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Analytical solutions for brine discharge plumes on a sloping beach

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ABSTRACT

Large scale seawater desalination plants are operated along the coasts and dispose of their brine waste stream by discharging into the sea. As the need for desalinated seawater is steadily increasing, more new desalination plants are planned to be constructed. If desalination plants are closely clustered together along the coastlines, the adverse environmental impacts of the brine effluent discharges from plants such as these are strongly inter-dependent. A far-field mathematical model for continuous brine discharges from two desalination plants on a uniformly sloping beach is presented. The analytical solutions are illustrated graphically to study the interaction of two brine discharge plumes. Asymptotic approximation will be made to the shoreline's brine concentration to evaluate the maximum salinity build-up in the coastal waters.

Keywords: Brine discharge; Desalination plant; Sloping beach; Two outfalls

1. Introduction

More than half of the world's seawater desalination plants are operated along the coasts of the Arabian Gulf (total capacity of 12 million m^3/day), Gulf of Oman (1 million m^3/day) and Red Sea (7.7 million m^3/day). As the need for desalinated seawater is steadily increasing, not only are the number of new large scale desalination plants constructed along the coastal areas growing, the existing plants are also gradually increasing their water production capacities. Seawater desalination plants generate two products, pure water and brine—a reject concentrate stream. The unwanted brine product is primarily seawater but at a more concentrated level, with the concentration factor as high as 2.5, which is to be disposed of by continuously discharging it back into the sea through an outfall. Therefore, like any large scale industrial process, unfortunately seawater desalination also has its potential environmental impacts [1].

Owing to the highly variable nature of the sea, we do not yet have a full understanding or description of the mixing processes of brine discharges from coastal desalination plants [2]. Immediately after release from the diffuser, vigorous and rapid dilution of concentrate brine is governed by the effluents buoyancy, momentum of the discharge and its interaction with the sea currents. At the end of this mixing stage, the established steady discharge brine plume then continues to drift away with the currents. Due to relatively shallow water depth, it is observed that the elongated brine plumes are spreading towards the shoreline and may cause an increase in salinity in the coastal waters [3]. The merging of two or more brine plumes adds further complexity; such situations are not uncommon along the coasts of the Arabian Gulf, Gulf of Oman and Red Sea, where large scale desalination plants often tend to be tightly clustered together [1]. Considering the growth in desalination plants and their capacity, there is an urgent need to evaluate the localized environmental impacts from the individual plant, and the cumulative strategic impacts compounded from the neighbouring plants.

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As large scale seawater desalination plants are built predominantly on the sloping sandy beaches, a mathematical model is developed using a two-dimensional advection–diffusion equation for continuous brine discharges from two coastal plants that incorporates the effect of sloping beach. The solution is presented graphically to study the interaction of two brine plumes. Asymptotic approximation will be made to the shoreline's solution to assess the salinity build-up in the coastal waters [4,5].

2. Long sea outfall on a sloping beach

As we are only concerned with the effect of seabed depth profile, for simplicity, the other complexities such as tidal motions, density and temperature, are ignored. The shoreline is assumed to be straight and the sea wide, and we consider the concentrated brine stream to be steadily discharged with a rate Q_1 from a long sea outfall at the position (0, αh_0), where h_0 is an arbitrary reference water depth. As for modern desalination plants the discharge is made via diffusers and utilizes the best available technology to promote rapid initial dilution, we also assume that the outfall's brine plume is vertically well-mixed over the water depth. Furthermore, for shallow coastal waters, the dispersion in the vertical direction occurs much faster than in the lateral direction.

The coastal (drift) current is assumed to be steady with a speed U and remains in the x-direction parallel to the beach at all times. The dispersion mechanisms are represented by eddy diffusivities, and diffusion in the x-direction is neglected, as the brine plumes in steady currents become very elongated in the x-direction. The variations in the y-direction of drift current U and coefficient of dispersivity D are assumed as the power functions only of water depth h, and for applications, we take U to be proportional to $h_0^{1/2}$ and D to $h_0^{3/2}$. These scalings are appropriate for a turbulent shallow-water flow over a smooth bed [5–7].

In a uniformly sloping beach, the water depth varies increasingly linear as $h(y) = my$, where the beach slope m and the beach is at $y = 0$, the far-field advection–diffusion equation for concentration c is

$$\frac{\partial}{\partial x}(hUc) - \frac{\partial}{\partial y}\left(hD\frac{\partial c}{\partial y}\right) = Q_1\,\delta(x)\,\delta(y - \alpha h_0), \tag{1}$$

with the boundary condition $hD\ \partial c/\partial y = 0$ at the beach $y = 0$, and c is assumed to be ultimately dissolved into the ocean. δ is the Dirac delta function. We define dimensionless quantities $y = y{*}h_0$, $x = x{*}h_0$, $c = c{*}Q_1/h_0^2U_0$ [3–5], and by setting $U = U_0 y{*}^{1/2}$ and $D = D_0 y{*}^{3/2}$, the analytical solution of (1) is given by

$$c{*} = \frac{\lambda}{mx{*}}\left(\frac{1}{\alpha y{*}}\right)^{3/4} \exp\left(-\frac{\lambda\left[y{*} + \alpha\right]}{x{*}}\right) I_{3/2}\left(\frac{2\lambda\sqrt{\alpha y{*}}}{x{*}}\right), \tag{2}$$

where $I_{3/2}$ is a modified Bessel function.

As the water depth is gradually decreasing towards the beach, the brine plumes are elongated and turning towards the beach, and the gentler the beach slope, the higher the build-up in concentration in the shallow water close to the beach [4,5]. This is expected since deeper water has a more efficient transport mechanism. The model parameter $\lambda = h_0U_0/D_0$ represents the brine plume elongation in the x-direction. To investigate the uncertainty in λ, Fig. 1 shows the possible values of λ for some relevant measured values of U_0 and D_0 [7] in a shallow water depth h_0 = 10m. Larger values of λ are mostly due to a stronger drift current U_0 with less longitudinal dispersivity D_0. For the graphical illustrations, the values of $\lambda = 1$ will be used in all plots, and note that larger the values of λ, the more elongated the brine plumes [4,5].

Again following [4,5], the appropriate measure for assessing the impact of brine discharges from coastal desalination plants would be the concentration levels at the beach. Letting $y{*} \to 0$ and replacing $I_{3/2}$ in (2) by its limiting form, we obtain the concentration at the beach

$$c{*}(0) \approx \frac{4}{3m\sqrt{\pi}}\left(\frac{\lambda}{x{*}}\right)^{5/2} \exp\left(-\frac{\lambda\alpha}{x{*}}\right). \tag{3}$$

By differentiating, the maximum value of the concentration at the beach is $c_{1m} \approx 0.61/m\alpha^{5/2}$, which occurs at $x_1{*} = 2\lambda\alpha/5$ downstream of the outfall. As shown in Fig. 2, c_{1m} does not depend on $x{*}$, and is inversely proportional to the outfall length α. Note that, for an existing outfall with length α, the location of the maximum concentration $x_1{*}$ is only governed by λ.

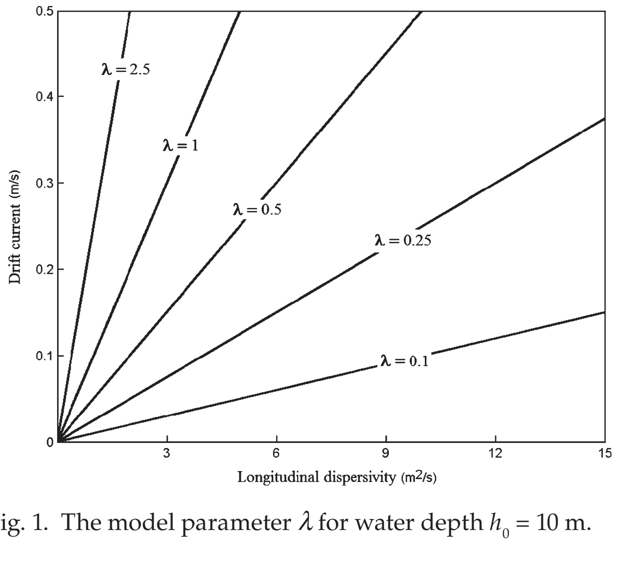

Fig. 1. The model parameter λ for water depth h_0 = 10 m.

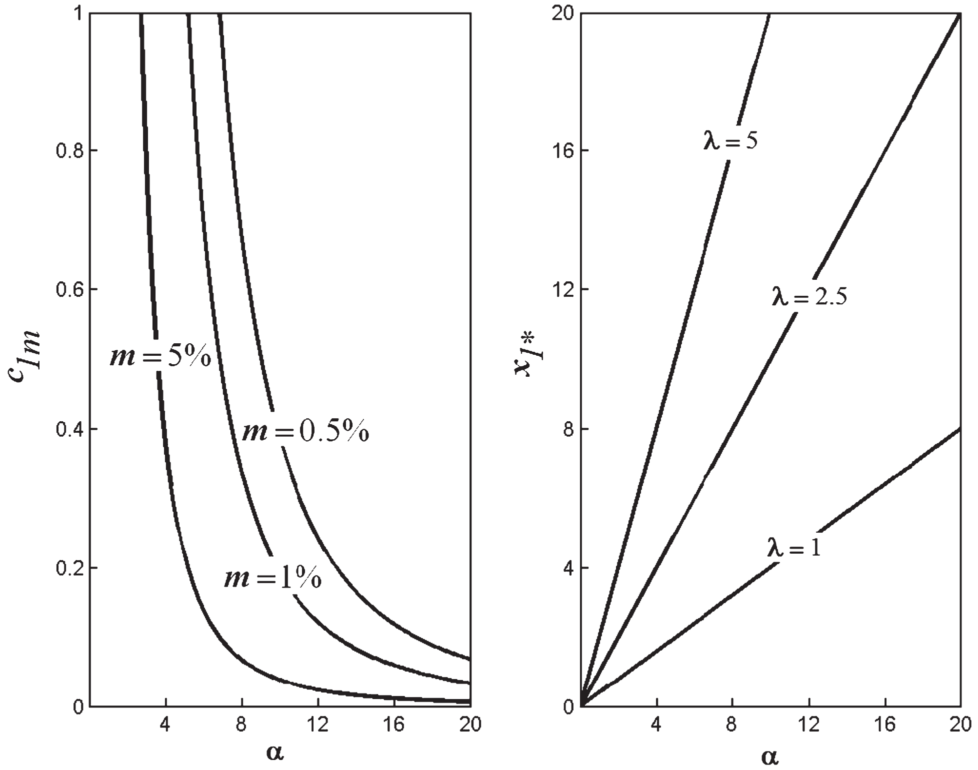

Fig. 2. The effects of varying outfall length α on the maximum concentration at the beach c_{1m} (left) and on the position of maximum concentration x_{1*} (right).

3. Two long sea outfalls

As illustrated in Fig. 3, we consider the concentrated brine stream to be steadily discharged from two large scale desalination plants at L distance apart at a rate Q_1 from the first (old) outfall at the position (0, αh_0) and at a different rate Q_2 from the second (new) outfall at the position ($-L$, βh_0). As an extension of the previous model [4,5], the two-dimensional advection–diffusion equation for the far-field plume concentration c from the two outfalls is

$$\frac{\partial}{\partial x}(hUc) - \frac{\partial}{\partial y}\left(hD\frac{\partial c}{\partial y}\right) = Q_1\,\delta(x)\,\delta(y-\alpha h_0) + Q_2\,\delta(x+L)\,\delta(y-\beta h_0). \tag{4}$$

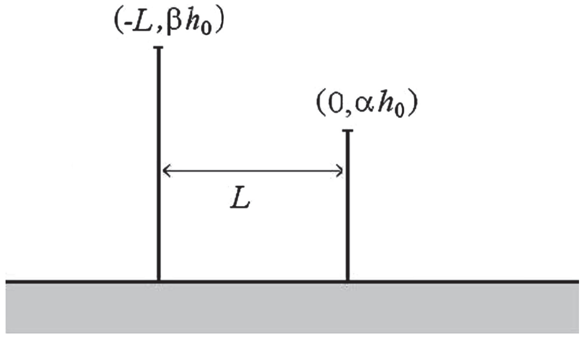

Fig. 3. Diagram of two brine outfalls.

In terms of the dimensionless variables, and by applying a linear superposition, the analytical solution of (4) is given by

$$c_* = \frac{\lambda}{m x_*}\left(\frac{1}{\alpha y_*}\right)^{3/4} \exp\left(-\frac{\lambda[y_* + \alpha]}{x_*}\right) I_{3/2}\left(\frac{2\lambda\sqrt{\alpha y_*}}{x_*}\right) + \frac{\lambda q_*}{m(x_* + \ell)}\left(\frac{1}{\beta y_*}\right)^{3/4} \exp\left(-\frac{\lambda[y_* + \beta]}{x_* + \ell}\right) I_{3/2}\left(\frac{2\lambda\sqrt{\beta y_*}}{x_* + \ell}\right), \tag{5}$$

where $q_* = Q2/Q1$ and $L = \ell h_0$. The interaction of two discharged brine plumes is also governed by the outfall lengths α and β, the separation distance between them ℓ, and the discharge ratio q_* of the second (new) outfall. For extremely large separation distances $\ell \to \infty$, the contribution of the new outfall is negligible; this is because the two plumes are well separated and become two individual plumes. However, for shorter distances $\ell \le \ell_* = 2\lambda\beta/5$, the two plumes are merging, and in the limit $\ell = 0$, we have a single outfall with two ports.

The effect of a sloping beach with slope $m = 0.01$ on the mixing and dispersal of brine discharges from two desalination plants with $q_* = 0.5$ is illustrated graphically by plotting the concentration contour of (5). Fig. 4a shows a typical merging brine plume drifting along the coastal waters from two outfalls where $\alpha = 8$ and $\beta = 10$

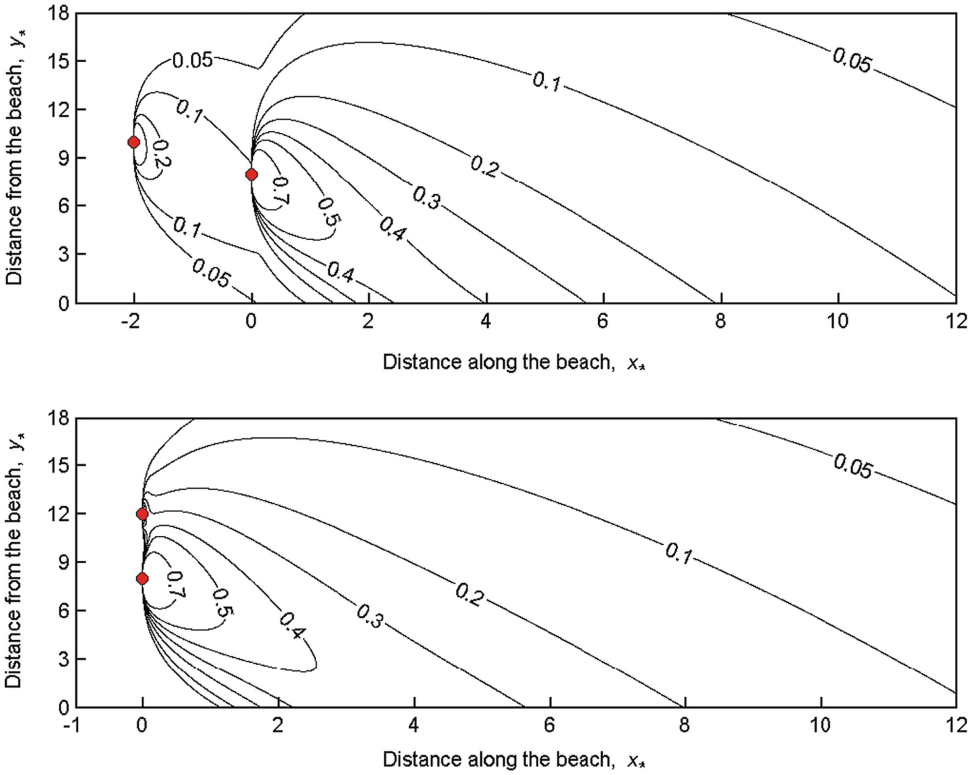

Fig. 4. Merging of two brine plumes when the separation distance $\ell = \ell_*/2$ (top) and when $\ell = 0$ (bottom).

with separation distance $\ell = \ell_*/2$. The case of a single outfall ($\ell = 0$) with two ports where $\alpha = 8$ and $\beta = 12$ is shown in Fig. 4b.

Next, the compounded concentration at the beach can be approximated by

$$c_*(0) \approx \frac{4}{3m\sqrt{\pi}}\left[\left(\frac{\lambda}{x_*}\right)^{5/2}\exp\left(-\frac{\lambda\alpha}{x_*}\right) + q_*\left(\frac{\lambda}{x_*+\ell}\right)^{5/2}\exp\left(-\frac{\lambda\beta}{x_*+\ell}\right)\right]. \tag{6}$$

As clearly shown in Fig. 4, the position of maximum compounded concentration at the beach occurs downstream of the first (old) outfall. Thus, the presence of the second (new) outfall will not change the location of maximum concentration x_{1*} of the first (old) outfall; however, it will contribute to the maximum value c_{1m} of concentration at the beach. Therefore, by substituting $x^* = x_{1*}$ to (6), the maximum value of compounded concentration at the beach can be written as

$$c_{2m} \approx c_{1m}\left[1 + q_*\left(\frac{\alpha\ell_*}{\alpha\ell_* + \beta\ell}\right)^{5/2}\exp\left(-\frac{5}{2}\left\{\frac{\beta\ell_*}{\alpha\ell_* + \beta\ell} - 1\right\}\right)\right]. \tag{7}$$

Note that, in the limit $\ell = 0$, the maximum value of the compounded concentration (7) reduces to

$$c_{2m} \approx c_{1m}\left[1 + q_* \exp\left(-\frac{5}{2}\left\{\frac{\beta}{\alpha} - 1\right\}\right)\right],$$

Fig. 5 shows the compounded concentration at the beach from two outfalls, where $\alpha = 8$ and $\beta = 10$ with $q_* = 0.5$ for three values of the separation distance $\ell = 0.5\,\ell_*$, $\ell = \ell_*$ and $\ell = 2\ell_*$. For comparison, the concentration at the beach from the first (old) outfall is also shown by the dotted line. The longer the separation

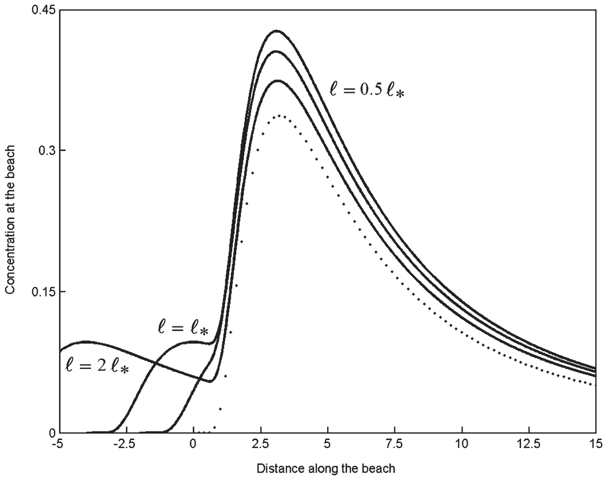

Fig. 5. The compounded concentration at the beach.

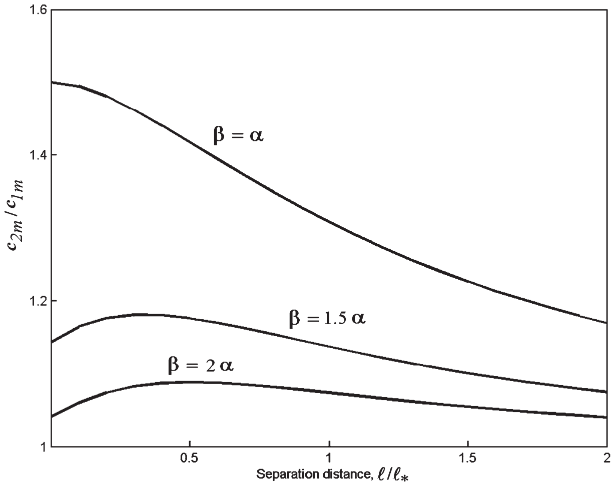

Fig. 6. The maximum value of the compounded concentration.

distance ℓ between the two outfalls, the smaller the contribution of the new outfall.

Finally, the maximum value of the compounded concentration at the beach is shown in Fig. 6 as a function of the separation distance between two outfalls with $q_* = 10$ for $\beta = \alpha$, $\beta = 1.5\alpha$ and $\beta = 2\alpha$. The only way to keep the effect of the (new) second outfall minimal is to build a longer new outfall. A reduction of 20% in the maximum value can be achieved by extending the new outfall length by 50%.

5. Conclusion

If two or more large scale seawater desalination plants are operated closely together along the coastlines, the maximum value of the compounded concentration at the beach is strongly dependent on the outfall lengths, the separation distance between the outfalls and the discharge ratio of each outfall. Therefore, when assessing the individual potential environmental impact of desalination plants, nearby plants' discharges should also be taken into consideration.

Modern ocean outfalls have been constructed with multiport diffusers to enhance initial dilution, which is essential in minimizing the impact of outfall plumes. In the limit $\ell = 0$, equation (6) can be extended for calculating the compounded concentration at the beach following discharges from a single outfall with multiport diffusers.

Nomenclature

c	–	concentration
c_{1m}	–	maximum concentration at beach
D	–	coefficient of dispersivity
D_0	–	reference value of D
h	–	water depth
h_0	–	arbitrary reference water depth
$I_{3/2}$	–	modified Bessel function
ℓ	–	separation distance
L	–	distance
m	–	beach slope
Q	–	rate of discharge
q_*	–	discharge ratio
U	–	drift current
U_0	–	reference value of U
x_1*	–	position of maximum concentration

Greeks

α, β	–	outfall lengths
λ	–	model parameter
δ	–	Dirac delta function

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