



## Flocculation model applied to adjust operating conditions as flow changes

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### ABSTRACT

Higher flows in the flocculation stage of water and wastewater treatment systems can be dealt with by adapting the operating conditions. This study investigates the changes required to maintain the flocculation efficiency. The present study developed a mathematical model that assimilates particle ratio and shear rate with particle aggregation/breakage kinetics. The model allows determining how higher flow rates may respond under new flocculation conditions. The kinetic coefficients of the model were derived from literature sources. The mathematical model enables selecting the most appropriate combinations of operating variables in order to produce specified efficiency ranges. Due to the number of operating variables, the number of possible solutions to the mathematical model may be very large. Therefore, an important result is that the model can select 0.1%–1.9% of the possible combinations in operating variables, thus minimizing the experimental effort required to verify the response in terms of operation changes.

*Keywords:* Mathematical model; Water treatment; Flocculation process

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### 1. Introduction

A very important stage in water and wastewater treatment is the coagulation-flocculation process, which is widely used due to its simplicity and cost-effectiveness [1]. Depending on the overall treatment scheme applied, coagulation-flocculation is usually included either as a pre- or post-treatment stage as illustrated in Fig. 1. Flocculation is essentially a physical phenomenon that changes the particle size distribution (PSD) by introducing energy to the system. Each solid-liquid separation technique requires a different treatment depending on the quality of the effluent, the reason why it is crucial to carry out studies for each case. Moruzzi and Reali [2] presented the influence of floc size and hydraulic detention time on the performance of a dissolved air flotation (DAF) pilot unit.

A common practice in water and wastewater treatment is to apply coagulation and flocculation to remove colloidal and small particles that settle slowly [3]. Biological wastewater treatment processes produce microorganisms that naturally flocculate themselves and other suspended matter, although it may be necessary to add coagulating agents to assist their flocculation in periods of poor performance [3]. Coagulants may also be added continuously for nutrient or suspended matter removal in physical-chemical wastewater treatment processes [3].

Therefore, increasing efficiency in the coagulation-flocculation stage is a key factor to improve the overall treatment efficiency [1].

The efficiency and mechanism of the coagulation-flocculation process depend on several factors, and the most relevant regards initial turbidity, pH, reagents (coagulant, adjuvant) dosage and type, system hydrodynamics in coagulation and flocculation stages, dissipated energy, temperature and alkalinity [4].

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Due to the complex interdependence of numerous factors inherent to the coagulation and flocculation processes, it is vital for a thorough understanding of the phenomena involved [5]. In this sense, Moruzzi and Oliveira [6] analyzed the flocculation process in continuous systems with chambers in series using the classical aggregation and breakage kinetic model proposed by Argaman and Kaufman [7]. The analysis consisted of performing system behavior simulations under different operating conditions, using different number of chambers and using fixed or variable velocity gradients in the units. The response variable analyzed in the simulations was the total retention time necessary to achieve a given flocculation efficiency. The number of chambers with values ranging from 1 to 6, velocity gradients of 20–60 s<sup>-1</sup> and flocculation efficiencies of 50–90 % were used. The authors concluded that the arrangement of chambers in series with fixed gradients allows reducing the total flocculation time up to a determined efficiency limit. However, for higher velocity gradients, it is not possible to operate the system with high efficiency due to the increased breakage of the flocs. A progressive decrease of  $G_f$  in flocculation chambers allows operating the system with higher  $G_f$  values as well as increasing flocculation efficiency when separation by sedimentation at conventional velocities is used.

Conventional plant designs have conservative retention times and loading rates. In general, inflow changes occur due to an expected diurnal variation. However, in water treatment, river-based plants are subject to the effects of rainfall events, which may cause a sudden rise in incoming flow. In wastewater treatment, the plant could be subject to unexpected industrial releases. These higher flows have called for the immediate response by many processing systems to situations that go beyond the original design (nominal flow rates), without implementing structural changes. This fact results in the system's critical operating condition, close to the maximum hydraulic capacity.

These situations require adaptation strategies for the new maximum capacity conditions in order to maintain the specified efficiency. Accordingly, adapting certain process variables such as the coagulant dosage and the coagulation pH, as well as adjusting the average velocity gradient of flocculation, are the main nonstructural measures commonly practiced.

However, this new operating condition requires conducting a set of bench scale tests in order to evaluate the best configuration of the manipulated variables. Therefore, the response time required for the operational intervention is almost always performed in the treatment system right away. Mathematical model-based simulations can help define the new and more efficient configuration to be practiced, increasing operational safety and minimizing the required experimental effort.

In this context, the present study applied new flocculation conditions required by a water treatment system for inflow changes based on experimental results, on classic design parameters and on a mathematical model that incorporates particle aggregation and breakage kinetics.

### 1.1. General considerations on mathematical modeling

Camp and Stein [8] introduced the concept of mean velocity gradient, which was then used as a design parameter for rapid mixing and flocculation units, defined by Eq. (1).

According to Camp and Stein [8], velocity gradients vary widely within a mixing chamber, representing complex turbulence mechanisms to transport destabilized particles.

$$G_m = \sqrt{\frac{P}{\mu V}} \quad (1)$$

where  $G_m$  is the mean velocity gradient (s<sup>-1</sup>),  $P$  is the useful power introduced to the system (N m/s),  $V$  is the useful volume of the flocculation chamber (m<sup>3</sup>) and  $\mu$  is the fluid dynamic viscosity (N s/m<sup>2</sup>).

The minimum mean velocity gradient value of flocculation ( $G_f$ ) for operating flocculation units is that which has no floc sedimentation inside the flocculator. Typical  $G_f$  values range between 10 and 100 s<sup>-1</sup>, depending on the technology used for solid–liquid separation and characteristics of the effluent. A minimum value of 20 s<sup>-1</sup> is generally used for  $G_f$  in order to keep the particulate material in suspension.

The maximum number of flocculation chambers is not set, but the minimum number required is usually three chambers in series, which is determined based on hydrodynamic recommendations that ensure the average hydraulic retention time required.

According to Di Bernardo and Dantas [9], sedimentation is a physical phenomenon in which any particle suspended in a liquid medium is accelerated by gravity until the viscous resistance forces, thrust and deformation are equal to the particle's weight force. From this moment on, the terminal velocity is constant and equal to the settling velocity.

With regards to the mathematical modeling of the flocculation process in continuous systems, Argaman and Kaufman [7] presented a model with two parameters ( $K_a$  and  $K_b$ ), which incorporates the particle aggregation and breakage kinetics, the two phenomena governing the process. Aggregation may occur after particle collision and breakage may occur by erosion or by fragmentation [10]. A dynamic steady-state is expected during flocculation as aggregation and breakage rates make PSD and particle structure stable over time [11]. The occurrence of a transition phase may also occur and low shear rate values result in large aggregates [12]. Aggregates size distribution can be indirectly followed by turbidity [13, 14]. The Argaman and Kaufman classical kinetic model was initially designed for chambers in series operated with fixed velocity gradients. When it is operated with different velocity gradients, the model is represented by Eq. (2) [6]. According to Bratby [15], obtaining the kinetic constants requires evaluating the settling performance by analyzing the remaining residual turbidity, which must be obtained for each settling velocity ( $V_s$ ) required. Thus, the  $K_a$  and  $K_b$  values reflect the coagulation and flocculation experimental conditions and the characteristics of the treated water or wastewater, consisting of apparent constants.

$$\frac{n_{i-1}}{n_i} = \frac{1 + K_a G_{f_i} \frac{T_f}{m}}{1 + \frac{n_0}{n_{i-1}} K_b G_{f_i}^2 \frac{T_f}{m}} \quad (2)$$

where  $n_{i-1}/n_i$  is the ratio of the number of primary particles (measured indirectly by the turbidity ratio ( $N_{i-1}/N_i$ )) according

to Bratby [16]), influent and effluent of the two consecutive flocculation chambers;  $K_{ai}$  and  $K_{bi}$  are the particle aggregation and breakage kinetic coefficients in the  $i$ -th flocculation chamber, respectively;  $T_f$  is the total flocculation time;  $m$  is the number of chambers in series; and  $i$  is an index indicating the upstream to downstream chamber analyzed ( $i = 1, 2, \dots, m$ ).

The term “primary particles” is used here to designate microflocs or other particles that may form aggregates from collisions with each other. Therefore, primary particles are the ones that do not readily settle.

According to Bratby [15], determining the aggregation and breakage coefficient in flocculation requires performing long-term coagulation, flocculation and sedimentation tests, long enough so that there is no significant improvement in the quality of the supernatant. There is, however, an enhanced mixture condition that performs better. The determination of the most suitable mixing condition can be performed by preparing removal efficiency curves according to the flocculation time for each velocity gradient used in order to relate the number of primary particles with the turbidity value. Methods for obtaining the kinetic coefficients from bench tests can be found in Bratby [6] and Brito [17].

The methodology developed by Bratby [15] was studied by Pádua [18] and Libânio [19], who concluded that a very long settling time is not suitable for determining design parameters, because it does not represent the reality of the physical-chemical treatment systems and because they do not significantly reduce flocculation efficiency. Although this measure contradicts the assumption of the mathematical model, which is based on the aggregation and breakage kinetics of primary particles, the use of times compatible with the settling rates practiced in real systems allows evaluating how the change in  $V_s$  affects the overall performance of the flocculation-sedimentation process.

Therefore, several combinations of flocculation operating conditions were analyzed to identify those that meet the specified efficiency when hypothetical influent flow changes occur in the treatment system and to evaluate the effects of these variations on the other variables such as apparent surface application rate ( $HLR_{ap}$ ), settling rate ( $V_s$ ), hydraulic retention time in each chamber ( $\theta_h$ ) and total flocculation time ( $T_f$ ).

The initial objective was to generate a surface for permutations of  $G_f$  (in  $10\text{-s}^{-1}$  increments) and  $m$  chambers. From this surface solution, we have the  $T_f$  values needed (along with a combination index) to obtain a specified efficiency  $E$ . Since there are a vast number of solutions, the final objective was to reduce the feasible solutions to a manageable number that can be confirmed by laboratory experiments or by actual operation. It appears that its use is to reduce the effort needed to respond to flow variation during operation. Flow variation would affect  $T_f$  and  $HLR_{ap}$  and therefore,  $E$ .

## 2. Methodology

A hypothetical treatment system was designed in the present study, in which mechanical flocculators in series are coupled to a conventional horizontal flow settler (Fig. 1). The inflow variations with respect to the reference value ( $Q'$ )

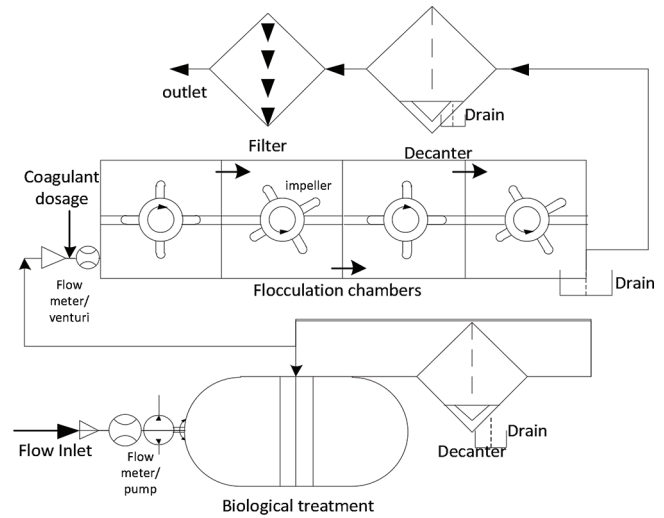


Fig. 1. Hypothetical water and wastewater treatment process flow sheet (in this illustration,  $m = 4$ ). For drinking water treatment biological treatment and the subsequent decanter are not applied.

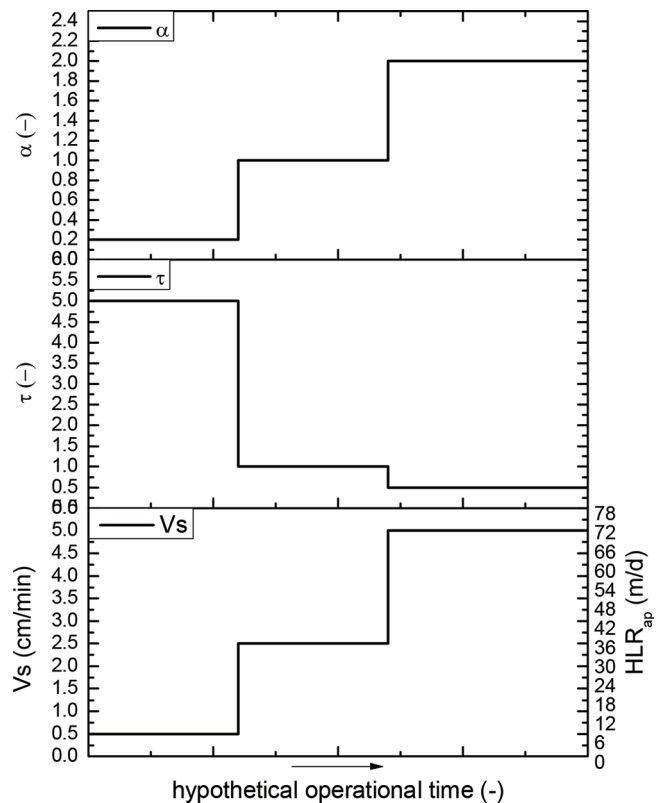


Fig. 2. Behavior of parameters  $V_s$ ,  $\tau$  and  $\alpha$  associated with influent flow changes in the treatment system.

were simulated by means of a parameter  $\alpha$  (Eq. (3)), the value of which depends on the settling velocity ( $V_s$ ), for which the kinetic constants were obtained, namely 0.5 ( $\alpha = 0.2$ ), 2.5 ( $\alpha = 1.0$ ) and 5 cm/min ( $\alpha = 2.0$ ). Fig. 2 shows the behavior of  $V_s$ ,  $\theta_h$  and  $\alpha$  associated with the inflow variations simulated in the time horizon projected for plant operation.

The design parameters  $HLR_{ap}$  and  $\theta_h$  are inversely proportional to each other, as seen in Eqs. (4) and (5), which also shows the relationship of these parameters with the affluent flow ( $Q$ ).

$$Q = \alpha \cdot Q^* \quad (3)$$

$$HLR_{ap} = Vs = \frac{Q}{A} \quad (4)$$

$$\theta_h = \frac{V_t}{Q} \quad (5)$$

$$\theta_h^* = \frac{V_t}{Q^*} \quad (6)$$

$$\tau = \frac{\theta_h}{\theta_h^*} = \frac{1}{\alpha} \quad (7)$$

where  $Q$  is incoming flow ( $\text{m}^3 \text{ s}^{-1}$ );  $Q^*$  is reference flow ( $\text{m}^3 \text{ s}^{-1}$ ), corresponding to  $HLR_{ap} = 36 \text{ m/d}$  ( $Vs = 2.5 \text{ cm/min}$ );  $\alpha$  is parameter for flow variation (dimensionless);  $HLR_{ap}$  is apparent hydraulic load rate in settler (results expressed in terms of  $\text{m}^3 \text{ m}^{-2} \text{ d}^{-1}$ );  $A$  is area in settler plant ( $\text{m}^2$ );  $\theta_h$  is total hydraulic retention time,  $\theta_h = T_f$  (s);  $\theta_h^*$  is reference total hydraulic retention time, corresponding to  $Q^*$  (s);  $t$  is dimensionless hydraulic retention time; and  $V_t$  is total volume of flocculation chambers in series ( $\text{m}^3$ ).

The new settings required by the treatment system were evaluated using five  $G_f$  values and between three and six chambers in series ( $m$ ), thereby generating 35, 70, 126 and 210 different combinations of  $G_f$ , respectively. The following restrictions were applied on the different combinations of  $G_f$  in the  $m$  chambers in series: (i)  $G_f$  of upstream greater than or equal to the downstream value ( $G_{fi} \geq G_{f(i+1)}$ ); (ii) number of chambers in series ( $m$ ) between three and six ( $m = 3, \dots, 6$ ); (iii)  $G_f$  values between 20 and 60  $\text{s}^{-1}$  ( $20 \text{ s}^{-1} \leq G_f \leq 60 \text{ s}^{-1}$ ), with  $\Delta G_f = 10 \text{ s}^{-1}$ .

Different  $R$  values ( $n_0/n_m$ ) were specified for the determination of  $T_f$  in Eq. (2) in order to simulate various efficiencies ( $E$ ) required for the flocculation-sedimentation process. For each value assumed by  $T_f$  during the iterations, the overall efficiency of the  $m$  flocculation chambers in series was calculated according to Eqs. (8) and (9).

$$\prod_{i=1}^m \left( \frac{n_{i-1}}{n_i} \right) = \left( \frac{n_0}{n_1} \right) \left( \frac{n_1}{n_2} \right) \left( \frac{n_2}{n_3} \right) \dots \left( \frac{n_{m-1}}{n_m} \right) = \left( \frac{n_0}{n_m} \right) = R \quad (8)$$

$$E = 1 - 1/R \quad (9)$$

where the ratio  $n_{i-1}/n_i$  in each flocculation chamber was calculated by the Eq. (2).

The basic assumptions made in the development of the mathematical model given by Eq. (2) are as follows [5]: (i) the particles are monodispersed, (ii) all particles are spherical and remain so after collisions, (iii) collisions occur only between two particles, (iv) the efficiency of collisions is 100%, and (v) completely mixed flocculation chambers in series of equal volume with a total flocculation time ( $T_f$ ). Furthermore, it was assumed that  $R$  can be measured by turbidity ratio ( $N_0/N_m$ ) [15].

The total flocculation time ( $T_f$ ) was determined using the Newton–Raphson method, described in Constantinides and Mostoufi [20], which determined the  $T_f$  value that satisfies the equation  $R_{\text{specified}} - R_{\text{calculated}} = 0$ . Overall efficiency values ranging between 0.5 (50%) and 0.9 (90%), equivalent to  $2 \leq R \leq 10$  ( $\Delta R = 1$ ), were specified.

The  $T_f$  values obtained made up the set of solutions for the conditions used. Next, the restrictions applied were based on the relations  $Vs$ ,  $\theta_h$  and  $\alpha$  represented by Eqs. (4), (5) and Fig. 2 in order to identify the combinations that fulfill the conditions imposed by the new operating configuration of the treatment system.

The solutions to the set of Eqs. (2), (4) and (5) were classified into three subsets, for which successive restrictions were applied, namely, possible solutions (SP), when there was convergence resulting from the numerical solution; determined solutions (SD), when the required efficiency was obtained, respecting the required values of  $Vs$ ,  $\theta_h$  and  $\alpha$ ; and feasible solutions (SF), when the answers obtained were within the usual flocculation time range commonly practiced in treatment systems (from 5 to 60 min).

The values for the aggregation and breakage kinetic coefficients ( $K_a(V_s)$  and  $K_b(V_s)$ ) related to the settling velocities 0.5, 2.5 and 5.0 cm/min were extracted from Brito [17] and Di Bernardo et al. [21] and are shown in Table 1. These settling velocities correspond, respectively, to the apparent surface application rates ( $HLR_{ap}$ ) of 7.2, 36.0 and 72.0 m/d, and were applied to the hypothetical system to simulate the inflow variations.

Table 1  
Aggregation ( $K_a(V_s)$ ) and breakage ( $K_b(V_s)$ ) coefficient kinetics for different settling velocities

$G_f(\text{s}^{-1})$	Brito [12]		Di Bernardo et al. [16]			
	$V_s = 0.5 \text{ cm/min}$		$V_s = 2.5 \text{ cm/min}$		$V_s = 5.0 \text{ cm/min}$	
	$K_a(-)^a$	$K_b(\text{s})$	$K_a(-)$	$K_b(\text{s})$	$K_a(-)$	$K_b(\text{s})$
20	$1.70 \times 10^{-4}$	$3.39 \times 10^{-8}$	$1.83 \times 10^{-4}$	$1.83 \times 10^{-7}$	$1.90 \times 10^{-4}$	$4.23 \times 10^{-7}$
30	$1.71 \times 10^{-4}$	$4.55 \times 10^{-8}$	$9.40 \times 10^{-5}$	$1.91 \times 10^{-7}$	$1.09 \times 10^{-4}$	$6.40 \times 10^{-7}$
40	$9.01 \times 10^{-5}$	$3.60 \times 10^{-8}$	$6.97 \times 10^{-5}$	$2.33 \times 10^{-7}$	$5.88 \times 10^{-5}$	$6.31 \times 10^{-7}$
50	$5.14 \times 10^{-5}$	$8.22 \times 10^{-9}$	$7.93 \times 10^{-5}$	$2.42 \times 10^{-7}$	$2.85 \times 10^{-5}$	$3.56 \times 10^{-7}$
60	$5.70 \times 10^{-5}$	$1.06 \times 10^{-7}$	$3.68 \times 10^{-5}$	$2.30 \times 10^{-7}$	$3.10 \times 10^{-5}$	$3.85 \times 10^{-7}$

<sup>a</sup>(-) dimensionless.



3. Results and discussion

Figs. 3–5 show the behavior profiles of total flocculation time ( $T_f$ ) for settling velocities of 0.5, 2.5 and 5.0 cm/min and different combinations of  $G_f$ . To better view the profiles, results were restricted to a subset of the total number of  $G_f$  combinations, i.e. for six chambers in series as this simulation results in wide range of efficiency and  $T_f$  values (full profiles

can be found in Manetta [22]). The other number of chambers in series showed a similar behavior to that shown, consisting of a pattern representative of the overall data. The combinations shown correspond to the 60–140 combinations from a total of 210 different combinations for six chambers in series. Figs. 1–3 show that the  $T_f$  values increase with increasing settling velocity. The maximum  $T_f$  values observed were 18.9, 56.2 and 544 min, respectively, for  $V_s$  values of 0.5, 2.5 and 5.0 cm/min. As the required efficiency increases for each  $V_s$ , the total flocculation time values also increase.

There were cases where the required efficiency for all investigated settling velocities was not achieved, thus, the higher the  $V_s$ , the higher the number of combinations in which the convergence of the numerical method was not verified, indicating there is no possible solution. The number of cases in which there is convergence of the numerical method decreases with the increasing efficiency required. Table 2 shows the number of combinations for which the  $T_f$  values can be determined, which in the set of possible solutions are the values to operate the system with three, four, five and six chambers in series.

The increased settling rate involves changing the total time required in order to maintain the desired efficiency and fulfill Eqs. (3) and (4), as well as the  $V_s$ ,  $\theta_h$  and  $\alpha$  relations resulting from the new configuration of the treatment system. To retain the efficiency, the  $G_f$  values should be modified to fulfill the change in the  $\theta_h$  value, and consequently in  $T_f$ . This can be seen for the 60/50/40/40/30 combination whose  $T_f$  is of 46.3 min for  $V_s$  of 2.5 cm/min and efficiency of 90%. Increasing the  $V_s$  to 5.0 cm/min, the  $T_f$  will be less than or equal to half the initial value, and in this condition the 50/30/20/20/20/20 combination whose  $T_f$  is of 10.4 min for 90% efficiency, the necessary requirements are met. However, it is not always possible to achieve the same efficiency required when the  $HLR_{ap}$  is changed by the influent flow, and this occurs more frequently when the settling velocity is higher.

The overall data consist of a number of combinations of 352 ( $m = 3$ ), 702 ( $m = 4$ ), 1262 ( $m = 5$ ) and 2102 ( $m = 6$ ) for each efficiency standard required. However, the number of possible solutions is restricted to the solution of Eq. (2) for each efficiency rate required.

Figs. 3–5 depict the occurrence of peaks and valleys which represent the variation around a given  $T_f$  value, where valleys correspond to the lower flocculation total time combinations for that particular efficiency. The region of the graph and data highlighted in Fig. 4 represent the  $G_f$  combination values and their respective  $T_f$  values in one of the peaks and valleys identified on the surface, obtained for the same efficiency (87.5%), for different  $G_f$  combinations in six flocculation chambers in series.

Furthermore, the peaks formed in Figs. 3–5 shift to smaller efficiency regions with the increasing  $V_s$ . In Fig. 1, for the  $V_s$  of 0.5 cm/min the peaks and valleys formed are well distributed, showing the tendency of  $T_f$  to increase according to increased efficiency, which may achieve efficiencies of around 90%.

In Fig. 4, with  $V_s$  of 2.5 cm/min, the peaks and valleys are more pronounced in the region which has efficiency of around 87.5%. In Fig. 3, with  $V_s$  of 5.0 cm/min, the peaks and valleys are concentrated in the region corresponding to the efficiency of approximately 75%.

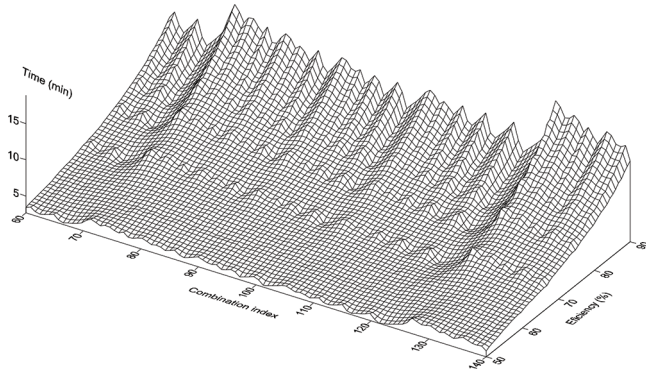


Fig. 3. Total flocculation time ( $T_f$ ) according to the required efficiency for  $V_s$  of 0.5 cm/min, and different combinations of  $G_f$  ( $m = 6$ ).

Combination	$T_f$ (min)	$E$ (%)
60/50/40/40/40/30	37.0	87.5
60/50/40/40/40/20	25.9	87.5
60/50/40/40/30/30	28.9	87.5

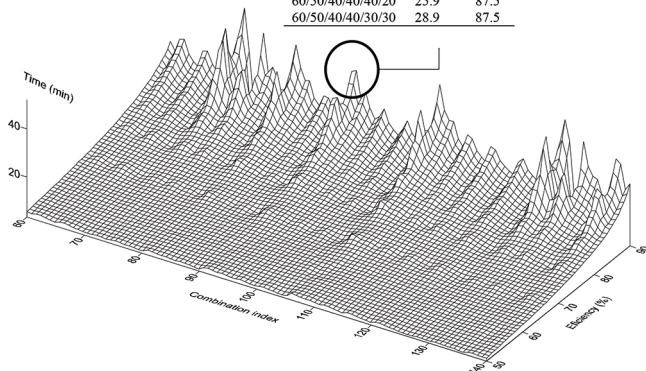


Fig. 4. Total flocculation time ( $T_f$ ) according to the required efficiency for  $V_s$  of 2.5 cm/min, and different combinations of  $G_f$  ( $m = 6$ ).

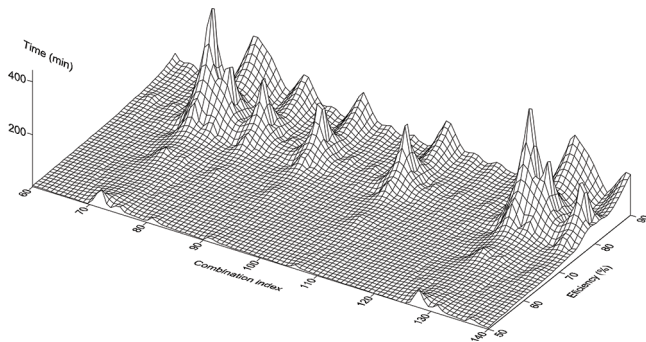


Fig. 5. Total flocculation time ( $T_f$ ) according to the required efficiency for  $V_s$  of 5.0 cm/min and different combinations of  $G_f$  ( $m = 6$ ).

Table 2

Number of combinations to determine the total time of flocculation according to the required efficiency standards and the number of chambers in series

$V_s$ (cm/min)	Efficiency (%)								
	50	66.7	75	80	83.3	85.7	87.5	88.9	90
<i>m</i> = 3									
0.5	35	35	35	35	35	35	35	34	34
2.5	35	34	34	34	34	31	25	25	25
5.0	31	25	25	25	15	15	15	15	15
<i>m</i> = 4									
0.5	70	70	70	70	70	70	70	69	69
2.5	70	69	69	69	69	65	55	55	55
5.0	65	55	55	55	35	35	35	35	35
<i>m</i> = 5									
0.5	126	126	126	126	126	126	126	125	125
2.5	126	125	125	125	125	120	107	107	107
5.0	120	105	105	105	70	70	70	70	70
<i>m</i> = 6									
0.5	210	210	210	210	210	210	210	209	20
2.5	210	209	209	209	209	202	185	185	18
5.0	203	182	182	182	128	128	128	128	12

Table 3

Number of possible solutions (SP) and determined solutions (SD) according to the required efficiency (%)

Efficiency (%)	50	66.7	75	80	83.3	85.7	87.5	88.9	90	% Average
<i>m</i> = 3										
SP	1,085	850	850	850	510	465	375	375	375	
SD	17	11	25	37	68	68	14	17	19	
%	1.6	1.3	2.9	4.4	13.3	14.6	3.7	4.5	5.1	6
<i>m</i> = 4										
SP	4,550	3,795	3,795	2,415	2,275	1,925	1,925	1,925	1,925	
SD	44	38	75	118	203	232	55	66	80	
%	1.0	1.0	2.0	3.1	8.4	10.2	2.9	3.4	4.2	4
<i>m</i> = 5										
SP	1,512	13,125	13,125	13,125	13,125	8,750	8,400	7,350	7,350	
SD	85	102	194	311	501	640	174	214	275	
%	0.6	0.8	1.5	2.4	5.7	7.6	2.4	2.9	3.7	3
<i>m</i> = 6										
SP	42,630	38,038	38,038	38,038	26,334	25,452	22,932	22,932	22,932	
SD	167	227	436	715	1,112	1,413	470	600	809	
%	0.4	0.6	1.1	1.9	4.2	5.6	2.0	2.6	3.5	2

Table 3 shows the number of SP and SD for the overall data generated by changing the number of chambers, namely, 3 and 6, respectively. The number of possible solutions is inversely proportional to the required efficiency. However, this behavior is not observed for SD. The number of SD generally represents a very small percentage (average of around 4%) of possible solutions, thus limiting the overall feasible solutions with the increasing flow, maintaining the same settling efficiency. The percentage of determined solutions tends to decrease with the increasing number of chambers, on average ranging from 6% for  $m = 3$ , 4% for  $m = 4$ , 3% for  $m = 5$  and

2% for  $m = 6$ . This fact reinforces the importance of the proposed methodology to minimize the response time and the experimental effort required to assess the new configuration due to the changing inflow.

There is, however, an additional restriction that can be applied to the restricted set of determined solutions. This refers to  $T_f$  values typically used in flocculators for physical-chemical treatment, presented as feasible solutions. Typical  $T_f$  values range between 20 and 50 min for hydraulic flocculators and between 10 and 40 min for mechanized flocculators. Table 4 was obtained simulating various  $T_f$  intervals

Table 4

Feasible solutions with restriction of  $T_f$  according to the flocculation time range ( $T_f$ ) commonly practiced for efficiencies ranging from 50% to 90%

$m$ $T_f$ (min)	3 [%] <sup>a</sup>	4 [%]	5 [%]	6 [%]
05 to 60	110 [1.9]	415 [1.7]	1,188 [1.4]	2,987 [1.1]
05 to 50	82 [1.4]	298 [1.2]	790 [0.9]	2291 [0.8]
10 to 50	13 [0.2]	34 [0.1]	131 [0.2]	248 [0.1]

<sup>a</sup>[%] Percentage of feasible solutions for the overall combinations, determined by the possible solutions for each  $T_f$  range in different numbers of chambers in series ( $m$ ).

Table 5

Examples of feasible solutions for restricted  $T_f$  between 10 and 50 min considering efficiency of 90% for four flocculation chambers in series

(Vs = 2.5 cm/min)					(Vs = 5.0 cm/min)				
$G_f$		$T_f$ (min)			$G_f$		$T_f$ (min)		
60	50	50	30	49.6	40	20	20	20	11.6
					30	30	20	20	11.9
					30	20	20	20	10.0
60	50	40	30	48.2	40	20	20	20	11.6
					30	30	20	20	11.9
					30	20	20	20	10.0
50	50	50	30	43.2	30	20	20	20	10.0
50	50	40	30	40.8	30	20	20	20	10.0
50	40	40	30	41.6	30	20	20	20	10.0
40	40	40	30	43.7	30	20	20	20	10.0

for situations in which the same efficiency is achieved with  $V_s$  increasing from 2.5 to 5.0 cm/min, which considered efficiencies of 50% to 90% for all numbers of chambers used. It is observed that the application of the restriction conditioned to the  $T_f$  value, commonly used in treatment systems, limits the number of solutions to even lower percentages (in the order of 0.1% to 1.9%), increasing the advantage of applying the mathematical model for the preliminary evaluation of the best conditions required, minimizing the response time, reducing costs and possibly increasing the success rates.

As an example, Table 5 shows combinations corresponding to alternative solutions for the increasing flow of  $V_s$  from 2.5 cm/min to 5.0 cm/min ( $\alpha = 2$ ), with  $T_f$  restricted between 10 and 50 min. The combinations lead to different  $G_f$  values arranged from the largest to the smallest in the flow direction. Of course, laboratory scale tests have to be carried out in order to confirm the results. One should also consider that the kinetic constants must be obtained for each water flow, for each  $G_f$  value. However, the application of the mathematical model as presented significantly restricts the overall solutions to be tested, significantly reducing the response time required for decision making.

#### 4. Conclusions

The settling rate greatly influences the efficiency of water treatment plants and the changes therein create particle settling disturbances. However, to fulfill the need for increased flow, without any structural changes and respecting the hydraulic capacity, operational changes can be executed, such as adjusting the average velocity gradients in the flocculation chambers in series, in order to fulfill the new conditions required by the increased load in the system, maintaining the clarification efficiency.

Laboratory studies are not discarded because the aggregation and breakage kinetic coefficients depend on the water quality under study and the average velocity gradient performed. However, as exemplified in this article, using the mathematical model can significantly reduce the set of feasible solutions. Applying the restriction conditioned to the  $T_f$  value, commonly used in treatment systems, limits the number of solutions for values ranging from 0.1% to 1.9% of the overall possible solutions, thereby demonstrating the advantage of applying the mathematical model for the preliminary determination of the best conditions required, with reduced response time, lower costs and with the increased likelihood of successful decisions.

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#### Symbols

- $A$  — Area in settler plant,  $m^2$
- $E$  — Flocculation efficiency, dimensionless
- $G_m$  — Mean velocity gradient,  $s^{-1}$
- $G_f$  — Mean velocity gradient of flocculation,  $s^{-1}$
- $HLR_{ap}$  — Apparent hydraulic load rate in settler,  $m^3 m^{-2} d^{-1}$
- $K_a$  — Aggregation coefficient, dimensionless
- $K_b$  — Breakage coefficient,  $s$
- $m$  — Number of chambers in series, dimensionless
- $n_0$  — Number of primary particles per unit volume at the inlet stream of the 1st chamber,  $cm^{-3}$
- $n_i$  and  $n_{i-1}$  — Number of primary particles per unit volume in the output of the  $i$ -th and  $(i-1)$ th flocculation chambers, respectively,  $cm^{-3}$
- $n_m$  — Number of primary particles per unit volume at the outlet stream of the last chamber,  $cm^{-3}$
- $P$  — Useful power introduced to the system,  $N m/s$
- $Q$  — Incoming flow,  $(m^3 s^{-1})$
- $Q^*$  — Reference flow, corresponding to  $HLR_{ap} = 36 m/d, V_s = 2.5 cm/min$
- $R$  — Parameter expressed by the ratio  $n_0/n_m$ , dimensionless
- $T_f$  — Total hydraulic retention time in the set of  $m$  chambers in series, min
- $V$  — Useful volume of the flocculation chamber (equal for all chambers),  $m^3$
- $V_t$  — Total volume of flocculation chambers in series,  $m^3$
- $V_s$  — Settling velocity,  $cm min^{-1}$

$\theta_h$	— Total hydraulic retention time, $\theta_h = T_r$ , min
$\theta_h^*$	— Reference total hydraulic retention time, corresponding to $Q'$ , min
$\tau$	— Dimensionless hydraulic retention time, dimensionless
$\alpha$	— Parameter for flow variation, dimensionless
$\mu$	— Fluid dynamic viscosity, N s/m <sup>2</sup>

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