

Determination of head loss progress in dual-media BOPS-sand filter using numerical modeling incorporated with matrix approach

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ABSTRACT

Rapid filtration is a well-known method in treating raw water for municipal water supplies. Burned oil palm shell (BOPS) was considered as a new potential filter media produced from oil palm shell. The filtration unit constructed consists of a mono-media filter of BOPS and sand as well as a dual-media filter of BOPS-sand. Effective size (ES) of BOPS media was specified at 0.6, 0.8 and 1.0 mm, whilst sand was 0.5 mm. Filtration process was run by passing through the synthetic raw water at two different flow rates of 3.62 m/h and 5.81 m/h. The effluent water quality in terms of total suspended solids removal is dependent on both filtration flow rate and effective sizes of granular media. The higher the flow rate and the higher the effective sizes of granular media produced, the lower the total suspended solids removal. The dual-media filter with ES of 0.8–1.0 mm for BOPS and 0.5 mm for sand at flow rates between 3.6–5.8 m/h was found to be the most appropriate combinations in resulting a high quality effluent, longer operation time as well as a balance head loss pattern for both BOPS and sand layers.

Keywords: Head loss; Dual-media filter; Matrix approach; BOPS; Sand

1. Introduction

Filtration process is a common method used for removal of particulate matter in water and wastewater treatment. Various types of filter media has been found effective in removing solid particles of a wide range of sizes up to 50 μm that readily exist in water [1]. Generally, surface waters contain microorganisms like algae, viruses, pathogens, sediment, clay, colloidal humid compound, and some other organic and inorganic particulate matter

[2]. Nowadays, most surface waters will undergo filtration process prior to municipal distribution.

Filtration happens when the water flow through a bed of filter media, at either low or fast speed. This study is focused on rapid filtration which had its origin in United States during the 1880s. Elements for modern design such as mechanical and hydraulic system to assist with cleaning media during backwashing were discovered during the decade. Currently, due to rapid development that leads to high turbid water and high water demand, a conventional mono-media sand filter is not sufficient enough in producing high filtration capacity. Therefore, a

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selection of a dual-media filter is found to be significant. Burned oil palm shell (BOPS) is one of alternative filter media with a great potential and effectiveness used in the dual-media BOPS-sand filter. Dual-media filters thus have the advantage of filtration efficiency and effectively utilizing pore space of grained media.

During filtration, the progress in deposition of suspended particles within a filter increases with time and alters the structure of the filter media as well as the nature of the surface interactions between particles and the collecting media. It creates a complex geometry of pores changes as deposition builds up. Moreover, the attachment process is dependent upon the interaction forces condition, contributed by the charge of the particles, filter grains and ionic chemicals being used in the influent. Therefore, the collection efficiency and the permeability of the media change with time [3].

There are several trajectory models developed to simulate the performance of deep-bed filter and evaluated using colloid filtration theory (CFT) such as YAO model by Yao et al. [4], RT model by Rajagopalan and Tien [5] and TE model by Tufenkji and Elimelech [6]. The YAO model considers three major mechanisms of diffusion, interception and sedimentation however he did not consider hydrodynamic interaction and Van der Waals attractive forces. In addition, RT model also considered London attractive forces in the mechanism of interception. This study applies the trajectory model approach developed by Tufenkji and Elimelech [6]. TE model was developed based on sphere-in-cell model. It encloses hydrodynamic interactions and universal van der Waals attractive forces in diffusion and sedimentation mechanism. Therefore, TE model was selected as compared to RT and YAO models.

The transport of solid particles from the water streamline to a filter media is typically dominated by three major mechanisms which are diffusion, interception and sedimentation. Smaller particles ($<1 \mu\text{m}$) are influenced by Brownian diffusion that will deviate the particles from the fluid streamline. Solid particles pass through the filter media layers by a distance of half the particle diameter or less possibly will get in contact or intercept the grained media. Particles with bigger size and higher density will tend to deviate from fluid streamline due to gravitational forces. Therefore, these three major mechanisms act as the basis of the TE equation [6]:

$$\eta_{\text{total}} = \eta_D + \eta_I + \eta_G = 2.4A_S^{\frac{1}{3}}N_R^{-0.081}N_{Ps}^{-0.715}N_{vdw}^{-0.052} + 0.55A_SN_R^{1.675}N_A^{0.125} + 0.22N_R^{-0.24}N_G^{1.11}N_{vdw}^{-0.053} \quad (1)$$

where η_D , η_I and η_G are the transport efficiencies due to diffusion, interception and sedimentation respectively. The detail description of each function in the equation is given in the Appendix.

The main goal of this particular study is to determine the effect of flow rate and different effective sizes of

BOPS filter used on the operational head loss progress in the filtration system. Data analysis using a new matrix approach incorporated with numerical modeling in simulating filtration efficiency, effluent concentration, and specific deposit. In addition, the results obtained will be further analyzed in determining operational head loss progress in each sub-layer of mono and dual-media filter involved. The specific objectives are to: (1) study the relationship between effective size of filter media, filtration flow rate, operational time, accumulation of deposited sediment and the progress of operational head loss; (2) determine the performance of matrix approach application on estimating operational head loss; and (3) determine the effectiveness of dual-media BOPS-sand filter in treating raw water.

2. Materials and methodology

2.1. Filter media preparation and characterization

In order to study the performance of dual-media filter of BOPS-sand in treating raw water and its influence on the operational head loss, some procedures were established to obtain the raw materials and the preparation of grained media as well as synthetic raw water prior to filtration process. Some physical properties such as morphology, density, porosity, specific deposit, effective size, and uniformity coefficient of BOPS have been studied and the same manner done with the sand as an existing commercial filter media. As a new filter media, the appropriateness in terms of effective sizes, uniformity coefficient and the best combination of BOPS and sand sizes in dual-media filter were primarily very important to be established before a design protocol or guideline could be recommended [7,8]. Filtration process was carried out using three different types of filters, namely, mono-media sand filter, mono-media BOPS filter as well as dual-media BOPS-sand filter and each filter was done in triplicates to enhance accuracy. The methodology consists of three major parts which are preparation of filter media as well as raw water, set up of a filtration system and analysis of data using numerical modelling incorporated with matrix approach.

Burned oil palm shell (BOPS) was manufactured from oil palm fruit shell which presents a by-product from local oil palm factory and abundantly available in Malaysia. The palm shell was burned in a furnace at about 300°C and then grounded into granules prior to sieve analysis to obtain the desired particles size. BOPS media are porous with low specific gravity of 1.30. The shape of BOPS granules prepared by crushing is angular with sphericity of 0.7 and initial porosity of about 0.49. The effective size (ES) and uniformity coefficient (UC) of BOPS chosen was 0.6, 0.8 and 1.0 mm and 1.5 respectively. Otherwise, sand media which is readily available in the local market was also sieved to obtain the preferable size

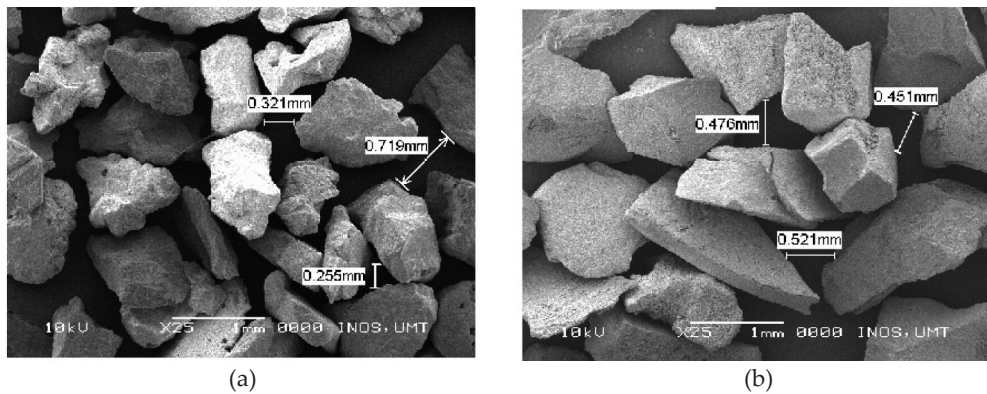


Fig. 1. The morphology of (a) sand and (b) BOPS media.

whose effective size is 0.5 mm and uniformity coefficient of 1.5 in the same manner as the BOPS media. The initial porosity of sand was 0.4 which is slightly less than BOPS. Fig. 1 shows the differences in morphology between sand and BOPS particles. It shows the shape, size and porosity of both particles. The shape affects how grains pack together in a bed. In addition, the shape also affects the size determination by sieve analysis. The sieve opening corresponds to the diameter of spheres media. Therefore, the sieve opening theoretically corresponds to the largest dimension of the smallest particle cross section through a sieve lengthwise for non-spherical media [7].

2.2. Filtration system

The filtration process was run using filter columns equipped with manometer tubes to monitor and record the head losses at various time intervals and depths of each filter media as indicated by Fig. 2. Synthetic raw water was prepared by adding kaolin clay powder into tap water at specified concentration of about 70–80 mg/L as total suspended solids (TSS). Moreover, TSS concentration was also observed at two different filtration flow rates of 3.62 m/h and 5.81 m/h to analyze the performance of mono-media BOPS, and sand as well as dual-media filter of BOPS-sand. The flow rates chosen were within the typical range of rapid filtration rate of 3–7.5 m/h.

2.3. Operational head loss

All raw data were obtained via simulation and experiment which will be compared to each other before it can be accepted as final results. Single collector efficiency (SCE) for sub-layers of both mono and dual-media filters was predicted using TE equation, Eq. (1). The SCE obtained was then used to simulate the progress of specific deposit by applying the matrix approach processes which will be shown in details in results and discussions. Finally, the determination of head loss was simulated by further analysis of specific deposit data. Operational head loss in filtration is directly proportional to the specific sedi-

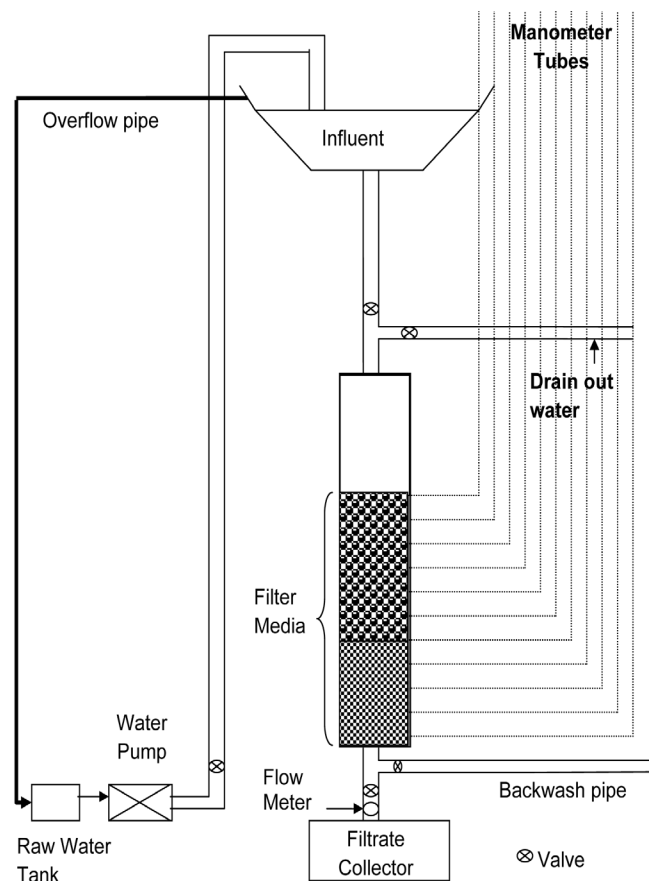


Fig. 2. A schematic diagram of a filtration system.

ment deposit in a particular sub-layer of the filter unit. Therefore, operational head loss changed linearly with running time begins with the initial head loss. This theory was also supported by previous studies done by Boller et al. [9] as in the following equation:

$$\frac{h_{Lt}}{h_{LO}} = 1 + k_{HL} \sigma_t \tag{2}$$

The progress of operational head loss was also predicted to deviate polynomially as revealed by Bai and Tien [10]. When the volume ratio of sediment deposit is less than 10%, the value of σ_3 and the following will be smaller and can be ignored. The relationship of operational head loss in polynomial method can be shown as below:

$$\frac{h_{Lt}}{h_{Lo}} = 1 + k_1\sigma_1 + k_2\sigma_1^2 + \dots \quad (3)$$

All details of Eqs. (2) and (3) are described in the Appendix at the end of this paper.

3. Results and discussion

3.1. Filtration efficiency and specific deposit

In order to determine the performance of BOPS and sand media in a dual-media BOPS-sand filter, the most important parameters such as filtration efficiency, coefficient, effluent concentration, and suspended solids removal $(C_o - C_e)/C_o$, were analyzed. According to Yao et al. [4], the theory on the accumulation of suspended particles on a single filter grain collector was incorporated with mass balance of the flux flow through a filter unit. The TE model which was developed by Tufenkji and Elimelech is similar to Yao model, where the total filtration efficiency includes three major filtration processes, i.e. diffusion, interception and sedimentation as shown in Eq. (1). Several assumptions have been made prior to solve the TE model. The average size and density of kaolin clay particles were assumed to be constant at 10 μm and 2200 kg/m^3 , respectively [11]. Due to kaolin clay particles and the outer layer of filter media both having negative charges, the possibility of clay particles to adhere to the filter media or so-called "attachment efficiency" or α , might be quite low (repulsive mode). Therefore, α was assumed to be in the range of 0.1–0.2 (or 10–20%) [12].

Based on mono-media filters analysis in Table 1, mono-media sand filter produces high suspended solid removal, $[(C_o - C_e)/C_o]$. However it was found to have a very low total capacity of specific deposit (v/v) , and this phenomenon is due to the physical properties of the sand media which have smaller void space and lower porosity in nature. Hence, it leads the mono-media sand filter to get clogged very rapidly. Nevertheless, the main advantage of the mono-media sand filter over the BOPS filter is the ability to produce a very high water quality effluent in terms of total suspended solids and turbidity instead of having a very short filtration operation time of just 5 h.

Even though the mono-media BOPS filter creates such low filtration efficiency, it has the ability to produce a very high capacity of specific deposit (v/v) and has as a result a very long filtration operation time. This phenomenon is contributed to the characteristics of BOPS that has bigger void space as well as higher porosity. Therefore, the main disadvantage of mono-media BOPS filter is

Table 1

Simulation of suspended solids removal and specific deposit at final operation time for mono and dual-media filters

Filter media	Flow rate (m/h)	TSS removal $(C_o - C_e)/C_o$	Specific deposit, (v/v)
Sand	3.62	0.787	0.013
(ES = 0.5 mm)	5.81	0.628	0.006
BOPS	3.62	0.399	0.092
(ES = 0.6 mm)	5.81	0.268	0.135
BOPS	3.62	0.396	0.190
(ES = 0.8 mm)	5.81	0.211	0.220
BOPS	3.62	0.350	0.270
(ES = 1.0 mm)	5.81	0.179	0.360
BOPS:Sand	3.62	0.574	0.047
(ES = 0.6:0.5 mm)	5.81	0.423	0.057
BOPS:Sand	3.62	0.510	0.071
(ES = 0.8:0.5 mm)	5.81	0.372	0.076
BOPS:Sand	3.62	0.448	0.094
(ES = 1.0:0.5 mm)	5.81	0.329	0.128

production of a rather low water quality effluent in terms of total suspended solids and turbidity. The study of both mono-media sand and BOPS filters was also supported by Jusoh et al. [3].

The main functions of developing a dual-media filter were to combine the advantages of sand media with the high water quality effluent and BOPS media with the high capacity of specific deposit reservoir and long period of total filtration operation time. The performance of dual-media BOPS-sand filter with effective sizes of 0.6, 0.8 and 1.0 mm for BOPS and 0.5 mm for sand media was determined and compared its relationship in terms of suspended solids removal and the quantity of specific deposit as illustrated in Table 1. Based on Table 1, suspended solids removal was found to be inversely proportional to the increasing of BOPS' effective sizes. Nevertheless, bigger effective size of BOPS used will provide higher capacity of specific deposit content to the filter and furthermore produce longer filtration operation time [13,14].

According to Table 1, at the same effective size of BOPS used, the suspended solids removal was also influenced by the variation of flow rate. The lower the filtration flow rate applied, the greater the reduction of suspended solids removal. This is because the lower flow rate contributed to the higher retention time or opportunity time of filtration.

3.2. Initial and operational head loss

The pressure of water flows within a designed filter depth increase with the increase of filter depth. According to Bai and Tien [15] and Stevenson [16], the head loss increases linearly with the filter depth. From this study, it was found that the initial head loss which was monitored

at time 0 h is directly proportional to the effective size of media. For example, the mono-media sand filter with a smaller effective size ($ES = 0.5$ mm) and lower porosity ($\epsilon = 0.4$) produced a high initial head loss, whereas the mono-media BOPS filter with a larger effective size ($ES = 0.6, 0.8$ and 1.0 mm) and higher porosity ($\epsilon = 0.49$) produced lower initial head loss. Furthermore, for the mono-media BOPS filter, as the BOPS effective size is increased, the initial head loss is decreased. Therefore, combination of both sand and BOPS media in a dual-media BOPS-sand filter produced a moderate initial head loss as can be seen in Table 2.

The total filtration operation time is one of the key factors in assessing a filter performance [17]. Fig. 3 shows the progression of head loss against the operation time of mono-media sand and BOPS filters as well as dual-media BOPS-sand filters. All the results exhibited

the same trend where the head loss increased with the filtration operation time. However, a mono-media sand filter ($ES = 0.5$ mm) has a very short operation time of about 5 h and the highest filtration increasing rate, K_L value of 126.37 mm/h while the mono-media BOPS filter has resulted in a very long total filtration operation time of 93 h and a lower K_L value of 6.58 mm/h. However, a combination of BOPS and sand media in a dual-media BOPS-sand filter still produced high water quality effluent and a relatively long total operation time of about 70 h and a low K_L value of 8.87 mm/h. Therefore, the dual-media BOPS-sand filters produce both high water quality effluent and a long total filtration operation time with is indicated by a lower K_L value.

3.3. Relationship of progressing specific deposit and head loss profile

Figs. 4, 5 and 6 show the profiles of head loss development in relation to the progress of specific deposit for dual-media BOPS-sand filters using BOPS effective sizes of 0.6, 0.8 and 1.0 mm at filtration flow rates of 3.62 and 5.81 m/h, respectively. In a dual-media BOPS-sand filter, the increasing of effective size of BOPS used leads to lower down the percentage of specific deposit in BOPS layer and at the same time raised up the percentage of specific deposit in sand layer. As the effective size of BOPS increased, suspended solids becomes easier to penetrate the BOPS layer and consequently, more suspended solids are entrapped in the sand layer. For example, at a specified velocity of 3.6 m/h and BOPS effective size of 0.6 mm used, the percentage of specific deposit in BOPS layer is rather high — 74%, while in sand layer it is just 26% as depicted in Fig. 4a. However, when larger effective sizes of BOPS were utilized such as 0.8 and 1.0 mm as indicated in Figs. 5a and 6a, the percentage of specific deposit in BOPS layer reduced to 61% and 47% while the percentage of specific deposit in the sand layer increased to 39% and 53%, respectively.

At low filtration velocity, the percentage of specific deposit indicates a larger quantity in the BOPS layer as compared to the higher velocity. On the other hand, percentage of specific deposit in the sand layer increased as the flow rate increased. This is due to the effect of high filtration velocity which will carry the suspended solids deeper through the BOPS media and therefore, the specific deposit in BOPS layer is decreased but in sand layer it is sufficiently increased.

Generally, in terms of specific deposit capacity in the dual-media BOPS-sand filter, the higher the BOPS effective size, the higher the accumulation of specific deposit. For instance, the capacity of specific deposit increased from 0.06 to 0.13 v/v for the dual-media BOPS-sand filters with BOPS effective sizes of 0.6–1.0 mm, respectively at the same filtration velocity of 5.8 m/h (Figs. 4–6).

Table 2

Initial head loss and head loss coefficient, K_L for mono and dual-media filters at filtration velocity of 5.8 m/h

Filter media	Initial head loss (mm)	Head loss coefficient, K_L , linear (mm/h)
Sand (ES = 0.5 mm)	33.13	126.37
BOPS (ES = 0.6 mm)	13.18	19.48
BOPS (ES = 0.8 mm)	8.26	10.28
BOPS (ES = 1.0 mm)	4.67	6.58
BOPS:Sand (ES = 0.6:0.5 mm)	15.17	21.38
BOPS:Sand (ES = 0.8:0.5 mm)	10.42	12.70
BOPS:Sand (ES = 1.0:0.5 mm)	6.15	8.87

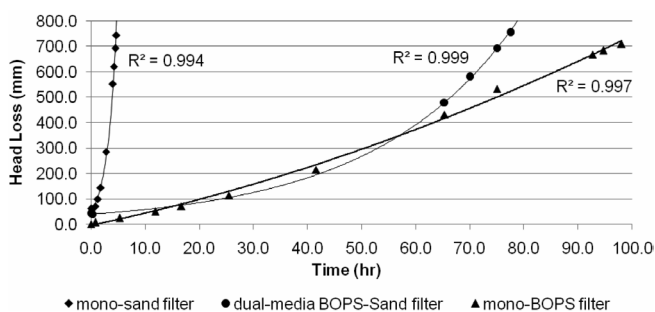


Fig. 3. Relationship between experimental head loss and operation time at $V = 5.81$ m/h, for mono-media sand filter with $ES = 0.5$ mm, mono-media BOPS filter with $ES = 1.0$ mm, and dual-media BOPS-sand filter with $ES = 1.0:0.5$ mm.

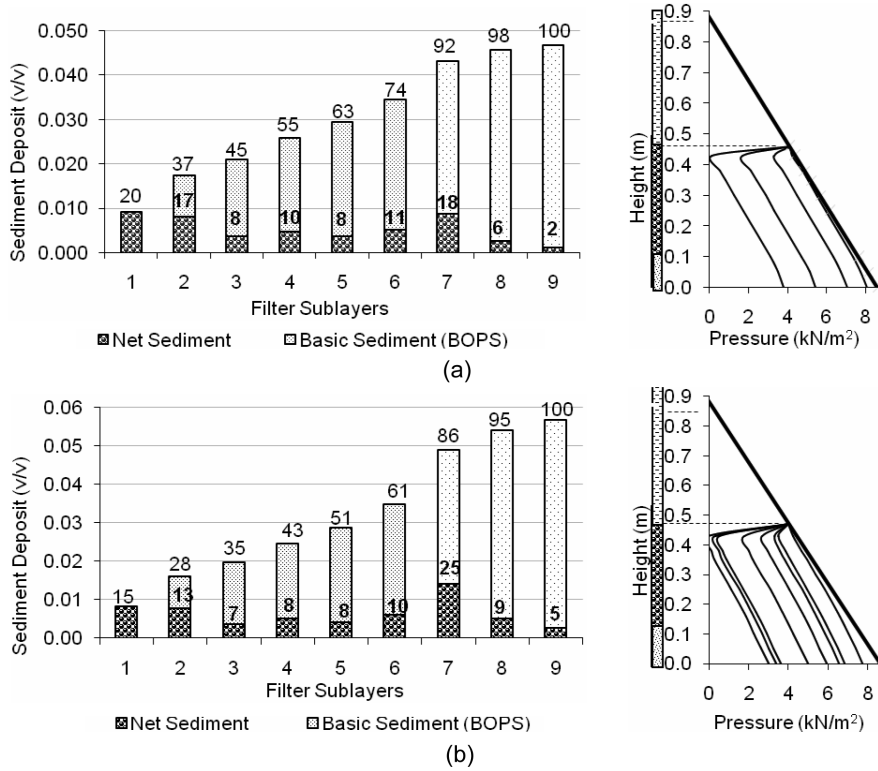


Fig. 4. Progress of specific deposit and head loss in a dual-media BOPS-sand filter with effective size of 0.6 : 0.5 mm; (a) at filtration velocity of 3.62 m/h and total operation time of 50 h; (b) at filtration velocity of 5.81 m/h and total operation time of 44 h.

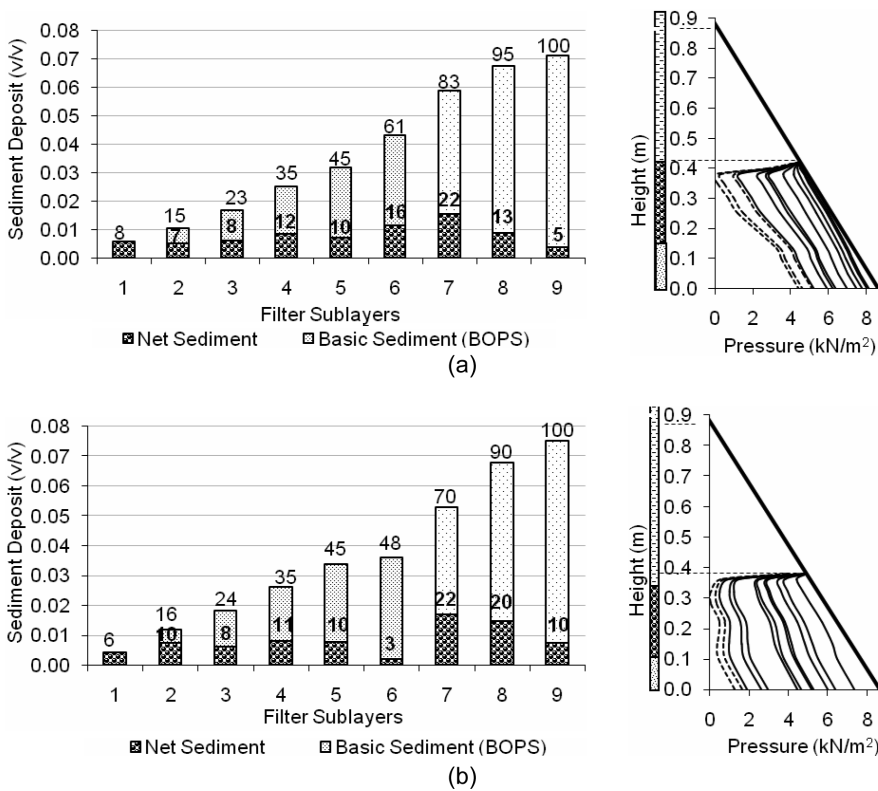


Fig. 5. Progress of specific deposit and head loss in a dual-media BOPS-sand filter with effective size of 0.8 : 0.5 mm: (a) at filtration velocity of 3.62 m/h and total operation time of 62 h; (b) at filtration velocity of 5.81 m/h and total operation time of 51 h.

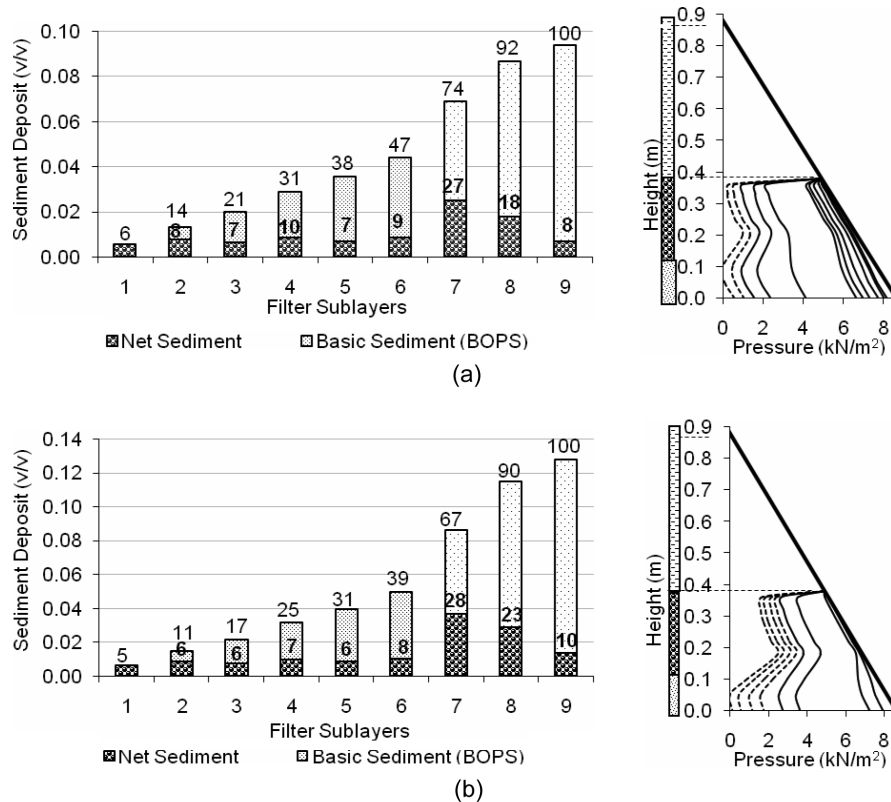


Fig. 6. Progress of specific deposit and head loss in a dual-media BOPS-sand filter with effective size of 1.0 : 0.5 mm: (a) at filtration velocity of 3.62 m/h and total operation time of 91 h; (b) at filtration velocity of 5.81 m/h and total operation time of 82 h.

Operational head loss depends on the progress of specific deposit and can be determined using Eqs. (2) and (3). The operational head loss of a mono-media sand filter increased very rapidly from the beginning to the end stage of operation time at a constant flow rate used [18]. As can be seen in Fig. 3, the mono-media sand reached the maximum level of head loss only within 5 h whereas the mono-media BOPS filter was able to produce such a long total filtration operation time before it reached the maximum level of head loss.

Figs. 4–6 also show the effect of effective size of BOPS and filtration velocity on the head loss profile in dual-media BOPS-sand filters. As indicated by Fig. 4, when the effective size of BOPS used was almost the same as sand which was 0.6:0.5 mm, the peak of the head loss curves occurred at the top of a few centimeters layer of BOPS media which means that the majority of the specific deposit accumulation occurred in the BOPS layer. Both head loss profiles, as shown in Figs. 4a and b, show a similar profile pattern at two different flow rates of 3.62 and 5.81 m/h.

According to Fig. 5a, with the effective size of 0.8:0.5 mm, at a low filtration velocity of 3.62 m/h, it shows the head loss profile which indicates a similar

pattern as Fig. 4. This is due to the low filtration velocity resulted in a high accumulation of specific deposit in the BOPS layer. However, in Fig. 5b with a higher filtration velocity of 5.81 m/h, there is a second curvature at the bottom layer due to the specific deposit which is able to penetrate deeper through the BOPS layer and is significantly entrapped in the sand layer. This phenomenon is due to the high flow rate of 5.8 m/h which allows the solid particles to penetrate deeper down the filter column and mostly entrapped in the sand layer. The peak of the head loss curvature is also indicated as the most active zone where the filter experiences heavy clogging in the corresponding layer.

The most apparent curvature of the head loss profile is illustrated by Fig. 6. Specifically, in Fig. 6a, at a velocity of 3.6 m/h, the head loss profile shows that the peaks of the curvature for both media touch the vertical line at the same time. It shows that the behavior of the head loss at BOPS and sand layers occur simultaneously. However, in Fig. 6b with a higher filtration velocity of 5.81 m/h, the peak of the curvature at the sand layer progressed faster as compared to the BOPS layer or the head loss curvature of sand media touch the vertical line earlier than BOPS. This is due to the higher filtration velocity which

brought the specific deposit to penetrate deeper into the sand layer. Referring to the head loss profile pattern, the combination of a dual-media BOPS-sand filter with the effective size of 1.0:0.5 mm resulted in the most effective filter media that can produce higher filter capacity, longer operation time and high water quality. Generally, the range of filtration velocity between 3.62–5.81 m/h is the most appropriate velocity for the effective size of 1.0:0.5 mm. It is found that sand media was able to capture most solid particles and get clogged earlier than BOPS media at flow rate of 5.81 m/h. Therefore, BOPS media prevented the filter from rapid clogging and hence less frequency of backwash was required.

At the end stage of the filtration period, the filter encountered a heavy media clogging and the breakthrough time for those filters was imminent. However, it is believed that certain layer of those filters had achieved negative pressure which will occur when the peak of curvature exceeds the vertical line and indicate the lifetime limit of the filter in order to begin the backwash process.

4. Conclusion

This study focuses on the progress of specific deposit and operational head losses in each sub-layer of dual-media BOPS-sand filters. Operational head loss progress in a granular media filter at a constant influent suspended solids concentration is strongly dependent on the filtration flow rate, effective size, porosity and type of the granular media used. In all dual-media BOPS-sand filters studied, a larger effective size of BOPS and high filtration velocity used contributed to the deeper penetration of the specific deposit through the BOPS layer and entrapped most into the sand layer underneath. Referring to all the head loss profile patterns, the combination of dual-media BOPS-sand filter with the effective size of 1.0:0.5 mm at the range of filtration velocity of 3.6–5.8 m/h resulted in the most effective filter media that can produce higher filter capacity, longer operation time and higher water quality effluent. Moreover, the “black box” analyses that incorporate matrix approach in simulating specific deposit and head loss development in filter system was successfully used to evaluate the performance of the most appropriate effective size and filtration velocity for dual-media BOPS-sand filter.

Symbols

A	— Hamaker constant
A_s	— Porosity function
C	— Concentration of suspended solids, mg/L
C_e	— Effluent concentration of suspended solids, mg/L
C_o	— Influent concentration of suspended solids, mg/L

d_c	— Average diameter of grain media, m
d_p	— Average diameter of kaolin clay particles, m
h_L	— Head loss, m
k_{HL}	— Filtration constant
kT	— Boltzman constant, 1.381×10^{-23} J/K
L	— Depth of filter, m
N_A	— Attraction number
N_G	— Gravity number
N_{Pe}	— Peclet number
N_R	— Aspect ratio
N_{vdw}	— Van der Waals number
t	— Time, s
v	— Superficial filtration velocity

Greek

α	— Attachment coefficient
δ_c	— Derivative of concentration
δ_t	— Derivative of time
δ_z	— Derivative of media thickness
$\delta\sigma$	— Derivative of specific deposit
$\varepsilon, \varepsilon_o$	— Porosity and initial porosity of media
γ	— Porosity function
η_{total}	— Total transport efficiency
η_D	— Transport efficiency due to diffusion
η_G	— Transport efficiency due to gravity
η_I	— Transport efficiency due to interception
λ	— Filter coefficient
μ	— Dynamic viscosity of water
ρ_p	— Density of suspended particles, kg/m^3
ρ_w	— Density of water, kg/m^3

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Appendix

$$A_s = \frac{2(1-\gamma^2)}{(2-3\gamma+3\gamma^2-2\gamma^2)} \quad (4)$$

where

$$\gamma = (1-\varepsilon)^{1/3} \quad (5)$$

$$N_R = \frac{d_p}{d_c} \quad (6)$$

$$N_{ps} = \frac{vd_c}{\left(\frac{4T}{3\pi d_p \mu}\right)} \quad (7)$$

$$N_{vdw} = \frac{A}{kT} \quad (8)$$

where

$$A = 1.6 \times 10^{-21} \quad (9)$$

$$N_G = \frac{g(\rho_p - \rho_w)d_p^2}{18\mu v} \quad (10)$$

$$\frac{\delta C}{\delta z} = \frac{-3(1-\varepsilon)\eta\alpha C}{2dc} \quad (11)$$

$$\lambda = \frac{3(1-\varepsilon)\eta\alpha}{2d_c} \quad (12)$$

$$C_e = C_0 \exp\left[\frac{-3(1-\varepsilon)\eta\alpha L}{2d_c}\right] \quad (13)$$

The accumulation of deposited sediment in the first sub-layer, second sub-layer until the last sub-layer at time t_2 is illustrated in the following Process One:

$$\sigma_{21} = \sigma_{11} + \Delta\sigma_{T2L1}$$

$$\sigma_{22} = \sigma_{21} + \Delta\sigma_{T2L2}$$

$$\sigma_{23} = \sigma_{22} + \Delta\sigma_{T2L3}$$

$$\sigma_{24} = \sigma_{23} + \Delta\sigma_{T2L4}$$

$$\sigma_{2n} = \sigma_{2n} + \Delta\sigma_{T2Ln}$$

The accumulation of deposited sediment in all sub-layers and at time t_3 is described in Process Two below:

$$\sigma_{31} = \sigma_{21} + \Delta\sigma_{T3L1}$$

$$\sigma_{32} = \sigma_{22} + \Delta\sigma_{T3L1} + \Delta\sigma_{T3L2}$$

$$\sigma_{33} = \sigma_{23} + \Delta\sigma_{T3L1} + \Delta\sigma_{T3L2} + \Delta\sigma_{T3L3}$$

$$\sigma_{34} = \sigma_{24} + \Delta\sigma_{T3L1} + \Delta\sigma_{T3L2} + \Delta\sigma_{T3L3} + \Delta\sigma_{T3L4}$$

$$\sigma_{3n} = \sigma_{2n} + \Delta\sigma_{T3L1} + \Delta\sigma_{T3L2} + \Delta\sigma_{T3L3} + \Delta\sigma_{T3L4} + \Delta\sigma_{T3Ln}$$