied by pipeline length L. Table 1 shows the actual past record of installation cost for pipeline C [yen/m]. As can be seen from Table 1, installation cost C has a linear relationship to the cross-section of the pipeline A [m^2]. Utilizing this data, we performed regression analysis on pipeline cost C as a function of A (A = $D2/4$), subsequently obtaining Eq. (1) as the regression model (correlation coefficient $r = 0.996$).

$$C = \alpha \cdot A + \beta = \alpha\left(\frac{\pi}{4}\cdot D^2\right) + \beta \qquad \alpha + 653.68,\ \beta = 79.45 \tag{1}$$

Furthermore, for the pipelines i (I =1,…,n) which form the water distribution network, when pipe diameter x_i [m] and pipeline length L_i [m] are input, the total cost needed for renewing the entire water distribution network TC [yen] can be calculated from Eq. (2) as follows. Note that the values of α and β are obtained from the regression analysis above.

$$TC = \sum_{i \in \text{pipes}} \left[\alpha\left(\frac{\pi}{4}\cdot x_i^2\right) + \beta\right] \cdot L_i = \alpha\frac{\pi}{4} \sum_{i \in \text{pipes}} L_i x_i^2 + \beta \sum_{i \in \text{pipes}} L_i \tag{2}$$

While the pipeline i (i =1,…,n) and the node j (j = 1,…,m) still meet hydraulic constraints, the question as to which pipe diameter x_i for each pipeline i is most desirable for selection is considered as a mathematical programming problem, in order to minimize TC. When formulated as an integer programming problem [5,6], the equations are as follows.

$$\left|\begin{array}{lll} \text{Minimize} & TC = \alpha\frac{\pi}{4}\sum_{i \in \text{pipes}} L_i x_i^2 + \beta \sum_{i \in \text{pipes}} L_i & \\ \text{Subject to} & V_{\mathrm{i}} \le V_i^* & (i = 1,\dots n) \\ & H_j \le H_j^* & (j = 1,\dots,m) \end{array}\right\} \tag{3}$$

Here, TC represents the total cost needed for renewing the pipeline [yen]; x_i and L_i are the pipe diameter [m] and pipeline length [m] respectively; V_i is the flow velocity [m/s] of pipeline i; and H_j is the effective water head at node j. Furthermore, V_i* represents the maximum flow velocity [m/s] in pipeline i; H_j* is the minimum effective water head needed to be allocated at node j; and D_i expresses the set of candidates for pipe diameter for pipeline i.

Focusing on the mathematically formulated problem of objective function TC, we know that the first element on the right side contains the square of x_i and is a nonlinear function. As for the constraint conditions, these include hydraulic restriction of the pipeline and node, as well as integer conditions for the variable x_i representing the pipe diameter. From the viewpoint of cavitation suppression, the fulfillment of conditions such as pipe flow velocity V_i being kept below V_i* and effective water head H_j at node j being kept above H_j* is necessary to achieve adequate water service management. Generally, pipe flow velocity V_i and effective water head H_j are calculated using a pipe

Table 1
Previous pipeline installation costs

Pipeline diameter D (×10^3 [m])	75	100	150	200	250	300	350	400	450	500
Cross section area A (×10^{-3}) [m^2])	4.4	7.9	17.7	31.4	49.1	70.7	96.2	125.6	159.0	196.3
Installation cost C (×10^3 [yen/m])	77	85	91	103	113	127	137	169	185	203

network analysis program, with the pipe diameter xi of every pipeline and the water demands at every node being defined as conditions. Since constraint conditions of the x_i are defined according to the diameter of the pipeline, x_i cannot be handled as a continuous quantity, meaning that it is necessary to add integer conditions for x_i. In this problem, the values capable of handling the variable x_i are defined by the set of pipe diameter candidates D_i for the pipeline, and a framework is formed for selection of an appropriate pipe diameter from this set D_i.

2.2. Evaluation indicators for pipe flow velocity stability

Eq. (3) is mathematically formulated, and takes into consideration optimization of the water distribution network during replacement planning while also minimizing pipeline construction cost. However, as mentioned earlier, any problems with changing the pipeline diameter are closely connected to quality of the water distribution service. During replacement of the water distribution network, while improved economical efficiency is important, fairness and stability of the water supply service are also objectives for consideration during planning.

Therefore, when considering problems in planning water supply network replacement, the fairness of service quality over the area of water distribution is a secondary objective function. In this paper, we examine the spatial gap quantification indicators [7] of pipeline flow.

$$IV = \sum_{\text{pipes}} L_i x_i^2 \left(V_i - V_{\text{avg}}\right)^2 \Big/ \sum_{\text{pipes}} L_i x_i^2 \tag{4}$$

Here, IV is an indicator for pipe flow velocity [(m/s)^2], and V_{avg} is the average flow speed [m/s] of the total pipeline network. The $L_i x_i^2$ in Eq. (4) is the pipeline capacity's weight, which is an indicator for the water-supplied population of each pipeline. IV shows the stability of pipe flow velocity (since pipe flow velocity varies from area to area). The larger the value of IV, the larger the regional variation, and the more difficult it is to maintain the quality of water flow. Smaller values for IV are most desirable. In addition, it is necessary to plan for both economic efficiency and versatility in pipeline construction, while handling the problem of optimization of this indicator IV as an objective function. The practical methodology will be explained in detail in the next section.

3. HGA model and multi-purpose optimization

3.1. Characteristics of the HGA model

To address the optimization problem formulated in Section 2, we here propose incorporation of a pipeline analysis program [8,9] into a genetic algorithm. This HGA model, intended specially for pipe diameter selection during pipeline network renewal planning, has the following two advantages. First, as a basic frame for the optimization algorithm, it utilizes a GA; thus, it supports a combination of discrete decision variables and nonlinear objective functions for the optimization problem. Secondly, at the point where the pipe network analysis meets GA, by performing parallel calculation of pipe flow velocity and hydraulic conditions of effective water head etc., established alternative planning can be verified from the perspective of engineering utility.

The coding method used in the HGA model for individual modeling is shown in Fig. 1. Namely, the variable vector X^k for pipe diameter x_i is simulated and encoded as the gene A^k belonging to an individual, using a binary{0,1} letter string sequence (bit sequence), thus being converted from "phenotype" to "genotype". This process is known as "coding" in GA, and the reverse process (going from "genotype" to "phenotype") is known as "decoding". The multiple individuals making up a given generation (the group of individuals), as they continue to repeat generational iteration, will generate the individuals of the next generation (the descendants) [10].

Fig. 2 shows an overview of the calculation flow of the HGA model proposed in this study. An evaluation of each generated individual's merits and demerits fitness fv is calculated using a fitness function and penalty method; basically, individuals with a higher *fv* value will be generated in greater numbers in the next generation, which is the objective of this model. Regarding the optimization problem of Eq. (3) (in other words, the problem of minimizing pipeline construction cost), this should be made to support ma$x_i$mization of the fitness value *fv*, and fitness function implemented as inverse to the objective function *TC* [*fv* (*TC*) =1/*TC*]. Furthermore, for each combination of pipe diameter x_i a pipe network analysis is performed. If an individual is generated which does not meet the constraint condition V_i and H_j, a penalty is assigned to this individual. Specifically, pipelines or nodes that violate constraint condition fv multiply by 0.1 for that particular location value, making them less likely to occur in the next generation.

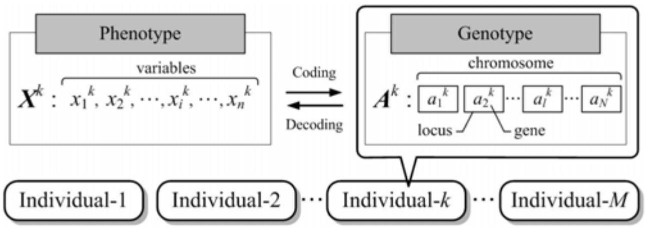


Fig. 1. Individual modeling.

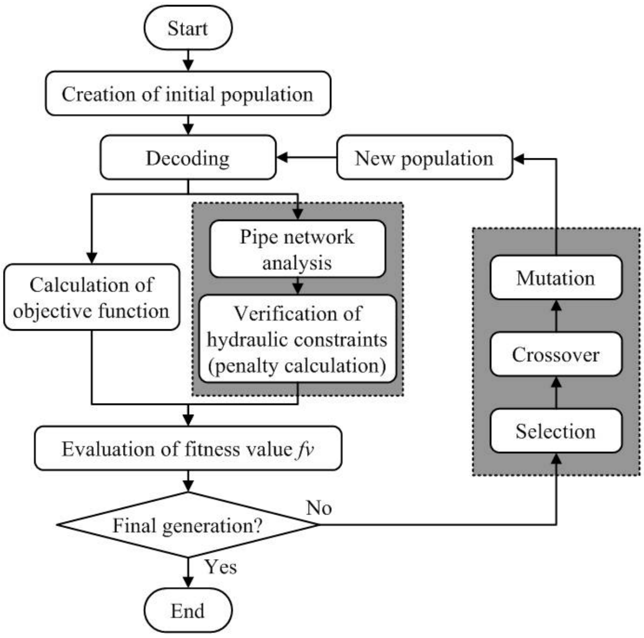


Fig. 2. Overview of HGA model calculation flow.

We adopted "roulette selection" for the selection process and one-point crossover (simple crossover) for the crossover process as the genetic operator built into the HGA model, and added the standard mutation process. However, in order to generate better individuals, we also simultaneously adopted a process of "elite preservation" where individual groups with the highest fitness are kept in the next generation.

3.2. Expanding on the multipurpose HGA model

Expanding on the proposed HGA model, we examine a scenario for planning replacement water networks that achieves a good balance of economic efficiency and stability of pipe flow velocity. We define this kind of multipurpose optimization model as a "multipurpose HGA model," as distinct from the single purpose optimization represented by the HGA model.

In the multipurpose HGA model, two objective functions (economic efficiency in pipeline construction and stability in pipe flow velocity) are considered together. Each value of the function is standardized, whereby the best case is set to "1" and the worst case is set to "0". In other words, the *TC* of Eq. (2) (indicating total cost of pipeline renewal [yen]) and the *IV* of Eq. (4) (indicating stability of pipe flow velocity [$(m/s)^2$]) can be standardized using Eqs. (5) [11].

$$Z_1 = \frac{TC - TC^{\max}}{TC^{\min} - TC^{\max}} \quad Z_2 = \frac{IV^{\max} - IV}{IV^{\max}} \tag{5}$$

Here, Z_1 expresses economic efficiency of pipeline construction and Z_2 expresses stability of pipe flow velocity; $TC^{\max}$ and $TC^{\min}$ express the maximum and minimum values (respectively) tolerated by decision-makers of the objective function *TC* [yen]; and $IV^{\max}$ expresses the maximum value of the objective function *IV* [$(m/s)^2$]. Using the two standardized objective functions Z_1 and Z_2, the fitness function of the multipurpose HGA model is defined in Eq. (6) below:

$$\begin{aligned} fv(Z_1, Z_2) &= Z_1 \times Z_2 \\ &= \left(\frac{TC - TC^{\max}}{TC^{\min} - TC^{\max}}\right) \times \left(\frac{IV^{\max} - IV}{IV^{\max}}\right) \end{aligned} \tag{6}$$

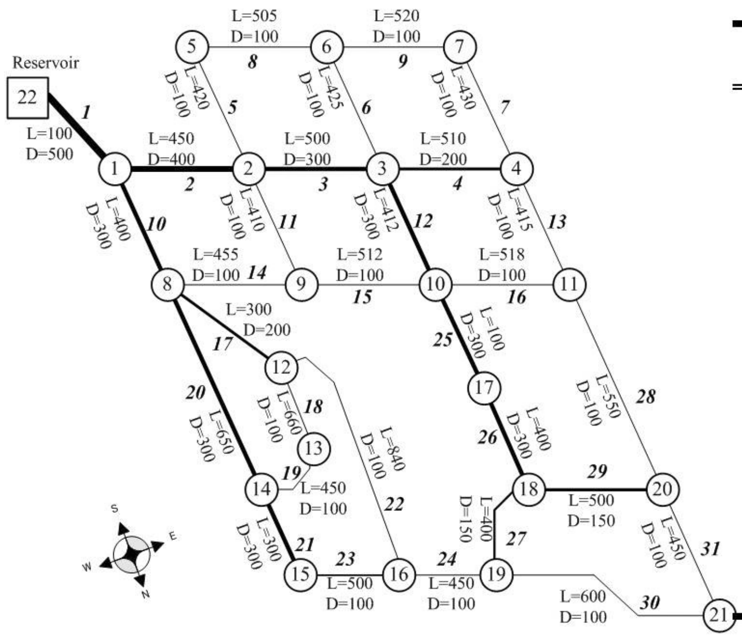
4. Case study

4.1. Overview of target area

The area selected for this case study measures about 2.3 km north–south and about 2 km east–west, with total area coverage around 4.6 km^2. The ground height is from around 50 m to 57 m, and is almost entirely flat. Total population of the case study area is around 24,000 persons. Population density is 5,200 person/km^2. Water demand hits a maximum hourly rate of 478.4 m^3/h (132.9 L/s) when the water consumption rate is 320 L/d per person. Fig. 3 shows a diagram of the target area's water distribution network. The target water distribution network is made up of pipelines at 100 mm or greater in diameter, and consists of one distribution reservoir, 21 node points and 31 pipelines. Pipeline No. 1 connects the reservoir with the pipeline network, and we have excluded this pipeline from the current study.

4.2. Application conditions for the optimization model

The pipe diameter candidates D_i for change or selection of each pipeline i are based on the present pipe diameter. Considering future reductions in water demand, we decided to include smaller diameter pipelines than at present. Table 2 shows proposed diameters corresponding to present diameters, as well as the encoding performed during the implementation of the HGA model (on the upper level). If water demand is expected to increase in future for a given area, a diameter larger than the current one is set as the diameter candidate. In Table 2, of the 30 selected pipelines, 18 pipes have a pipe diameter if 100 mm. These are represented using 1 bit. In cases where pipeline diameters greater than 150 mm are allocated, 2 bits, and pipeline diameters of 100mm are allocated 1 bit,



Node no.	Water demand [L/sec.]
1	4.0
2	5.7
3	7.7
4	6.4
5	3.7
6	4.3
7	4.2
8	2.2
9	2.0
10	6.9
11	6.0
12	4.9

Node no.	Water demand [L/sec.]
13	6.0
14	10.8
15	3.4
16	7.7
17	7.5
18	14.5
19	3.3
20	12.0
21	9.7
22	(Reservoir)
Total	132.9

Fig. 3. Water distribution for target area in case study.

Table 2
Diameter options and binary numbers

Present diameter	Options for pipe diameter			
400mm	400mm	300mm	250mm	200mm
300mm	300mm	250mm	200mm	150mm
200mm	200mm	150mm	100mm	75mm
150mm	200mm	150mm	100mm	75mm
Binary number	00	01	10	11

Present diameter	Options for pipe diameter	
100mm	100mm	75mm
Binary number	0	1

a single individual gene is represented using 42 bits (= 2×12+1×18), and the size of the solution space (the combination of solutions) reaches around 4.4 trillion (=242).

During implementation of the proposed method, hydraulic constraint was V_i^* =3.0 [m/s] and H_j^* = 15 [m] for pipelines and nodes, and pipeline length L_i was given as the present pipeline length. It was assumed during replacement planning that the pre-replacement configuration of the pipeline network was to be preserved. Furthermore, in the multipurpose HGA model, for the maximum and minimum values of objective function tolerated by decision-makers, $TC^{\max}$ [yen] was the total cost when the current pipe diameter was selected and renewal performed, and $TC^{\min}$ [yen] and $IV^{\max}$ [$(\mathrm{m/s})^2$] represented the total cost and indicator for the stability of pipe flow velocity obtained after the HGA model was applied. Additionally, as the results in the next section show, GA parameters were determined by taking into consideration previous research [5]; the number of individuals was set at 500; number of generations was 1000; crossover rate was 0.8; and mutation rate was set to 0.07.

5. Results and discussion

Table 3 shows the contents of the alternative proposals obtained by applying both models. With the hybrid HGA model which aims to minimize costs, although four pipelines located far from the distributing reservoir

Table 3
Results from implementing optimization model

Pipeline number	1	2	3	4	5	6	7	8	9	10	11
Present diameter	500	400	300	200	100	100	100	100	100	300	100
HGA model	(500)	250	250	100	75	75	75	75	75	200	75
Multipurpose HGA model	(500)	300	300	150	100	75	75	75	75	250	75
Difference in downsizing		Δ		Δ						Δ	

Pipeline number	12	13	14	15	16	17	18	19	20	21	22
Present diameter	300	100	100	100	100	200	100	100	300	300	100
HGA model	200	75	75	75	75	75	75	75	150	150	75
Multipurpose HGA model	200	100	100	100	100	150	75	75	150	150	100
Difference in downsizing						Δ					

Pipeline number	23	24	25	26	27	28	29	30	31
Present diameter	100	100	300	300	150	100	150	100	100
HGA model	100	75	200	200	100	75	150	100	100
Multipurpose HGA model	100	100	150	150	75	100	75	100	75
Difference in downsizing			★	★	★		★		★

Note 1: Thick frames indicate pipelines where downsizing is possible compared to the current diameter.
Note 2: "Δ" indicates pipelines where downsizing is possible, but diameter chosen will be larger than the HGA model.
Note 3: "★" indicates pipelines where downsizing is possible, but diameter chosen will be smaller than the HGA model.

Table 4
Comparison of economic efficiency and pipe flow velocity for pipeline construction

	If replace with present diameter	HGA model (cost-minimization)	Multipurpose HGA model (both cost and stability)
Total cost of pipeline renewal *TC* [million yen]	1,353	1,224	1,247
Stability of pipe flow velocity *IV* [(m/s)^2]	0.062	0.291	0.061

(pipelines 23, 29, 30 and 31) were excluded, for the other 26 pipelines results were obtained proposing a pipe diameter smaller than the current size. In the multipurpose HGA model, since stability of the pipe flow velocity was added as an objective, for pipelines 2, 4, 10 and 17 (located near the distributing reservoir) the process of downsizing was de-emphasized in order to improve this aspect. However, for pipelines located far away from the distribution reservoir (pipelines 25–27, 29 and 31), as can be observed, in cases where it was possible to downsize pipe diameter within a range capable of satisfying hydraulic constraints on pipelines and nodes, stricter downsizing (in comparison to the cost-minimization proposal) is to be implemented.

Next, as an integrated evaluation of each model, Table 4 shows the relationship between the total cost of pipeline renewal *TC* [million yen] and the indicator of stability of pipe flow velocity *IV* [(m/s)^2]. Focusing on the economic efficiency of pipeline construction, compared to the case where present pipe diameter is preserved and renewal undertaken, use of the multipurpose HGA model together with the HGA model can likely reduce costs by more than 100 million yen. However, while the HGA model shows some deterioration in stability of pipe flow velocity, (from the current rate of 0.062 to a new rate of 0.291 $[(m/s)^2]$), the multipurpose HGA model, on the other hand, shows a slight improvement over selecting the present pipe diameter.

From the above, it is clear that the multipurpose HGA model proposed in this study can provide alternative proposals for improving the economic efficiency of water distribution network replacement planning without sacrificing quality of water supply or stability of pipe flow velocity.

6. Conclusions

We mathematically formulated problems with water distribution network replacement planning while satisfying hydraulic constraints on pipeline and nodes and taking minimization of pipeline construction cost as an objective. Furthermore, we added a secondary objective function to the optimization problem, defining an indicator for stability of pipe flow velocity. As an optimization method for the formulated problems, we proposed combining a pipe network analysis program with a genetic algorithm (HGA model). Moreover, we examined the use of this multi-purpose HGA model in optimizing two factors (improved economic efficiency and stability of pipe flow velocity), and explained the processes of standardization and integration of differing objective functions. To evaluate the effectiveness of the model proposed, a water distribution network with one distributing reservoir, 21 nodes, and 31 pipelines was selected and a case study performed. During implementation of the multi-purpose HGA model, not only was pipeline construction cost successfully decreased, but stability in pipe flow velocity was simultaneously achieved.

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