Desalination and Water Treatment
www.deswater.com

34 (2011) 295–303
October

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doi: 10/5004/dwt.2011.2858

Design and tuning of feedback controllers: effects on proteins ultrafiltration process modeled by a hybrid system

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Received 3 September 2010; Accepted 3 January 2011

ABSTRACT

The focus of the paper was the description of a feedback control system that, based on the predictions of a previously formulated hybrid neural model, allowed improving the performance of proteins UF, carried out in pulsating conditions. The behavior of three classical feedback controllers, i.e., proportional (P), proportional integral (PI) and proportional-integral-derivative (PID), were compared and analyzed in different situations. The characteristic equation defining each type of controller was added to the already developed hybrid model in order to obtain the true closed-loop responses, thus allowing achieving a proper design and an accurate tuning of the controllers. It was observed that when no control action occurred, the permeate flux tended to progressively decay and that a proportional control was capable to reduce this decay only to a limited extent. The differences between the actual permeate flux and the desired set-point tended, instead, to nil when a properly tuned PI or PID controller was utilized. The selection of the best values for the adjustable parameters of the controller was eventually attained by a time-integral performance criterion, i.e., the minimization of the integral of the time-weighted absolute error (ITAE), using, as a starting point, the values provided by the application of the Ziegler-Nichols tuning method.

Keywords: Hybrid neural modeling; Membrane; Permeate flux decay; PID; ITAE; Ziegler-Nichols

1. Introduction

Pulsating conditions represented a simple and rather effective method aimed at reducing the permeate flux decay occurring during membrane filtration [1,2]. In a previous paper [3], a hybrid system modeling the behavior of BSA ultrafiltration in pulsating conditions was presented. The developed hybrid neural model consisted of two different parts: a classic theoretical model and a very simple artificial neural network, together concurring, in an iterative scheme, at the obtainment of the time evolution of permeate flux decay, $J_p(t)$, as a function of the system inputs. The theoretical model was characterized by the solution of momentum, continuity and convection-diffusion equations in the module channel, coupled to the Brinkman and the continuity equation written to determine the velocity field within the membrane, which was assumed to completely reject BSA. The neural model, consisting of two end-layers with one neuron each and one hidden layer with five neurons, was used to determine the complex functional relationship existing between the BSA concentration adsorbed on the membrane surface, $C_a(t)$, and the additional resistance due to membrane fouling, $R_{ad}(t)$. A schematic representation of the hybrid neural model is shown in Fig. 1. Five different inputs were identified: the transmembrane pressure (TMP), the current value of feed flow rate (Q), the feed concentration of BSA solutions

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Presented at the AMS6/IMSTEC10, The 6th conference of the Aseanian Membrane Society in conjunction with the 7th International Membrane Science and Technology Conference, Sydney Australia, November 22–26, 2010

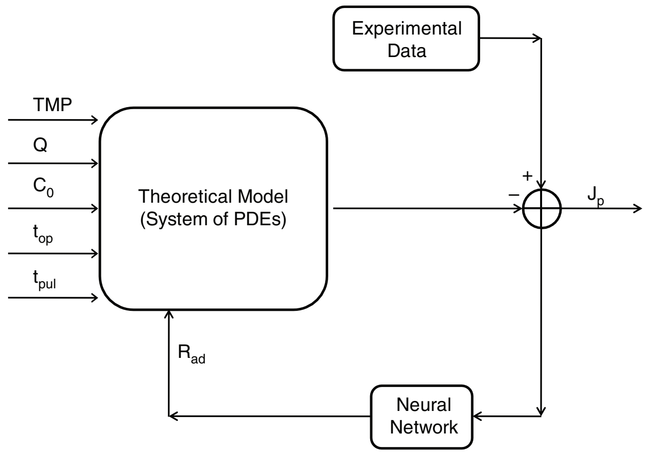

Fig. 1. Block diagram of the formulated hybrid model (TMP: trans-membrane pressure; Q: feed flow rate; C_0: feed concentration of BSA solutions; t_{pul}: pulse time; t_{op}: operating time; $J_p(t)$: permeate flux decay).

and, finally, the duration of each square wave upstream pressure release, i.e., the pulse time (t_{pul}) and the duration of upstream operating pressure restoration, i.e., the operating time (t_{op}) during which UF was actually performed. The permeate flux decay represented the sole output of the system. The main advantage of the developed hybrid model actually regarded the possibility of describing some well-assessed phenomena by means of a theoretical approach, leaving the analysis of other aspects, very difficult to interpret in a fundamental way, to a rather simple cause-effect model. The developed model, therefore, incorporates both an *a* priori knowledge of the UF process and an adaptive neural network aimed at identifying the difficult-to-model (uncertain) part of the UF dynamics. The proposed hybrid system gave very accurate predictions of the behavior of UF modules operating in pulsating conditions and demonstrated a more efficient alternative to both fundamental and pure neural network (NN) modeling. As compared to NN, the developed hybrid model exhibited much better extrapolation properties; the validity of its predictions, in fact, could be extended well outside the range of the training data.

Generally speaking, a control system is called on to satisfy three general classes of needs, e.g., suppressing the influence of external disturbances, ensuring the stability of the process, optimizing the performance of the process. Two of the major issues regarding feedback controllers were their design, i.e., what kind of controller should be used to control a given process, and their tuning, i.e., how to select the best values for the adjustable parameters of the controller. Among feedback controllers, both Proportional-Integral (PI) and Proportional-Integral-Derivative (PID) controllers were extensively used in process industries. The use of feedback controllers is rather common in membrane technology. Feedback controllers were proposed to decide either when to perform the membrane backwash [4] or the duration of backwash process [5]. Both proportional–integral (PI) and proportional–integral–derivative (PID) control algorithms were adopted in water desalination to regulate the process flow rates and to adjust the operating pressure so as to achieve a desired rate of clean water production [6]. A feedback controller was proposed to determine the dosing strategy of a coagulant in an UF process so as to promote a reduction of hydraulic resistance of the fouling layer deposited on the membrane surface. It was found that the developed control system performed well, exhibiting a significant reduction of coagulant consumption as compared to the traditional dosing strategies [7]. A feedback control system was integrated with a SEC-based energy optimization algorithm in order to maintain RO system operation at energy-optimal conditions. The developed controller made use of multiple system variables and of a user defined permeate production rate. The optimal operating set-points were calculated so as to minimize the specific energy consumption of a reverse osmosis desalination system, thus satisfying the process and control system constraints [8]. A robust PID controller was developed for a proton exchange membrane fuel cell (PEMFC) system. The authors modeled the PEMFC as a multivariable system and applied the identification techniques to obtain the system's transfer function matrices, whereas both the system variations and the disturbances were regarded as uncertainties. Based on the evaluation of stability, performance, and efficiencies, the proposed robust PID controllers showed to be effective [9]. The tuning of a feedback controller can be achieved in several ways, depending on the dynamics and on the main features of the system to be controlled, and various methods were developed and proposed in the last decades. Many researchers provided the settings of PI and PID controller for various process models and different performance criteria. Ziegler and Nichols [10] proposed a classical method to tune feedback controllers; the method does actually represent a rather good starting point that is still used as a preliminary design by many. Another alternative to preliminary design of feedback controllers was that proposed by Cohen and Coon [11]. These well-known tuning relationships were developed to provide closed-loop responses with one-quarter-decay ratio as the performance criteria. Lopez, Murrill, and Smith [12] provided the PID tuning relationships using a minimum error integral performance criterion. Other tuning techniques based on achieving a desired closed-loop response were also proposed [13,14]. Another popular approach was the tuning of PI or PID controllers by the specification of gain margin and phase margin, both representing a significant measure of robustness [15]. In particular, the phase margin, being related to the damping of the system, was used as a performance measure [16]. Most of the proposed tuning techniques, however, were based on a strong simplification hypothesis, i.e., first-order dynamics plus time delay approximation of the actual process dynamics, and, therefore, were not capable to ensure the best control performance. Many processes of physical significance, in fact, exhibit significant non-linear dynamics. If these processes deviate only slightly from steady-state conditions, the effects of nonlinearities might not be severe and traditional control schemes provide satisfactory control performance. On the contrary, if the processes are required to work over a wide range of operating conditions, conventional linear control approaches are not capable to handle the system non-linearities. In these conditions, a mathematical model providing reliable predictions of the actual system behavior is essential for the implementation of high-performance control systems [17]. The presence of uncertainties and parameters change, in fact, can determine a mismatch between the formulated model and the true process. This degrades the control performance and may lead to serious stability problem, especially when the process is non linear. In recent years, advanced tuning methods coupled to optimization algorithms received more consideration in the literature, since they provide better stability and robustness of the controlled system [18]. The utilization of modern optimization techniques and the formulation of reliable models of the actual dynamics of the system under study make possible to tune a feedback controller using the actual transfer function of the plant, thus optimizing the true closed-loop performance [19]. Among the proposed performance criteria, the integral of the time-weighted absolute error (ITAE) was definitely one of the most popular performance index for the design and the tuning of different control systems [20,21].

The present paper was organized as follows:

Starting from the already-described hybrid neural model [3], a feedback control system was developed and exploited with reference to the UF of BSA solutions, carried out in pulsating conditions. It is expected that the reliability of the hybrid model, as provided by the combination of a set of transport equations and of a neural network aimed at estimating the uncertainties and the parameters change typical of UF process, does indeed allow overcoming the actual limitations, as reported in the literature, related to the implementation of proper control systems.

The behavior of three classical feedback controllers, i.e., a proportional (P), a proportional integral (PI) and a proportional-integral derivative (PID), were compared analyzing the closed-loop responses of the controlled systems in three different situations:

1) The set point, i.e., the permeate flux, $J_{p,SP}$, was kept constant throughout the experiment, except when each pulse occurred (Fig. 2). The control system, therefore, was called on to modify the manipulated variable so as to avoid or limit the permeate flux decay;
2) The disturbance, i.e., the feed concentration, C_0, did not change, whereas the set-point was modified according to a specific pattern (servo problem). Two positive steps and $J_{p,SP}$ two negative steps occurred, at definite times, as far as was concerned (Fig. 3);
3) The disturbance underwent a change (Fig. 4) according to a definite pattern (regulator problem), whereas the set-point assumed the same pattern as described in Case 1.

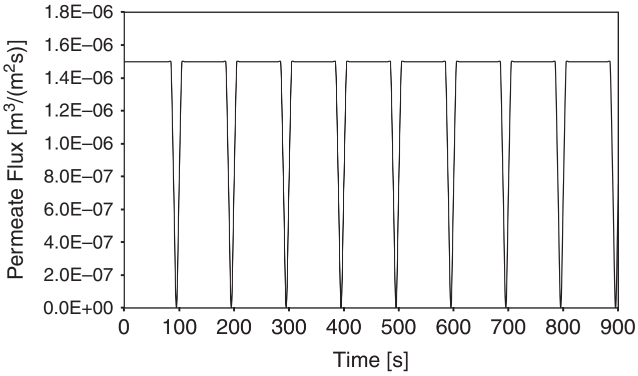

Fig. 2. Case 1 – Uniform Set-point – The set point, $J_{p,SP}$, is kept constant throughout the experiment.

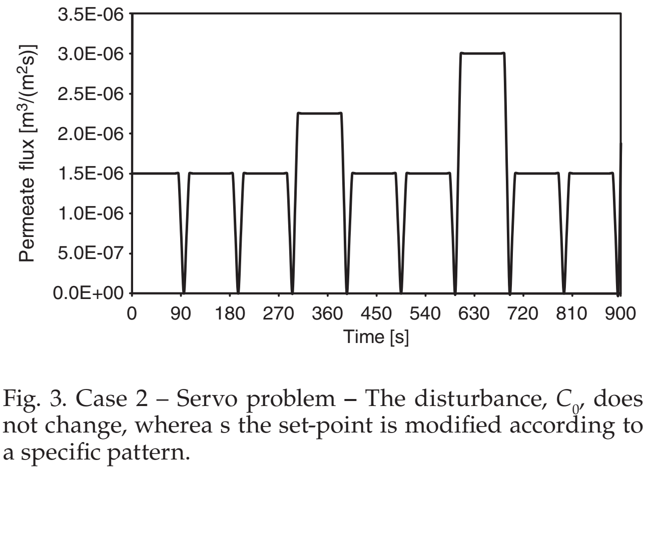

Fig. 3. Case 2 – Servo problem – The disturbance, C_0, does not change, wherea s the set-point is modified according to a specific pattern.

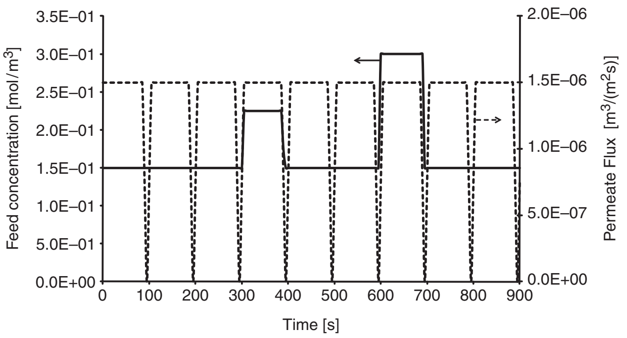

Fig. 4. Case 3 – Regulator problem – Disturbance pattern – The disturbance undergoes a change according to a definite pattern.

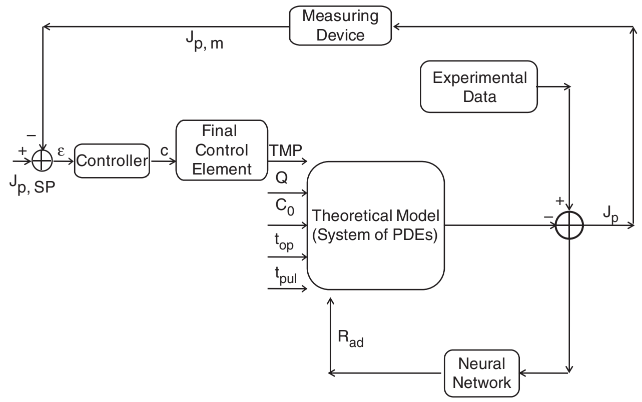

Fig. 5. Block diagram of the proposed feedback control system.

Then, the tuning of both PI and PID controllers was actually achieved by the minimization of the integral of the time-weighted absolute error (ITAE). As a starting point for the selection of the best value for the adjustable parameters of the controller, the well-known Ziegler-Nichols tuning rule was adopted. The methodology proposed in the present paper, although implemented with reference to traditional controllers, definitely represents a novelty in membrane science and technology; in fact, due to the neural part of the hybrid model, it is possible designing robust control systems for a wide spectrum of non-linear processes subject to uncertainties and parameters changes.

2. Feedback controller development

Since the previously developed hybrid neural model described rather well the actual behavior of BSA ultrafiltration in pulsating conditions [3], it seemed worthy to develop a feedback control system that, on the basis of hybrid model predictions, allowed suppressing the influence of external disturbance and optimizing the performance of membrane operations. Among the five inputs of the system, the inlet concentration was considered as the sole disturbance since it influenced the time evolution of adsorbed BSA and – therefore – strongly affected the permeate flux decay. TMP was, instead, chosen as the sole manipulated variable that, properly instructed by the control block, should allow maintaining the permeate flux as much similar as possible to the desired set-point pattern. Feed flow rate (Q), pulse time (t_{pul}) and operating time (t_{op}) were fixed at definite values before performing each simulation. The block diagram corresponding to the developed control system is shown in Fig. 5. The characteristic equations (Eqs. (1–3)) relating the controller output, $c(t)$, to the difference, $\varepsilon(t)$, existing between the set-point value, $J_{p,SP}(t)$, and the actual permeate flux, were added to the developed hybrid model in order to ascertain how the type of controller and its adjustable parameters affected the closed-loop responses of the controlled system:

P controller:

$$c(t) = K_c\ \varepsilon(t) + c_s \tag{1}$$

PI controller:

$$c(t) = K_c\ \varepsilon(t) + \frac{K_c}{\tau_I}\int_0^t \varepsilon(t)\, dt + c_s \tag{2}$$

PID controller:

$$c(t) = K_c\ \varepsilon(t) + \frac{K_c}{\tau_I}\int_0^t \varepsilon(t)\, dt + K_c\ \tau_D\ \frac{d\varepsilon(t)}{dt} + c_s \tag{3}$$

It is worthwhile observing that both the dynamics of the measuring device and that of the final control element were assumed to be negligible. For the latter, in particular, this means that its dynamics is simply described by its static, dimensional gain, a, relating the control signal, $c(t)$, to the manipulated variable, TMP(t), as:

$$TMP\left(t\right) = a \cdot c\left(t\right) \tag{4}$$

Generally speaking, P controller is very simple but the offset tends to zero only as the proportional gain, K_c, tends to infinity, thus leading to stability problems. PI controller is more complicated than P controller but the offset can be eliminated; moreover, as K_c increases, the response becomes faster but more oscillatory to set point changes; finally, the response is faster as the reset time, τ_I, decreases, for constant values of K_c. As far as the PID controller is concerned, its closed-loop response has the same qualitative dynamic characteristics as those resulting from PI control alone; however, the introduction of the derivative mode, accounted for by the parameter τ_D, brings a stabilizing effect to the system.

Appendix 1 reported the transport equations, actually a system of PDEs, used to formulate the hybrid neural model and briefly described the implementation of each control system. The simulations of UF process, controlled by either P or PI or PID controller, were performed by Comsol Multiphysics 3.4 that allowed determining the influence of controller parameters on the system behavior.

3. Feedback controller tuning

Actually, on developing the present feedback control system it was necessary not only to choose the type of controller, but also to select the best values of its adjustable parameters. Controller tuning methods should ensure stable closed-loop responses and the attainment of given objectives. All the methods proposed in the literature for the tuning of a feedback controller were based on the knowledge about the process to be controlled. This knowledge was generally formulated in terms of a transfer function, i.e., a mathematical relationship usually expressed in the Laplace's domain, relating the output(s) of the process to its input(s) [21]. In the case of BSA ultrafiltration performed in pulsating conditions, the knowledge about the complex process dynamics is definitely precise since it was provided by the developed hybrid model, which demonstrated offering very accurate predictions of the time evolution of permeate flux as a function of the significant process inputs.

As a starting point for the selection of the best value for the adjustable parameters of the present control system, the Ziegler-Nichols (Z-N) tuning rule was adopted; then, the parameters were improved by the minimization of the integral of the time-weighted absolute error (ITAE) that allowed achieving the final tuning of the developed feedback controllers.

Unlike other proposed methods, the Z-N tuning technique is a closed-loop procedure and, therefore, fits better in those cases, like the present one, in which the controlled system (Fig. 5) can be simulated over a wide range of both process conditions and controller parameters. According to the rule, a PID controller was tuned by firstly setting it to the P-only mode and with the feedback loop closed; a set point change was introduced and the proportional gain was then varied until the system oscillated continuously. The gain providing the continuous oscillations was referred to as the ultimate gain (K_u) and the oscillation period was termed as the ultimate period (P_u). Starting from the calculated K_u and P_u values the feedback controller parameters were, eventually, determined according to the following Table 1.

Table 1
Settings for feedback controllers as provided by the Z-N tuning rule

Controller	K_c	τ_I	τ_D
P	$K_u/2$	–	–
PI	$K_u/2.2$	$P_u/1.2$	–
PID	$K_u/1.7$	$P_u/2$	$P_u/8$

The actual shape of the complete closed-loop response, from time t = 0 until a definite time, t_{fin}, considered as representative of system dynamics and chosen – in the present case – as equal to 15 min, was used for the formulation of a dynamic performance criterion based on the minimization of the ITAE, defined as [21]:

$$ITAE = \int_0^{t_{fin}} t \left| \varepsilon(t) \right| dt \tag{5}$$

where $\varepsilon(t) = J_{p,SP}(t) - J_p(t)$. With respect to other similar time-integral performance criteria proposed in the literature, the ITAE criterion was chosen since it is capable to suppress errors that might persist for long times. The minimization of ITAE was performed by available numerical algorithms implemented within the exploited simulation software.

4. Results and discussion

4.1. Analysis of the closed-loop responses

The closed-loop responses of the three types of feedback controllers are shown in Fig. 6 and compared to

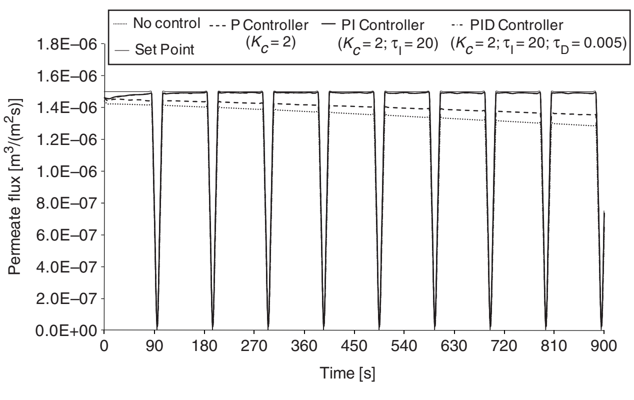

Fig. 6. Closed-loop responses of the three types of feedback controllers and comparison to the uniform set-point pattern (Case 1).

the uniform set-point pattern (Case 1). The input variables were set equal to: $Q = 6.67\ 10^{-5}$ m^3/s, $t_{pul} = 10$ s, $t_{op} = 90$ s and were never changed from now on, even though they represented the actual process inputs. It is worthwhile observing that, with no control action, the permeate flux tends to progressively decay and that a proportional control is capable to limit this decay only to a certain extent, since a significant increasing-with-time $\varepsilon(t)$ can be observed. Any observable difference between the set-point pattern and the actual permeate flux, instead, tends to nil when either a PI or a PID controller is utilized. Actually, PI controller exhibits much better performance as compared to P controller and similar responses, which overlap throughout the considered time horizon, to PID controller. However, PI controller is much simpler to operate and to tune than PID. For this reason, the behavior of PI controller was analyzed in more detail since it gave high-quality responses and required a lower computational effort, as compared to PID, to achieve a proper tuning. The effect of reset time, τ_I, on permeate flux decay is shown in Figs. 7 and 8 in the case of a servo- and of a regulator problem, respectively. In both the situations, it is observed that, for a constant value of $K_c = 2.2$, the control action is faster and more effective when τ_I is decreased from 44 s to 7.33 s. When a lower value of τ_I is chosen, the actual trend of permeate flux decay reproduces rather well each required change on set-point pattern (Fig. 7) and is not actually affected by the imposed step changes on BSA feed concentration (Fig. 8).

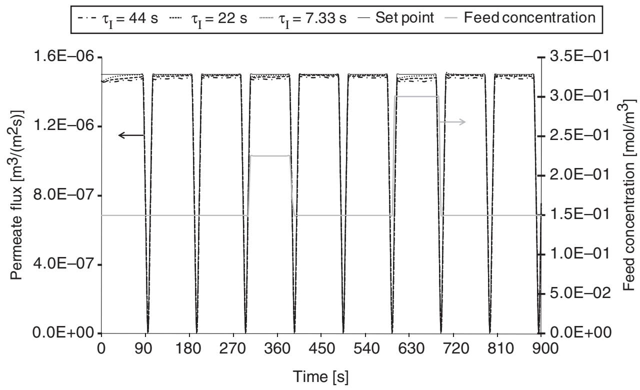

Fig. 8. Effect of the reset time on the performance of PI controller (Regulator problem).

Table 2
Settings for feedback controllers calculated by the Z-N tuning rule

Controller	K_c	τ_I	τ_D	ITAE
PI	2.049	4.17	–	0.041
PID	2.652	2.5	0.625	0.044

4.2. Ziegler-Nichols analysis

The developed control system exhibits good performance in all the tested conditions; however, it is necessary to perform a more rational choice of the controller parameters so as to obtain improved closed-loop responses of the present membrane system. The Ziegler-Nichols tuning technique is initially used. The controlled system shown in Fig. 5 is simulated using a P control only and introducing a set-point change; then, K_c is modified deliberately until a continuous oscillation of permeate flux is observed. The value of the proportional gain providing this oscillation, K_u, is equal to 4.508, whereas the ultimate oscillation period, P_u, is equal to 5 s. According to the Z-N tuning rule, the values of the parameters are determined for both a PI and a PID controller (Table 2). The corresponding ITAEs, as calculated by Eq. (4), in a case in which both the controllers are called on to reproduce the set-point change shown in Fig. 3 (servo problem) are reported too.

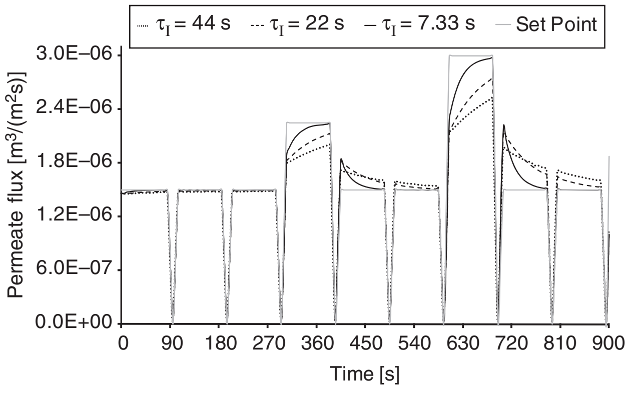

Fig. 7. Effect of the reset time on the performance of PI controller (Servo problem).

4.3. ITAE analysis for optimal control tuning

Starting from the calculated Z-N parameters and according to the performance criterion based on the minimization of the ITAE, a new set of optimized controller parameters is eventually obtained with reference to the previously described servo problem. The minimization of the objective function, ITAE, is performed numerically repeating the simulations with different values of the controller parameters, which are changed in an iterative scheme, until a minimum is reached. Table 3 summarizes the so obtained values of K_c, τ_I and τ_D referred to both a PI and a PID controller, together

Table 3
Optimized settings for PI and PID controllers obtained by minimizing the ITAE

Controller	K_c	τ_I	τ_D	ITAE
PI	2.724	1.911	–	0.0153
PID	2.928	2.272	0.033	0.0169

with the corresponding value of minimum ITAEs. It is worthwhile observing that the controller parameters, as estimated by the present improved technique, are rather different from those calculated according to the Z-N tuning rule. A considerably lower value of ITAE is also obtained, thus proving the effectiveness of the proposed tuning methodology. Fig. 9 shows a comparison among the closed-loop responses obtained using either the Ziegler-Nichols tuning rule or the tuning rule based on the minimization of ITAE. It can be observed that the control action is faster and more effective when the improved controller parameters are used, since the actual trends of permeate flux decay reproduce very well each imposed change on the set-point. On the contrary, the performance of Z-N PI controller is characterized by a much larger $\varepsilon(t)$, especially when the steps on set-point pattern actually occur. The time evolutions of the manipulated variable, TMP, relative to the PI controller tuned by either the Z-N or the proposed technique are shown in Fig. 10. In both cases, TMP tends to increase continuously over the considered time horizon in order to contrast the decay of permeate flux, which, instead, would be enhanced by the absence of any control action. However, due to the faster responses characterizing the set of controller parameters as provided by

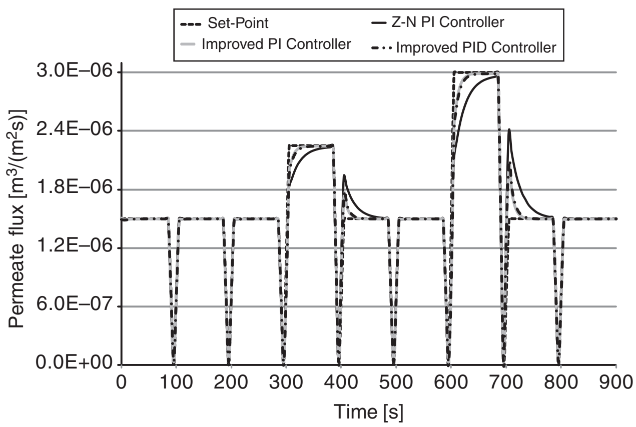

Fig. 9. Comparison among the closed-loop responses obtained using the Ziegler-Nichols tuning rule or the improved tuning rule for both PI and PID controllers (Servo problem).

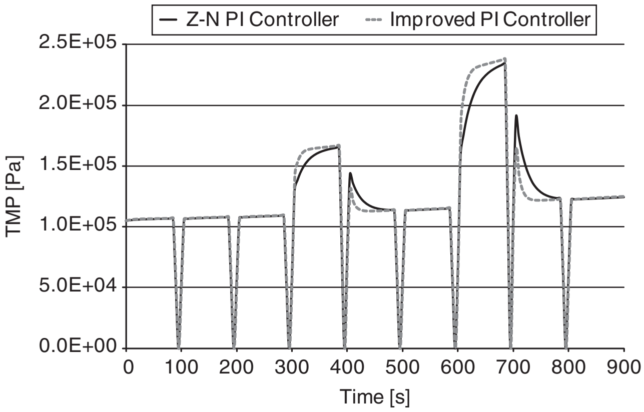

Fig. 10. Time evolution of the manipulated variable obtained by a PI controller tuned by either the Ziegler-Nichols or the proposed improved technique (Servo problem).

the proposed tuning technique, the TMP variations are much steeper than those corresponding to the Ziegler-Nichols rule, especially when the step changes on set-point actually occur.

5. Conclusions

The present paper showed a novel, effective and versatile methodology for the design and the tuning of feedback controllers aimed at improving the performance of membrane processes. A hybrid neural model provided the necessary knowledge about the complex process dynamics and represented the basis on which to develop the feedback control loop. It was proved that both PI and PID controllers were capable of reproducing quite well the desired set-point pattern, even though PI controller, as compared to PID, was actually much simpler to operate and required a lower computational effort when it had to be tuned. Finally, a more rational choice of the controller parameters was proposed. Starting from a set of controller settings, as provided by the Ziegler-Nichols tuning rule, an improved criterion, based on the minimization of the ITAE, was developed so as to obtain a new set of optimized controller parameters. The utilization of this calculated set of improved settings allowed obtaining faster and more effective control actions, as compared to the classical Z-N tuning rule. Moreover, the corresponding ITAEs, computed accounting for the actual shape of the closed-loop responses, were about threefold lower when the improved controller settings were used.

On the basis of the obtained results, it can be concluded that the methodology proposed in the present paper has several advantages, which can be successfully exploited during the implementation and the optimization of a feedback control system. These advantages can be briefly summarized as follows:

- The proposed control system was developed starting from the predictions of a reliable model that exhibited good performance over a wide range of process and operating conditions;
- The neural part of the hybrid model, due to its main characteristics and particularly to its adaptive nature, allowed identifying the difficult-to-model (uncertain) part of the UF dynamics, i.e., the actual value of the additional resistance due to membrane fouling. The proper estimation of any uncertainty and parameters change was definitely necessary so as to avoid a deterioration of the control system performance;
- No simplification, e.g. linearization, approximation of the true process dynamics to a first-order plus time delay dynamics, etc., was performed;
- The minimization of the integral of time-weighted absolute error (ITAE) allowed tuning the developed feedback controller on the basis of the actual transfer function of UF process, thus optimizing the true closed-loop performance.

The proposed methodology, based on the formulation of a proper hybrid neural model and on the optimization of a definite performance criterion, is very general as it may be exploited to implement and optimize the performance of feedback controllers operating on different kinds of membrane processes.

Appendix 1

Formulation of the hybrid neural model and implementation of the control system:

Model formulation

Momentum transport for the BSA solution flowing in the module:

General equation of motion in un-steady-state conditions and continuity equation:

$$-\nabla\cdot\eta\left(\nabla\mathbf{u}+(\nabla\mathbf{u})^T\right)+\rho(\mathbf{u}\cdot\nabla)\mathbf{u}+\nabla p+\rho\frac{\partial\mathbf{u}}{\partial t}=0 \tag{A.1}$$

$$\nabla\cdot\mathbf{u}=0 \tag{A.2}$$

where ρ denoted the liquid density (assumed constant with time), u was the velocity vector, η was the viscosity of BSA solution and p the pressure developing in the module.

Momentum transport in the membrane

Brinkman equation in un-steady state conditions combined to the continuity equation:

$$-\eta_1\nabla^2\mathbf{u}_1+\frac{\eta_1}{k(t)}\mathbf{u}_1+\nabla p_1+\rho_1\frac{\partial\mathbf{u}_1}{\partial t}=0 \tag{A.3}$$

$$\nabla\cdot\mathbf{u}_1=0 \tag{A.4}$$

where η_1 and ρ_1 denoted the viscosity and the density of the fluid flowing within the membrane, u_1 the velocity vector and p_1 was the pressure.

The membrane permeability, denoted as $k(t)$, was actually a function of time since an additional resistance, R_{ad}, due to the adsorption of BSA on the membrane surface, arose.

Mass balance equation

Assuming that the membrane under study exhibited total rejection towards BSA, the calculation of rejected species concentration was performed solving, in the module channel only, the convection-diffusion equation:

$$\nabla\cdot\left(-D_{AB}\nabla C+C\,\mathbf{u}\right)+\frac{\partial C}{\partial t}=0 \tag{A.5}$$

where, C was the BSA concentration and D_{AB} was the diffusion coefficient of BSA in water.

Boundary conditions

Apart from the non-permeable boundaries where no-slip conditions applied and no mass flux could be obtained, the following boundary conditions were employed:

Interface between the membrane and the channel

The components of velocity vectors were continuous.

Moreover, on the basis of the osmotic pressure model, the pressure p_1, evaluated on the "membrane-side", was equal to the pressure p, calculated on the "channel-side", diminished by the difference of osmotic pressure, $\Delta\Pi(x,t)$, between the membrane surface and the permeate:

$$p_1=p-\Delta\Pi(x,t) \tag{A.6}$$

where:

$$\Delta\Pi(x,t)=\alpha\left[C_w(x,t)-C_p\right]+\beta\left[C_w(x,t)^2-C_p^2\right]+\gamma\left[C_w(x,t)^3-C_p^3\right]+\ldots\ldots \tag{A.7}$$

$C_w(x,\ t)$ and C_p were, respectively, the local concentration of rejected species at the membrane surface and the concentration of BSA in the permeate, whereas α, β and γ were the virial coefficients calculated for BSA solutions.

Module inlet section:

Momentum balance: inlet solution velocity:

$$u_x=U_0(t),\ u_y=0 \tag{A.8}$$

Mass balance: inlet solution concentration:

$$C = C_0 \quad \text{(A.9)}$$

Module outlet section:

Momentum balance: superimposition of the pressure:

$$p = p_{\text{out}}(t) \quad \text{(A.10)}$$

Mass balance: convection prevailing over diffusion:

$$-D_{AB}\nabla C = 0 \quad \text{(A.11)}$$

Both outlet pressure, $p_{out}(t)$, and feed velocity, $U_0(t)$, were varied in order to reproduce, as a function of t_{op} and t_{pul} values, negative pulses for TMP(t) and positive pulses for $Q(t)$, according to periodic square-wave profiles.

Membrane outlet section:

Superimposition of the atmospheric pressure:

$$p_1 = p_{atm} \quad \text{(A.12)}$$

The trans-membrane pressure, TMP(t) was defined as:

$$TMP(t) = \frac{p_{in} + p_{out}(t)}{2} - p_{atm} \quad \text{(A.13)}$$

where p_{in} was the inlet pressure.

By a material balance in the membrane, a relationship between the amount of BSA adsorbed on membrane surface, C_a [Kg/m^2], and the diffusive and the convective contributions to mass transport was obtained:

$$\left(-D_{AB}\nabla C + C\mathbf{u}\right)_w + \frac{\partial C_a(x,t)}{\partial t} = 0 \quad \text{(A.14)}$$

The subscript w indicated that the flux vector was evaluated at the membrane wall.

R_{ad}, which contributed to the definition of permeability $k(t)$, was expressed as a function of C_a by means of a neural network consisting of only one hidden layer with five neurons and two end-layers with one neuron each.

The so-developed hybrid neural model allowed both simulating the UF process behavior over a wide range of process and operating conditions and implementing a feedback controller as schematized in Fig. 5.

On performing the simulations of the controlled UF process, the manipulated variable, TMP(t), was defined as a function of the closed-loop response, $J_p(t)$. In the case of a PID controller and assuming that the dynamics of the measuring device was negligible, it was obtained:

$$c(t) = K_c\left[J_{p,SP}(t) - J_p(t)\right] + \frac{K_c}{\tau_I}\int_0^t \left[J_{p,SP}(t) - J_p(t)\right]dt + K_c\,\tau_D\,\frac{d\left[J_{p,SP}(t) - J_p(t)\right]}{dt} + c_s \quad \text{(A.15)}$$

Assuming that the dynamics of the final control element was negligible, in particular, its dynamics was simply described by its static, dimensional gain, a, relating the control signal, $c(t)$, to the manipulated variable, TMP(t), was expressed as:

$$TMP(t) = a \cdot c(t) \quad \text{(A.16)}$$

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