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Propagation of uncertainties in water distribution systems modeling

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ABSTRACT

Optimal design and systematic uncertainty modeling of a municipal Water Distribution System (WDS) aim to minimize operational & construction costs, while meeting demands with required pressure levels and water quality. Study about the propagation of uncertainty through the hydraulic model provides a basis for general model improvement in an efficient and economic way. The present study introduces a novel technique for uncertainty analysis of a WDS, which synchronizes the hydraulic network and water quality solving capabilities of EPANET with an interface to MATLAB. It treats the pipe friction factors and the nodal demands as input fuzzy variables whereas the output fuzzy parameters include pipe discharges, nodal heads and chlorine concentrations. The methodology involves decomposition of the given uncertainties into a set of all possible combinations of input parameter values, numerical analysis of the hydraulic network, calculation of extremities of each unknown variable at each alpha cut level and the final construction of the respective fuzzy membership functions. Four distinct types of uncertainties associated with a hydraulic network are identified. Besides these, a sample hydraulic network is conceptualized to investigate the responses of different head-loss formulae to the variations in uncertainty.

Keywords: Water quality; Uncertainty analysis; *α*-cut level; Water distribution system; Roughness; Fuzzy set; Demand uncertainty

1. Introduction

A reliable municipal Water Distribution System (WDS) has been defined in the literature which can satisfy the consumer demands in quality and quantity under both normal and adverse working conditions [1,2]. Optimal design study and systematic uncertainty modeling of a WDS are aimed at to minimize operational and construction costs, while meeting quantity requirements of maintaining adequate nodal heads, pipe discharges, and water quality requirements (e.g., concentrations of residual chlorine and fluoride). These parameters rely

on several factors, like pipe roughness, nodal demands, pump characteristics, pipe lengths, diameters, amount of disinfectants, etc. The present study primarily investigates the effects of variations in pipe friction and the nodal demands on pipe discharges and nodal heads. Interestingly, pipe roughness and nodal demands can't be treated as deterministic parameters. For instance, pipe roughness is governed by the period of network usage while complex processes like corrosion, erosion, etc., constantly degrading the pipe surfaces. Even the demands at various nodes vary with time. Such complex correlation inhibits by precise prior estimation of these

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two factors. Furthermore, the cost and time involved in their measurements restrain the formulation of any proper database that could enable their representation in terms of probabilistic language. Besides inadequacy of information, imprecision in parameter values due to numerous factors precludes the use of mathematical tools. However, fuzzy set theory [3] helps translate scanty, imprecise information and linguistic jargons associated with independent network parameters to mathematical functions which can then be employed in an optimization framework to determine uncertain information of dependent parameters. Once the magnitude of the uncertainty in heads, discharges, and water quality requirements are quantified as fuzzy numbers, an engineer can use that information in several ways. Mainly safety factors can be included in design of network improvements. To enhance model predictions, additional field data can be collected which will lead to more precise model parameters. Finally, different network representations can be evaluated by comparing their predictive uncertainties [4].

Lansey et al [5] were the first to suggest an analysis that directly considered the uncertainties of WDS modeling. A chance constrained nonlinear programming technique was suggested to restrict the probability of not satisfying the demand and pressure at each node to some acceptable level of tolerance. Bao and Mays [1] proposed reliability analysis of pipe networks considering randomness of uncertain parameters through Monte Carlo simulation. Xu and Goulter [6] suggested linear probabilistic model based on first-order Taylor's series expansion for quickly carrying out large number of simulations for a WDS. Xu and Gaulter [7] used first-order reliability method for uncertainty modeling of a WDS, but the approach was only suitable for situations with small variations of uncertain parameters. The accuracy of first-order model deteriorated when large variations in the values of uncertain parameters were involved due to the effect of nonlinearity in the hydraulic model. Rivelli and Ridolfi [8] framed non-linear fuzzy equations for uncertainty modeling of WDS and solved using the concepts of fuzzy numbers in an optimization framework. Vamvakeridou-Lyroudia et. al. [9] have performed multiobjective optimization by combining fuzzy reasoning with genetic algorithms. Gupta and Bhave [10] proposed a technique based on usual hydraulic network analysis to determine the membership functions of dependent parameters. Their method relied on monotonic relationships between independent and dependent parameters, which might become invalid due to the inherent nonlinearity in hydraulic networks. A clustering and global variance-based sensitivity methodology is proposed by Pasha and Lansey [11] to account for spatial inconsistencies found in the results of their previous studies of uncertainty modeling of WDS. Uncertainties in WDS modeling creep in due to: (1) used formula [Hazen-Williams (HW), Darcy-Weisbach (DW) or Chezy-Manning (CM)] [12] for hydraulic simulation; (2) coefficients in the formula and (3) imprecise knowledge of the values of different parameter [13]. There is no literature available on uncertainty analysis of WDS that considers all these uncertainties, that leads to the motivation of the study presented in this paper. The knowledge of propagation of uncertainty can be used in developing Decision Support System (DSS) to answer the repair vs. replace dilemma for water distribution networks [14,15].

The objective of the present study is to understand the sources of uncertainty in water distribution system modeling so that better modeling and monitoring methodologies can be developed. The study investigates how the uncertainties in pipe friction and the nodal demands, considered formula & their parameters for hydraulic simulation addressed by fuzzy numbers engender uncertainties in water quality, pipe discharges and nodal heads. Unlike traditional efforts, the present study develops a generic methodology applicable to any WDS network, which processes the independent parameter uncertainties through a unique algorithm that is executed by EPANET [12] and MATLAB (http://www.mathworks.com) [16]. EPANET is public domain tool developed by USEPA that simulates hydraulic and water quality behavior of pressurized pipe networks very accurately. It tracks the flow of water in each pipe, the pressure at each node and the concentration of a chemical species throughout the network during a simulation period comprising of multiple time steps. The present method utilizes all the features of EPA-NET for uncertainty modeling, which brings flexibility in simulating the network more realistically. The Programmer's Toolkit accompanying it enables a programmer controlled & systematic network analysis using C/C++, MATLAB, Visual Basic, etc. In the present study MATLAB is chosen since it offers superior software-user interface and enhanced flexibility in controlling the execution of Toolkit's functions. The uncertainty induced due to lack of precise correlation between the friction factors obtained from different formulae is also addressed in the present study using the 'hydraulics' look-up menu provided in the EPANET software. This furnishes an option of choosing one out of Hazen-Williams (HW), Darcy-Weisbach (DW) and Chezy-Manning (CM) or Strickler resistance formula for the head loss computations. Uncertainty of Type C is linked to the task of selecting the best headloss formula for a given network. The present work is an extension of the model developed by Shradhanand and Karmakar [17] considering water quality aspect of WDS.

The paper is organized into four sections: Section 2 gives an introduction to pipe network hydraulics and points out the sources of uncertainty; Section 3 describes the proposed methodology elaborately in few subsections; Section 4 elaborates the model application; Section 5 explains the results and discussion; and finally Section 6 presents concluding remarks.

2. Pipe network hydraulics and sources of uncertainty

Flow in a WDS satisfies two basic principles: (1) conservation of mass and (2) conservation of energy. Conservation of mass states that, for a steady system, the flow into and out of the system must be the same and this holds for the entire network and for individual nodes. A mass balance equation is written for each node in the network:

$$\sum Q_{\rm in} - \sum Q_{\rm out} = Q_{\rm demand} \tag{1}$$

where Q_{in} and Q_{out} are the flows in pipe entering or exiting the node and Q_{demand} is the user demand at that location. These demands are uncertain as they are estimated from the local user base that can not be predicted exactly since they vary continually. In addition, the demand is typically represented as a lumped demand for users near the node. The second governing equation is a form of conservation of energy that describes the relationship between energy loss and pipe flow. There are three very popular head loss formulae in the hydraulics of WDS, i.e., Hazen–Williams (HW), Darcy–Weisbach (DW) or Chezy–Manning (CM) formula. A common representation for the three formulae:

$$h_L = Aq^B \tag{2}$$

where h_L is head loss (length), q is flow rate (volume/time), A and B are resistance coefficient and flow exponent, respectively. The expressions for various formulae with the values of resistance coefficient and flow exponent are tabulated in Table 1, where C, f and n are Hazen–Williams (HW), Darcy–Weisbach (DW) or Chezy–Manning (CM) roughness coefficients; d and L are diameter and length of pipe, respectively.

In USA and India the HW formula is widely used, England prefers DW formula, while the CM is preferred in certain parts of European continent [13]. The head loss in a pipe calculated by different formulae gives different values and all these answers should be taken as correct in practice.

Sources of uncertainty involved in WDS modeling and analysis may be classified in four categories: *Type A* (lack of precise measurements) — this uncertainty provides a snapshot of uncertainty in a network parameter

Table 1 Pipe head-loss formulae for full flow (head-loss in feet and flow rate in cfs) [12]

Formula	Resistance co-efficient (A)	Flow exponent (B)
Hazen–Williams	$4.727 \ C^{\scriptscriptstyle -1.852} \ d^{\scriptscriptstyle -4.871} \ L$	1.852
Darcy–Weisbach Chezy–Manning	0.0252 fd ⁻⁵ L 4.66 n ² d ^{-5.33} L	2 2

at a particular instant of time. It stems from a variety of factors. For instance, pipe friction factors represent pipe roughness that prominently depends on the wall friction encountered by a flowing fluid and the fluid friction (viscous drag force). The former relies on precise knowledge of average height of protrusions along pipe's entire inner periphery which is practically infeasible. Further, imprecision associated with viscous force promotes uncertainty in pipe friction factors. Type B (time induced uncertainty in pipe roughness): As explained above, the instantaneous wall friction accounts for the physical condition of pipe's inner surface at that moment. However, due to the complex biological and chemical reactions like organic growth, tuberculation, scale formation, corrosion, etc. constantly deteriorating the surface quality, wall friction becomes a casual function varying with the period of network usage. The effect of these processes on pipe roughness being indeterminate, they introduce a time-dependent uncertainty which is classified as type B uncertainty. It is well accepted that with age, water carrying pipes are susceptible to an increase in pipe roughness which implies an increase in Darcy's friction factor or a decrease in Hazen-Williams roughness coefficient. Variations in friction factors over sufficiently long periods have been documented by some researchers [18,19]. Type C (due to lack of precise correlation between the friction factors obtained from different formulae): The 'hydraulics' look-up menu provided in the EPANET software furnishes an option of choosing one out of Hazen-Williams (HW), Darcy-Weisbach (DW) or Chezy-Manning (CM) resistance coefficient for the head loss computations. Though results from all the three are empirically correct, the formula that yields results closest to the field observations is the most appropriate for the particular network. Owing to distinct theoretical origins, the results from the three formulae must be scaled appropriately prior to comparison. This necessitates a precise correlation between friction factors which in turn requires precise flow and hydraulic diameter measurements. Type D (model uncertainty): all the three head-loss formulae are empirical relations derived from distinct theoretical backgrounds. The Hazen-Williams formula is computationally the most convenient and hence the most widely used. The Darcy-Weisbach formula incorporates both viscous drag and wall friction and is the most theoretically correct. It applies to all liquids and is valid over all flow regimes. However, extensive computational overhead linked to the estimation of f (Darcy's friction factor) based on velocity-diameter combination make it less popular. Owing to such fundamental dissimilarities each formula transmits the uncertainties from the input to output uniquely resulting in non-coinciding results. In the present study, all these uncertainties are modeled using the concepts of fuzzy set theory.

A classical (traditional) set either 'wholly' embraces an element or 'wholly' excludes it, whereas a fuzzy set endorses 'partial membership' of an element. Fuzzy sets in practice are often understood as fuzzy numbers [20] and are represented through membership functions. Mathematically, if X is the set of discourse, then fuzzy set A in X is defined as the set of all ordered pairs such that,

$$A = \left\{ \left[x, \mu_A(x) \right], x \in X, \mu_A(x) \in [0, 1] \right\}$$
(3)

Here, $\mu_A(x)$ is the membership function that expresses the degree of membership of '*x*' in terms of a value between 0 and 1. For present analysis, uncertain parameters assume triangular membership functions. The membership function values for pipe friction factors and nodal demands have been borrowed from well documented literature [10,19,21]. The α -level (alpha-level) cut [20] of a fuzzy number A is defined as a set of those elements which have 'at least α membership':

$$A_{\alpha} = \left\{ x \in X, \mu_{A}(x) \le \alpha \right\}$$

$$\tag{4}$$

In the present study, the values of pipe roughness and nodal demand are assumed as fuzzy numbers with known extreme variations and triangular distributions following the assumptions of Revelli and Ridolfi [8] and Gupta and Bhave [10]. Such a triangular fuzzy number (A) can be mathematically described as:

$$\boldsymbol{\mu}_A(\boldsymbol{x}) = \boldsymbol{0}, \boldsymbol{x} < \boldsymbol{a}_1 \tag{5}$$

$$\mu_A(x) = \frac{x - a_1}{a_3 - a_1}, \ a_1 \le x \le a_3 \tag{6}$$

$$\mu_A(x) = \frac{a_2 - x}{a_2 - a_3}, \ a_3 \le x \le a_2 \tag{7}$$

$$\mu_A(x) = 0, \ x > a_2 \tag{8}$$

where a_1 , a_2 , a_3 are scalar parameters, a_1 and a_2 form the base of the triangle (also known as the support) showing maximum variation of A, while a_3 defines the crisp value showing most possible value. Next section describes the proposed methodology based on the fuzzy sets theory to model the uncertainties in WDS analysis.

3. Proposed methodology

The pipe roughness and nodal demand of the entire WDS are considered as triangular fuzzy numbers and at each alpha-level they are considered as two independent events. Following procedure is followed to generate membership functions of each pipe flow, nodal head and chlorine concentration.

3.1. Procurement of data

First step for any network uncertainty analysis involves construction of the test network in the EPANET's

'Network Map' and exporting the same in form of .inp file to the MATLAB's work directory. The devised MATLAB code is executed thereby initiating a chain of events. Initially, a part of the data required for uncertainty analysis is acquired from the user while the remaining parameters are fetched by exercising several toolkit's functions on the .inp file of the sample network. Among the figures sought from user, chief are the number of alpha levels desired for the network uncertainty analysis, the membership functions of pipe friction factors and the nodal demands. These are stored in an array "memberfn" whose physical layout depicted in Fig. 1, reflects the code execution procedure. Along with the data keyed in by the user, the program fetches the number of nodes and pipes in the given network by evoking a series of functions like ENopenH(), ENgetlinkcout(), ENgetnodecount(), etc.

3.2. Computing the α -cut interval of every input membership function

Considering a triangular membership function, a horizontal line corresponding to any particular α level shall inevitably intersect the function's curve at two distinct points thereby forming an α -cut interval.

These upper and lower bound points of each α -cut interval (corresponding to each of the prescribed α levels) are calculated for all the input membership functions. These are stored in an array named intersectval, whose data storage pattern is shown in Fig. 2.

Here, two columns are allotted per input fuzzy vari-



Fig. 1. Physical layout of the array memberfn and the pattern in which values are stored.



Fig. 2. Schematic representation of data storage pattern in the array intersectval.

able. Hence, the columns total up to twice the sum of the number of pipes and nodes. The number of rows equals the number of α levels. All the required data related to a particular input variable is summarized in a sub-array comprising of two columns and the corresponding rows. In each such columnar duplet, the first column stores the values of lower bounds (a, c, e, g, i) whereas the second column stores the upper bound values (b, d, f, h, i) corresponding to different α cut intervals. Such border (extreme) values for α cut intervals formed at each alpha level are stored row-wise.

3.3. Decoupling the uncertainty in various input parameters in the form of all possible combinations of various extreme values of α cut intervals corresponding to all variables

Unlike the traditional techniques developed so far for uncertainty analysis of a pipe network, this model handles the uncertainty of input variables in a unique fashion. It is well recognized that at any particular α level, the input fuzzy parameters can adopt a value from its respective α -cut interval whose extreme values are stored in intersectval. There remain two options for the value to be assigned to each input variable- the upper bound or the lower bound of the corresponding α -cut interval. Further, a total of *n* pipes and *m* nodes shall yield (n + m)input fuzzy parameters. Consequently, there are $2^{(n+m)}$ possibilities in which values can be assigned to input fuzzy parameters. In this manner, the input parameter uncertainties in the hydraulic network are decomposed into a set of $2^{(n+m)}$ combinations of permissible parameter values. All these combinations are figured out and stored

in an array called combinations. This array is structurally analogous to a truth table representing a system with the nodal demands and pipe friction factors as the inputs. Such an array generation is carried out at each α level. The above discussion is pictorially summarized in Fig. 3.

3.4. Computing the unknown parameters for a particular alpha level

At each α level, all the values of input parameters present in one particular row of the array combinations are assigned to the respective network elements using the toolkit's functions like ENsetnodevalue() and ENsetlinkvalue(). Thereafter the network is solved with the help of function ENsolveH(). This is followed by retrieving the values of the unknown parameters with the aid of toolkit's functions like ENgetnodevalue() and ENgetlinkvalue() combinations. Such a procedure ensures that all the feasible combinations are taken care of. It gives $2^{(n+m)}$ values for each unknown parameter which are stored in a separate array. For instance, in the developed code, the array nodehead stores all the $2^{(n+m)}$ values for the head at each node. In a similar fashion, the array linkflow stores all the $2^{(n+m)}$ values for the discharge through each pipe. These arrays are schematically represented in Fig. 4.

3.5. Sorting the arrays linkflow and nodehead to find the maximum and minimum of the unknown parameters for a particular α level

Towards the end, the software computes maximum and minimum of any unknown variable from the arrays

Columns for <i>n</i> pipes	Colum	ns for <i>m</i> nod	es One of the possible
		<u> </u>	combinations of
LB _{pipe1} LB _{pipe n}	LB _{node1}	LB _{node} m	lower bound
LB _{pipe1} LB _{pipe n}	LB _{node1}	UB _{node} m	intervals
LB _{pipe1} LB _{pipe n}	UB _{node1}	LB _{node} m	corresponding to all variables
LB _{pipe1} LB _{pipe n}	UB _{node1}	UB _{node} m	
LB _{pipe1} UB _{pipen}	LB _{node1}	LB _{node} m	
LB _{pipe1} UB _{pipe n}	LB _{node1}	UB _{node} m	
LB _{pipe1} UB _{pipe n}	UB _{node1}	LB _{node} m	Total number of possible combinations
LB _{pipe1} UB _{pipe n}	UB _{node1}	UB _{node} m	of extreme values at
UB _{pipe1} LB _{pipe n}	LB _{node1}	LB _{node} m	
UB _{pipe1} LB _{pipe n}	LB _{node1}	UB _{node} m	Number of rows =
UB _{pipe1} LB _{pipe n}	UB _{node1}	LB _{node} m	2 ****
UB _{pipe1} LB _{pipe n}	UB _{node1}	UB _{node} m	UB _{pipe1} : Upper bound of
UB _{pipe1} UB _{pipe n}	LB _{node1}	LB _{node} m	friction factor of pipe 1
UB _{pipe1} UB _{pipe n}	LB _{node1}	UB _{node} m	LB _{pipe1} : Lower bound of
UB _{pipe1} UB _{pipe n}	UB _{node1}	LB _{node} m	the alpha cut level of a friction factor of pipe 1
UB _{pipe1} UB _{pipe n}	UB _{node1}	UB _{node} m	

Array combinations corresponding to a particular alpha interval

Fig. 3. Schematic representation of data storage pattern in the array combinations.

linkflow and nodehead constructed at each alpha level (Fig. 5). These form the boundary values for the various α -cut intervals of the respective desired parameters.

3.6. Plotting the membership functions of unknown parameters

This is the conclusive step of network uncertainty analysis. The boundary values obtained in the previous step are used to construct a plot of membership functions for the corresponding desired parameters (pipe flows and nodal heads). Subsequently, these curves are subjected to meticulous investigation the results of which are presented in the following section.

4. Application

This section introduces a test network, which is utilized throughout the present study to illustrate the proposed procedure for uncertainty modeling. The network is a simple 5 pipes, 4 nodes system and is structurally same to the work of Rivelli and Ridolfi [8]. The data for various nodes and pipes has been shown in Fig. 6. Several sets of computations for uncertainty analysis are

conducted on the network by using the above procedure with various input membership functions (Tables 2 and 3) and different head-loss formulae. The data on variation in pipe roughness values are based on literature [18,19,22]. The concentration of chlorine at supply node (node 1) is considered as 0.5 mg/L. It is to be noted that the pipe friction factor (f) is being considered as uncertain hydraulic parameter following the applications by Rivelli and Ridolfi [8] and Gupta and Bhave [10]. The deterministic value (crisp value of the fuzzy number) of *f* is considered as 0.85 (in Table 2) for demonstration purpose. The value of friction factor depends on the absolute pipe roughness, Reynolds number and pipe discharge. The source of uncertainty is actually the absolute roughness of the pipe wall.

A rigorous and well planned uncertainty analysis drill is conducted for the network using the proposed computational procedure. The exercise involved several sets of computations performed using a variety of input membership functions and head-loss formulae. A thorough investigation culminated in the birth of four distinct types of uncertainties coupled to a hydraulic network. The first two are intrinsic to any pipe network while



Fig. 4. Schematic representation of data storage pattern in the arrays linkflow and nodehead.

Table 2 Triangular membership functions for pipe friction factors (identical for all 5 pipes)

% Uncertainty	Hazen–Williams (C)	Darcy–Weisbach (f)	Chezy–Manning (n)
10%	(121.5 135 148.5)	(0.765 0.85 0.935)	(0.01215 0.0135 0.01485)
20%	(108 135 162)	(0.68 0.85 1.02)	(0.0108 0.0135 0.0162)

Table 3

Triangular membership functions for nodal demands with 10% uncertainty

Uncertainty	Node index	Triangular membership function		
10%	1	(4.7681	5.2979	5.8277)
	2	(9.5349	10.5943	11.6537)
	3	(7.3102	8.1224	8.9346)

the latter two are related to the head loss computation approach adopted for the network's hydraulic analysis. The following section explores the ins and outs of each of these types.

5. Results and discussions

The uncertainties in pipe discharges considering uncertainty in roughness and nodal demand values are shown in Fig. 7 as fuzzy membership functions. A careful comparison between the results obtained using various head-loss formulae available with EPANET are also performed. The uncertainties of types C and D prevent a perfect match of crisp values (or the peaks of the membership functions) for flows of any pipe as shown in Fig. 7.

The flow is highest for pipe 1 for which the peaks of different discharge membership functions are noticeably spaced apart from each other. On the contrary, the peaks of discharge membership functions for pipes 2 and 3 (which have quite low discharges) almost overlap over one other. This observation points out a direct correlation



Fig. 5. Schematic representation of calculation of the extreme values of α^* -cut corresponding to head at node 1.



Fig. 6. Sample network (not to scale).

between the quantum of discharge and the uncertainty due to absence of precise correlation between various friction factors of different head loss formulae. Similarly, Fig. 8 shows the uncertainty in chlorine concentration, which clearly shows the nonlinearity of water quality simulation model of EPANET.

Response of the three head-loss formulae to fluctuations in input parameter uncertainty is studied by gradually increasing the support widths (in steps of 10% of the crisp value) of their fuzzy sets. A wider support of fuzzy set implies enhanced uncertainty in the corresponding parameter. In Fig. 9, 10% and 20% uncertainties in a parameter indicate that its fuzzy set has supports (0.9N, 1.1N) and (0.8N, 1.2N) respectively, where N is the crisp value. Suitable membership functions for uncertainty analysis are substituted from Tables 2 and 3. On increasing the friction factor uncertainty from 10% to 20%, with nodal demands fixed at crisp values, output fuzzy sets exhibit up to 238% surge in their support widths. Particularly, the Hazen-Williams and Chezy-Manning models are 'hyper sensitive' to variations in friction factor uncertainties. However, the Darcy-Weisbach formula is quite sensitive to nodal demand uncertainty variations while it responds feebly to those of pipe friction factors. Thus, it is suitable for analyzing a network wherein the friction factor data is less reliable than the nodal demand values. If the other two formulae are used, then even a



Fig. 7. Comparison of membership functions for pipe flows obtained by using different head loss formulae.



Fig. 8. Uncertainties in chlorine concentrations at nodes 2 and 3.

slight discrepancy in the already uncertain friction factor data will be unnecessarily amplified thereby deteriorating the reliability of final results.

The next bar diagram (Fig. 10) represents the maximum chlorine concentration obtained at different alpha levels for nodes 2 and 3. The *x* axis represents the 5 alpha levels (where level 5 indicates the crisp value of fuzzy number) and *y* axis represents the chlorine concentration in mg/L. The figure shows that, the concentrations at nodes 2 and 3 for five alpha levels obtained from DW equation are lower than that obtained from HW and CM.

The present study formulates a generic methodology of uncertainty modeling of WDS considering pipe roughness and nodal demand as uncertain parameters. Due to space constraint this section is limited to the above mentioned discussion on derived results.

6. Conclusions

The present study performs an exhaustive uncertainty analysis on water distribution network considering hydraulic and water quality characteristics. Uncertainty in a hydraulic network is being categorized into four different categories. Type A illustrates uncertainty at a given time instant due to inadequacy of or imprecision in data values. Type B studies the influence of complex ageing processes. Uncorrelated friction factors belonging to different head-loss equations introduce type C uncertainty. Different head-loss equations being theoretically distinct to empirical relations which uniquely process the input uncertainties giving rise to type D uncertainty.

The proposed method bypasses the use of optimization model and yet accounts for the inherent non-linearity



Fig. 9. Responses of different head-loss formulae to fluctuations in independent parameter uncertainties (ND – nodal demand, FF – friction factor).



Fig. 10. Uncertainty in chlorine concentration values (in mg/L) of nodes 2 and 3.

in a hydraulic network by decoupling the uncertainty at every α -level into a set of 2^{m+n} (for *m* nodes and *n* pipes) possible sequences of fuzzy set extremes. Most prominently, the software computations are independent of the physical network layout. This has been achieved by avoiding 'network specific' computations like categorization of network into loops/trees, formulation of loop equations,

etc. employed by conventional fuzzy uncertainty analysis techniques. Besides that EPANET facilitates (a) uncertainty analysis of realistic networks with valves, tanks, as well as control schemes (b) convenient alterations (if required) in the hydraulic network design.

The overestimating character of Hazen–Williams and Chezy–Manning is attributed to their restricted applica-

bility. Besides, the Chezy–Manning formula responds almost identically towards uncertainty variation in either of the two independent parameters. The Hazen–Williams is appropriate for network analysis if the nodal demand data lack consistency. However, the Darcy–Weisbach formula is quite sensitive to nodal demand uncertainty variations while it responds feebly to those of pipe friction factors. Thus, it is suitable for analyzing a network wherein the friction factor data is less reliable than the nodal demand values.

Besides the above, the model along with the devised code can be of immense assistance for deciding maintenance related issues like which pipes are to be replaced, the minimum number of pipes to be replaced for maximum reduction in uncertainty, etc. It can be used even to figure out the most economic way for maximum reduction in uncertainty, which can be resulted by changing pipes, increasing pumping pressure, installation of new pumps, etc. Besides reliability studies, it can assist in cost optimization through uncertainty reduction.

The methodology can be used to identify the propagation of uncertainty in a water distribution model. Although, it involves the simulation of all the combinations between the different values that each of the uncertain parameters of the model can assume. Real circumstances can deal with models containing thousands of pipes and nodes, this implies that the number of simulations to be performed would be so huge that it could become impossible to perform. To avoid this drawback, the number of simulations to be conducted must be considerably reduced and this can only be achieved by assuming that uncertainty relies only on a few of the model parameters (the remaining parameters are assumed to be deterministic) and these are restricted to a small set of values. The trapezoidal membership functions can be used in future to address uncertainty in input parameters, which will simultaneously model type A and type B uncertainties. The methodology can be extended in future by including the uncertainties associated with water quality parameters [11]. The values of pipe wall chlorine decay and the bulk water chlorine decay may be considered as uncertain parameters, which can be modeled intrinsically within EPANET. The propagation of uncertainty induced by wall and bulk decay parameters on chlorine decay and residuals can also be traced.

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