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Dropsize function during dropwise condensation in relation to heat transfer intensification – Statistical approach

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ABSTRACT

In the present trial a suggested statistical model is proposed to find the count and size distribution of droplets in dropwise condensation. This in turn, together with heat transfer through a single droplet is important for a thorough understanding of such mode of condensation. Growth due to natural (direct) condensation on the liquid/vapor interface and that by random coalescences are both considered. Fitting with early published experimental data is made for comparison.

Keywords: Dropwise condensation; Dropsize distribution function; Heat transfer

1. Introduction

Dropwise mode of condensation has aroused considerable interest of many investigators [1–18] because during this process the heat transfer coefficient is much higher than that during filmwise mode of condensation [1].

A thorough understanding of the dropwise condensation process requires good estimations of heat transfer through a single droplet together with a dropsize distribution function.

$$F(R) = \frac{\partial N}{\partial R} m^{-3}, \tag{1}$$

$$N = \int F(R) \mathrm{d}R m^{-2} \tag{2}$$

defined as the number of droplets of average radius (R), per unit area per unit radius interval. Where, $N(R)$, m^{-2} represents the number of droplets per unit area having radii in the size range between $R - \frac{\Delta R}{2}, R + \frac{\Delta R}{2}$. With average value ($R$).

The resolution power of the optical system from one side [2] and the capacity and speed of modern computers [3] which is not sufficient to simulate realistically high site density (nucleation centers) on the condensing surface, on the other side, put limitation on the trial to predict the dropsize distribution function.

Moreover, the coalescence between the neighboring droplets, complicates the trials to predict the distribution function analytically. Some of the suggested empirical formulas for the function $F(R)$ introduced by different authors [4–8] have been partially successful in predicting the heat transfer coefficient during the dropwise condensation process.

Due to the above-mentioned complexities, theoretical prediction of the distribution function is still required. The aim of the present article is to introduce a trial to predict the required function through a statistical approach.

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2. Derivation of the basic equations

It is well known that, vapor condenses on a hydrophobic surface in the form of a large number of initial droplets having critical radius ($R_{\min}$) determined by Thomson equation [9]:

$$R_{\min} = \frac{2\sigma T_s}{L\rho_1(T_S - T_W)},$$

where T_S, K is the absolute temperature of the saturated vapor at a given pressure, T_W, K is the absolute temperature of the condensing surface, σ, J/m^2 is a surface tension force of saturated vapor at a given pressure, L, J/kg is the latent heat of vaporization and ρ_1, kg/m^3 is the density of the liquid.

The droplets grow either by natural growth or through coalescence with neighboring droplets.

As a result a wide range of dropsizes are present on the condensing surface.

The drop grows until it attains a maximum radius ($R_{\max}$), then it starts to roll down the surface, sweeping it. Thus new generation of small droplets appear on the nucleation centers screened before by the departing drop. Thus a certain dropsize distribution function $F(R)$ is established on the condensing surface.

The process in general is transient process [10], but it can be studied as quasistationary if the time-averaged characteristics of the process remain constant. For simplicity, it is assumed that drops have hemispherical form.

2.1. Derivation of F(R) considering natural growth only

In the following an expression for $F(R)$ considering natural growth only (growth due to condensation of vapor molecules on the surface of the considered droplet) is considered.

The rate of volume increase for a single droplet is given as:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{2}{3}\pi R^3\right) = 2\pi R^2 \frac{\mathrm{d}R}{\mathrm{d}t} = 2\pi R^2 W(R). \tag{3}$$

The function $W(R)$, m/s is the natural growth rate function.

The heat transfer coefficient (h) in terms of both function $W(R)$ and $F(R)$ is given as [10]:

$$h = \frac{q}{\Delta T} = \frac{1}{\Delta T}\int 2\pi R^2\, W(R)\rho\, L\, F(R)\, dR, \tag{4}$$

where q, W/m^2 is the rate of heat flux through a unit area of the condensing surface.

Following a model introduced by El-adawi [8], according to which, the condenser surface is assumed to consist of neighboring boundaries containing droplets with successive increasing average radii R_i, R_{i+1}, R_{i+2}......

Making analogy with electricity, the continuity equation can be written in the form:

$$\mathrm{div} N(R,t) = \frac{\partial F(R,t)}{\partial t}. \tag{5}$$

For a quasistationary distribution function, Eq. (5) gives the relation:

$$W(R)F(R,t) = -\frac{\partial N(R,t)}{\partial t}. \tag{6}$$

This equation shows that the number of droplets leaving per second the boundary R_i equals $W(R_i)F(R_i)$.

To maintain a quasistationary dropsize distribution function on the surface [8] a dynamical equilibrium must be realized.

This condition can be formulated in the form:

$$\begin{aligned}\frac{2}{3}\pi R_{i+1}^3 F(R_{i+1})W(R_{i+1}) &= 2\pi R_i^2 W(R_i)N(R_i)\\ &\quad + 2\pi R_{i+1}^2 W(R_{i+1})N(R_{i+1}).\end{aligned} \tag{7}$$

Differentiating both side with respect to R_i one gets, the relation:

$$\frac{\partial}{\partial R_i} 2\pi R_i^2 W(R_i)N(R_i) = 0. \tag{8}$$

Hence

$$N(R_i) = \frac{C_1}{W(R_i)R_i^2}, \tag{9}$$

where C_1 is a constant of integration.

From Eq. (9), we get the following expression for the dropsize distribution function:

$$F(R_i) = \frac{\partial N(R_i)}{\partial R_i} = -C_1 \frac{\{\, W'(R_i)R_i^2 + 2R_i W(R_i)\,\}}{\{\, W(R_i)R_i^2\,\}^2} \tag{10}$$

Taking into account that the total temperature drop from vapor at its saturation temperature to the solid condensing surface is the sum of temperature differences due to curvature, interfacial mass transfer, and conduction [2,11,12]

$$\Delta T = \Delta T_i + \Delta T_{\mathrm{cap}} + \Delta T_{\mathrm{con}}.$$

It is possible to write the following expression for the growth rate velocity:

$$W(R) = \frac{\Delta T - \dfrac{2T_s\sigma}{L\rho_1 R}}{2\pi R^2\left(\dfrac{1}{2\pi\alpha_i R^2} + \dfrac{1}{4\pi\lambda_i R}\right)\rho_1 L} \tag{12}$$

where α_i is the instantaneous interfacial heat transfer coefficient W/m^2, and λ_1 is the thermal conductivity coefficient of the liquid W/mK.

Substituting from (12) into (10) one gets for $F(R)$, the following expression:

$$F(R_i) = -C_1 \frac{\left[\left(\frac{T_s\sigma}{L^2\pi\left(\frac{1}{2\pi\alpha_i R_i^2}+\frac{1}{4\pi\lambda_l R_i}\right)R_i^4\rho_l^2} - \frac{\Delta T - \frac{2T_s\sigma}{LR_i\rho_1}}{L\pi\left(\frac{1}{2\pi\alpha_i R_i^2}+\frac{1}{4\pi\lambda_l R_i}\right)R_i^3\rho_l} - \frac{\left(-\frac{1}{\pi\alpha_i R_i^3}-\frac{1}{4\pi\lambda_l R_i^2}\right)\left(\Delta T-\frac{2T_s\sigma}{LR_i\rho_1}\right)}{2L\pi\left(\frac{1}{2\pi\alpha_i R_i^2}+\frac{1}{4\pi\lambda_l R_i}\right)^2 R_i^2\rho_l}\right)R_i^2 + \left(\frac{\Delta T-\frac{2T_s\sigma}{L\rho_1 R_i}}{2\pi R_i^2\left(\frac{1}{2\pi\alpha_i R_i^2}+\frac{1}{4\pi\lambda_l R_i}\right)\rho_1 L}\right)2R_i\right]}{\left[\frac{\Delta T-\frac{2T_s\sigma}{L\rho_1 R_i}}{2\pi R_i^2\left(\frac{1}{2\pi\alpha_i R_i^2}+\frac{1}{4\pi\lambda_l R_i}\right)\rho_1 L}R_i^2\right]^2} \tag{13}$$

One further condition is required to determine the value of C_1.

The condition expresses the fact that the area covered by all droplets existing on the condensing surface must be less or equal to unity (1 m^2) i.e.:

$$\int_{R_{\min}}^{R_{\max}} \pi R^2 F(R)\mathrm{d}R \le 1. \tag{14}$$

Considering the equality sign as a limiting case one gets:

$$C_1 = \frac{-1}{\left[\frac{L^2(2\lambda_1+\alpha_i R)\pi R^2\rho_1^2}{2\alpha_i\lambda_1(-LR^2\Delta T\rho_1+2RT_s\sigma)}\right]_{R_{\min}}^{R_{\max}}} \tag{15}$$

In literatures, one finds two limiting cases for $W(R)$, namely:

1. $W(R) = \text{const}$. This case is applicable for very small droplets where the interfacial mass transfer between the liquid and vapor phases controls the growth rate. In such a case Eq. (10) shows that $F(R) \propto \frac{1}{R^3}$
2. For droplets of large size, where the growth rate is determined by heat conduction through the droplet [13].

$$W(R) = \frac{\text{const}}{R} = \frac{8.3\lambda\Delta T}{L\rho_l R} \tag{16}$$

For such a case, Eq. (10) shows that $F(R) \propto \frac{1}{R^2}$.

These results are in agreement with published experimental data [9] on dropsize distribution function.

Computations are carried out considering the same operating conditions as [9] to find $F(R)$ for the following parameters:

$T_s = 373^{\circ}\text{C},\ \Delta T = 0.277\,\text{K}, R_{\max} = 1,250$ micron,

$R_{\min} = 0.077$ micron, $L = 2.256 \times 10^6$ J/kg

$\lambda_l = 0.681$ W/m.K, $\alpha_i = 15.33 \times 10^6$ W/m^2.K,

$\rho_1 = 958.3 kg/m^3,\ \sigma = 0.058$ N/m.

Eq. (15) shows that $C_1 = -4.4 \times 10^{-8}$ m/s

The obtained computed values of $F(R)$ are compared with the corresponding experimental values given by Graham [9].

The comparison is illustrated graphically in Fig. 1.

One can conclude that the fitting is not satisfactory. This can be attributed to the fact that our model till now considers natural growth rate only and it does neglect the role of coalescence between neighboring drops. In the following this effective factor will be considered.

2.2. *Dropsize distribution function* $F(R)$ *taking the coalescence between droplets into consideration.*

For such a case the dynamical equilibrium can be formulated as follows:

$$dN_+ + d\nu_+ = dN_- + d\nu_-, \tag{17}$$

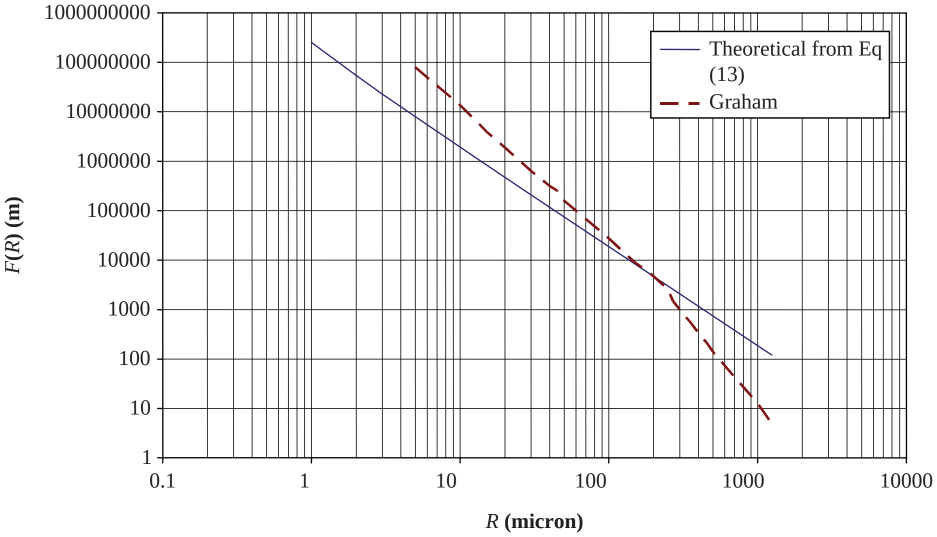

Fig. 1. Comparative illustration between the computed values [Eq. (13)] and the corresponding experimental values [9] of the function $F(R)$.

where dN_+ represents the number of droplets crossing the boundary of the considered zone from the preceding zone due to natural growth. dN_- represents the number leaving the same zone due to natural growth. While, $d\nu_+$ is the number of droplets entering the considered zone from the preceding one due to coalescence process between droplets of smaller sizes. Finally, $d\nu_-$ is the number of droplets leaving the considered zone due to coalescence of the zone droplets with each others or with neighboring droplets of any size.

According to the arguments given in the already discussed in Section 1 the function dN_- is given as:

$$dN_{-k} = W(R_k)F(R_k). \tag{18}$$

(1) Determination of dN_{+k}:

Let us assume that it is inversely proportional to the time required for a droplet to grow naturally from $R_{\min} \cong 0$ up to R_k, i.e:

$$dN_{+k} = \gamma_k \left(\int_0^{R_k} \frac{\mathrm{d}R}{W(R)} \right)^{-1}, \tag{19}$$

where γ_k is the constant of proportionality, γ_k depends on the density of the nucleation centers and on the operating conditions governing the condensation process.

(2) Determination of $d\nu_+$:

When two neighboring droplets come to contact, they coalescence instantaneously. At this moment let the radius of one of them is R_i while the radius of the other one is R_j. Thus one gets a droplet with radius R_k such that

$$R_k = \sqrt[3]{R_i^3 + R_j^3}. \tag{20}$$

The time required to realize this event satisfies the following inequality:

$$\tau_{ij} \le \int_{R_{\min}}^{R_i} \frac{\mathrm{d}R}{W(R)} + \int_{R_{\min}}^{R_j} \frac{\mathrm{d}R}{W(R)} \tag{21}$$

Assuming that $d\nu_{+k} \propto \frac{1}{\tau_{ij}}$ and is proportional also to the existing number of both types of droplets i.e.:

$$d\nu_k \approx N(R_i) \cdot N(R_j) \tag{22}$$

One can thus write:

$$d\nu_{+k} = \sum_{i,j} \alpha_k \left(\int_{R_{\min}}^{R_i} \frac{\mathrm{d}R}{W(R_i)} + \int_{R_{\min}}^{R_j} \frac{\mathrm{d}R}{W(R_j)} \right)^{-1}, \tag{23}$$

α_k represents the constant of proportionality and R_i ranges from $R_{\min}$ up to $\sqrt[3]{R_k^3 - R_{\min}^3}$ while R_j ranges from $\sqrt[3]{R_k^3 - R_{\min}^3}$ down to $R_{\min}$.

(3) Determination of $d\nu_-$:

To determine the number per unit area per second that leaves the zone of R_k due to coalescence, one assumes that it is proportional to $d\nu_{-k}$ in the form:

$$d\nu_{-K} = \beta_k \sum_{i,j} \alpha_k \left(\int_{R_{\min}}^{R_i} \frac{\mathrm{d}R}{W(R_i)} + \int_{R_{\min}}^{R_j} \frac{\mathrm{d}R}{W(R_j)} \right)^{-1}, \qquad (24)$$

where β_k is the constant of proportionality.

Let us indicate the following cases:

- For $\beta_k = 1$, both numbers of droplets entering and leaving the zone R due to coalescence are equal. This is equivalent to the case when one neglects the role of coalescence in the determination of the shape of the distribution function.
- For $0 < \beta_k < 1$, For such a case, the number that disappears in the considered zone due to coalescence is less than the number of droplets entering the considered zone due to coalescence.

To realize the dynamical equilibrium, the accumulation of droplets is balanced by the process of natural growth. For the considered case the number of droplets entering the considered zone due to natural growth will be less than the corresponding number emigrating (leaving) the zone due to natural growth and vice versa.

- A similar explanation can be given for the case when $\beta_k > 1$.

Substituting the expressions for $dN_-, dN_+, d\nu_- and\ d\nu_+$ given by equation (18), (19), (23) and (24) in the dynamical condition (17) and rearranging the terms, one gets for the distribution function the following expression:

$$F(R) = \frac{1}{W(R)} \left(\gamma_k \left(\int_0^{R_k} \frac{\mathrm{d}R}{W(R)} \right)^{-1} + \sum_{i,j}^{R_k} \alpha_k (1-\beta_k) \left(\int_0^{R_i} \frac{\mathrm{d}R}{W(R)} + \int_0^{R_j} \frac{\mathrm{d}R}{W(R)} \right)^{-1} \right) \qquad (25)$$

For the limiting cases, when $W(R) = \frac{C_3}{R}$ one gets:

$$F(R) = \frac{2\gamma_k}{R_k} + \alpha(1-\beta) \sum_{R_i,R_j}^{R_k} \frac{2R_k}{R_i^2 + R_j^2}, \qquad (26)$$

While for the case $W(R) = const = C_2$ one gets:

$$F(R_k) = \frac{\gamma_k}{R_k} + \alpha(1-\beta) \sum_{R_i,R_j}^{R_k} \frac{1}{R_i + R_j} \qquad (27)$$

Let the functions $\psi(R) = \alpha(1-\beta)$ for large droplets and $\varphi(R) = \alpha(1-\beta)$ for small droplets, together with the function $\gamma_k(R)$. These functions are determined through statistical treatment of the experimental data published by Graham [9] as follows:

The critical radius after which coalescence may take place is determined assuming a square array of the nucleation centers [2] with a definite suggested number ζ^2. In our computation this number is taken as 0.25×10^{12} [14,15], this gives a threshold value of $R_{\text{th}} = 1 \times 10^{-6}$ m = 1 micron.

Between $R_{\min}$ and R_{th}, the process is wholly due to natural growth. For such a case $\beta_k = 1$ and one gets $F(R) = \frac{\gamma_k}{R}$:

For each experimental value $F(R)$ [9] for small droplets, the function $\gamma_k(R)$ is obtained corresponding to the radius R.

The scatter of such a function makes it possible to suggest a semi-empirical formula to predict it in the form: $\gamma_k(R) = AR^{-B}$. The parameters A and B are determined through least square fitting technique [19]. For larger values of "R" one can find the values of both functions $\phi(R)$, and $\psi(R)$ and in the same way one can obtain the formulas to predict them.

For a certain regime one gets the following expressions:

$$\gamma(R) = 0.301(R)^{-1.69} \qquad (28)$$

$$\phi(R) = -2.3244 \times 10^{-5}(R)^{-2.698} \qquad (29)$$

$$\psi(R) = -3.884 \times 10^{-5}(R)^{-2.61} \qquad (30)$$

For the same regime considered before the obtained computed values of the distribution function is compared with the experimental data published by Graham [9] and are illustrated graphically in Fig. 2, good fitting is now obtained.

3. Conclusions

The obtained results make it possible to indicate the following conclusions:

1. In general the dropsize distribution function $F(R) \sim R^{-n}$.
2. Colescence plays an important role in determination the considered function.
3. The statistical trend is promising to understand the mechanism of dropwise mode of condensation.

Nomenclature

c_1	constant of integration
$F(R)$	droplet distribution function, m^{-3}

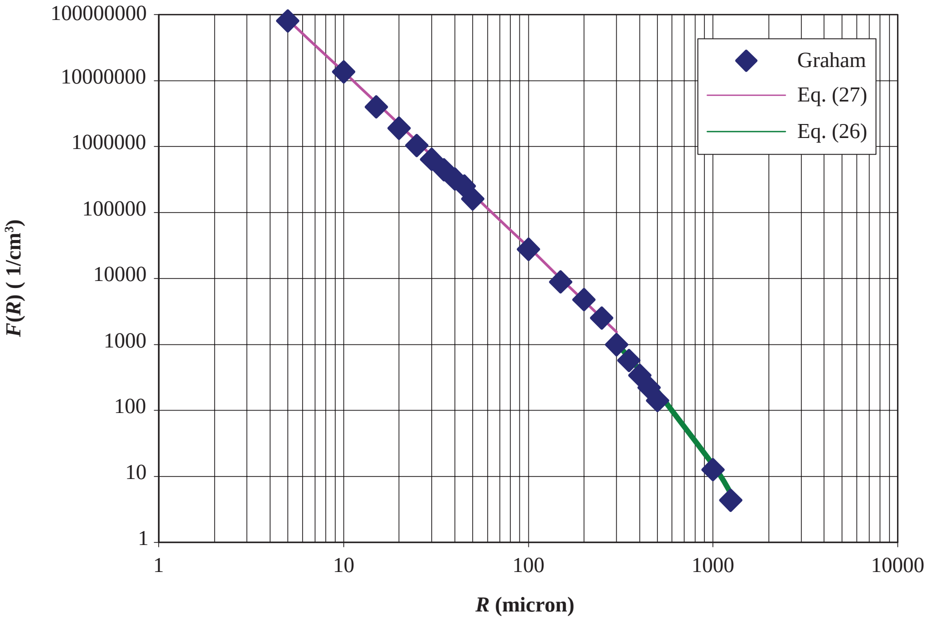

Fig. 2. Comparative illustration between the computed values [Eq. (26), Eq. (27)] and the corresponding experimental values [9] of the function $F(R)$.

h	Heat transfer coefficient, W/m^{2}K
L	latent heat of vaporization, J/kg
N(R)	total number of drops per unit area with radii between R$-\frac{\Delta R}{2}, R+\frac{\Delta R}{2}$, m^{-2}
dN_+,dN_-	defined in the text,
q	heat flux, W/m^2
R	droplet radius, m
R_{th}	threshold radius value (defined in the text), m
T	absolute temperature, K
T_{s}	the absolute temperature of the saturated vapor, K
T_{w}	the absolute temperature of the condensing surface, K
ΔT_i	the temperature difference associated within interfacial resistance, K
ΔT_{cap}	the capillary depression of equilibrium saturation temperature, K
ΔT_{con}	the temperature difference within conduction resistance through the droplet, K
t	time, s
$W(R)$	the natural growth rate of the droplet, m/s

Greek letters

α_K,	function of proportionality,
α_i,	interfacial heat transfer coefficient, W/mK
β_K,	function of proportionality,
$\gamma_K(R)$	defined in the text,
λ,	the thermal conductivity of the condensate, W/mK
$d\nu_+, \mathrm{d}\nu_-$,	defined in the text,
ξ^2,	total number of nucleation centers per unit area, m^{-2}
ρ,	liquid density, $^{\mathrm{kg}}/_{\mathrm{m}^3}$
σ,	the surface tension, J/m^2
$\Phi(R)$,	defined in the text,
$\boldsymbol{\Psi}(R)$,	defined in the text,

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