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Soil water diffusivity obtained from visual inspection experiment and comparison with γ-ray measurements

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ABSTRACT

Diffusivity is one of the main soil hydraulic properties. It is a critical parameter for the prediction of water transport within the vadose zone. The aim of this paper was to establish the soil water diffusivity of a soil sample using transformed soil moisture profile. Whisler et al. proposed a method, which requires knowledge of the complete soil moisture profile at fixed distances on the soil column. This article uses this method, which is more appropriate nowadays according to the available measuring instruments, for verification purposes. Our laboratory developed a visual method during horizontal experiment, which is simple and takes into consideration profile length observations, sorptivity, initial and final moisture content in order to calculate diffusivity. The method is based on the utilization of a complex empirical function either with four or three constants to generate the transformed soil moisture profile by treating the process as an optimization problem. The required conditions to compute the constants of the empirical function are: (a) the analytically computed sorptivity should agree with the experimental one and (b) the beginning and the end of the transformed soil moisture profile should agree with the final and the initial water content correspondingly. Once an analytic function for the transformed soil moisture profile is determined, then diffusivity is calculated analytically. Integral continuity is preserved throughout the process. The regenerated profiles, which were determined with the visual method, were verified with measured data points from γ-ray measurements during the horizontal absorption experiment and the results were very satisfactory.

Keywords: Diffusivity; Sorptivity; Soil water properties; Soil moisture profile; Horizontal absorption

1. Introduction

Soil hydraulic properties are very important in the prediction of water flow. The necessary properties are hydraulic conductivity [1,2], diffusivity and specific capacity [3]. Measurement of these properties is complex, time consuming and requires quite expensive instruments.

Recently, there is a tendency to consider soil water diffusivities of unsaturated soils as one significant soil hydraulic property. A method for measuring hydraulic conductivity and soil water diffusivity was described initially using pressure plate outflow data by Gardner [4]. Bruce and Klute [5] utilized horizontal absorption to relate soil water diffusivity to the volumetric water content. Their method is based on the Boltzmann transformation [6] and measurements of the water content slope distribution curve along the soil column. Since accurate determination of the slope is very difficult, errors arise also in the determination of soil water diffusivity. Whisler et al. [7] introduced a method that used the same theoretical analysis as that in the Bruce and

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Klute method [5], but the diffusivity is based on the water distribution as a function of time at a fixed position instead of the water content distribution with distance at a specific time. Clothier et al. [8] utilized a fitting function from the ones proposed by Philip [9] to approximate the water content distribution curve. This led to an analytical function for the description of the water diffusivity, but this function may not apply to all soil types. McBride and Horton [10] developed a method of determining water diffusivity using an empirical fitting function for the soil moisture profile from horizontal absorption experiments, but require cumbersome calculations. Shao and Horton [11] developed a method to estimate soil water diffusivity by using an analytical solution to horizontal redistribution based on general similarity theory. They assumed a power function relationship between soil water diffusivity and soil water content, which may not apply to all soil types. Šimůnek et al. [12] used a parameter estimation approach to analyze horizontal infiltration data to obtain the diffusivity water content function. They utilized numerical inversion in order to gain additional information about the water retention curve and the hydraulic conductivity function. Wang et al. [13] presented an analytical method to determine Brooks and Corey model [14] parameters from horizontal infiltration. Wang et al. [15] utilized cumulative infiltration, infiltration rate and wetting front distance in order to estimate the soil water diffusivity. However, the assumption is a good approximation only when soil water content is close to saturation and so far limited number of soils has been used to test the assumption. Ma et al. [16] developed an analytical method for estimating soil hydraulic parameters and they tested their method on 19 numerical soils and not experimental data. They used the assumption of exponential flux distribution to determine Brooks and Corey model [14] parameters. They utilized an experimentally revised shape coefficient to guarantee agreement of water content, soil tension, water flux distribution and cumulative infiltration estimated by the analytical method with those calculated by HYDRUS-1D software.

The aim of this paper is to verify the visual method proposed by Evangelides et al. [17] using the experimental procedure of Whisler et al. [7]. The original method for obtaining diffusion coefficient proposed by Bruce and Klute [5] is suitable only if the moisture profile of the whole soil sample can be determined at a fixed time. This was feasible when the soil column was sectioned and water content of each section was measured gravitationally. Also, another drawback of the method was that the whole process had to be repeated in order to obtain the profile at different times, which required drying and repacking of the soil sample with questionable results for the homogeneity of the sample during all steps. Whisler et al. method [7] is simpler and easier to apply when there is only one measuring soil moisture device since it requires the whole soil moisture profile at specific distances, which in accordance means that the soil sample does not need removal and repacking. Both methods utilize Boltzmann transformation, but Bruce and Klute [5] considers that time (*t*) is fixed, while Whisler et al. [7] considers fixed distance (*x*). In this paper, the visual method was used in order to establish the soil water diffusivity. This method uses profile length observations, sorptivity measurements, initial and final moisture content. The objective is to use a complex empirical function to generate the transformed soil moisture profile by treating the process as an optimization problem, and finally to extract the diffusion coefficient directly from the transformed soil moisture profile. The optimization process is such that integral continuity is maintained. The empirical functions for generating the transformed soil moisture profile were one with four constants [18] and a simpler one with three constants [17]. Soil moisture measurements with γ-ray were carried out as described by Whisler et al. [7] in order to verify the obtained transformed profile. Finally, the diffusion coefficient was obtained analytically. The results were very satisfactory.

2. Theory

The one-dimensional horizontal movement of water in unsaturated soil can be described by the following equation [5,7]:

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\left[D(\theta_x)\frac{\partial\theta}{\partial x}\right] \qquad 0 < x < \infty \tag{1}$$

where θ is the water content (L^3/L^3), D is the diffusivity (L^2/T^1), x is the position (L) and t is time (T). Eq. (1) implies that Darcy's law is valid for unsaturated flow, whereas it is assumed that a unique relationship exists between the pressure head and the water content [19]. The initial and boundary conditions for horizontal absorption are:

$$\begin{aligned} &\theta(x,t) = \theta_i; \quad x \geq 0,\ t = 0 \\ &\theta(x,t) = \theta_0; \quad x = 0,\ t > 0 \\ &\theta(x,t) = \theta_i; \quad x \to \infty,\ t > 0 \end{aligned} \tag{2}$$

where θ_i is the initial water content (L^3/L^3) and θ_0 is the final water content (L^3/L^3).

By introducing the Boltzmann transformation $\lambda = xt^{-1/2}$, which assumes that the water content θ_x is a single valued function of λ, Eq. (1) is transformed into the ordinary differential equation:

$$-\frac{1}{2}\lambda\frac{d\theta}{d\lambda} = \frac{d}{d\lambda}\left[D(\theta_x)\frac{d\theta}{d\lambda}\right] \tag{3}$$

with the following boundary conditions:

$$\begin{aligned} &\theta_x = \theta_i; \quad \lambda \to \infty \\ &\theta_x = \theta_0; \quad \lambda = 0 \end{aligned}$$

Integrating Eq. (3) between the limits θ_i and θ_x yields:

$$D(\theta_x) = -\frac{1}{2}\frac{1}{\left(\dfrac{d\theta}{d\lambda}\right)_{\theta_x}}\int_{\theta_i}^{\theta_x}\lambda\, d\theta \tag{4}$$

According to Philip [20], sorptivity (S) is given as:

$$S = \int_{\theta_i}^{\theta_0}\lambda\ d\theta \tag{5}$$

The cumulative infiltration (I) to the wetting front can be expressed as:

$$I = \int_{\theta_i}^{\theta_0} x \; d\theta \tag{6}$$

Eqs. (5) and (6), using Boltzmann transformation, become:

$$S = \frac{I}{\sqrt{t}} \tag{7}$$

Assuming an empirical function with four constants (A, B, C and D) for the transformed soil moisture profile [18]:

$$\theta(\lambda) = A - B \tan^{-1}(C\,\lambda - D) \tag{8}$$

or

$$\lambda(\theta) = \frac{1}{C}\left[\tan\left(\frac{A-\theta}{B}\right) + D\right] \tag{9}$$

Sorptivity (Eq. (5)) using Eq. (9) becomes:

$$S = \int_{\theta_i}^{\theta_0} \lambda \; d\theta = \frac{D}{C}\left(\theta_0 - \theta_i\right) + \frac{B}{C}\ \ln\left(\frac{\left|\cos\frac{\theta_0 - A}{B}\right|}{\left|\cos\frac{\theta_i - A}{B}\right|}\right) \tag{10}$$

Diffusivity (Eq. (4)) as a function of θ using Eq. (9) becomes:

$$D(\theta_x) = -\frac{1}{2}\frac{1}{\left(\frac{d\theta}{d\lambda}\right)_{\theta_x}}\int_{\theta_i}^{\theta_x}\lambda\; d\theta$$

$$= 0.5\frac{1+\left(\tan\left(\frac{A-\theta_x}{B}\right)\right)^2}{B\ C}\left[\frac{D}{C}\left(\theta_x - \theta_i\right) + \frac{B}{C}\ \ln\left(\frac{\left|\cos\frac{\theta_x - A}{B}\right|}{\left|\cos\frac{\theta_i - A}{B}\right|}\right)\right] \tag{11}$$

which gives diffusivity analytically at any θ_x between θ_i and θ_0, once A, B, C and D are determined.

Assuming an empirical function with three constants (a, b and c) for the transformed soil moisture profile [17]:

$$\theta(\lambda) = -\left[\theta_0 + a \tan^{-1}(b\ \lambda + c)\right] \tag{12}$$

or

$$\lambda(\theta) = \frac{1}{b}\left[\tan\left(-\frac{\theta+\theta_0}{a}\right) - c\right] \tag{13}$$

Sorptivity (Eq. (5)) using Eq. (13) becomes:

$$S = \int_{\theta_i}^{\theta_0} \lambda \; d\theta = -\frac{c}{b}\left(\theta_0 - \theta_i\right) + \frac{a}{b}\ \ln\left(\frac{\left|\cos\frac{2\theta_0}{a}\right|}{\left|\cos\frac{\theta_i + \theta_0}{a}\right|}\right) \tag{14}$$

Diffusivity (Eq. (4)) as a function of θ using Eq. (13) becomes:

$$D(\theta_x) = -\frac{1}{2}\frac{1}{\left(\frac{d\theta}{d\lambda}\right)_{\theta_x}}\int_{\theta_i}^{\theta_x}\lambda\; d\theta$$

$$= -\,0.5\frac{1+\left(\tan\left(-\frac{\theta_x+\theta_0}{a}\right)\right)^2}{a\ b}\left[\frac{c}{b}\left(\theta_x - \theta_i\right) + \frac{a}{b}\ \ln\left(\frac{\left|\cos\frac{\theta_i + \theta_0}{a}\right|}{\left|\cos\frac{\theta_x + \theta_0}{a}\right|}\right)\right] \tag{15}$$

which gives diffusivity analytically at any θ_x between θ_i and θ_0, once a, b and c are determined. Eqs. (12) and (13) are simpler using three constants under determination, but they require knowledge of the final moisture content (θ_0 or θ_s) of the sample.

3. Materials and methods

The physical problem was studied in the laboratory using a plexiglass cylindrical column, 100 cm long, 6 cm inside diameter and placed horizontally. The bulk densities and the moisture content were measured by γ-ray absorption method [21–23]. The device of γ-ray contained a 300 mCi Americium-241 source. The Americium source and the photomultiplier detector (including an NaI crystal and preamplifier) were set on a platform connected to a stepper motor. In this way, one can follow the development of water profiles in the column at different distances.

Soil sample was filtered through a 1 mm sieve in order to remove foreign particles, dried in 105°C for 24 h and crumbled. Then, it was packed in a transparent plexiglass column with 100 cm length and 6 cm inside diameter.

The soil sample was graded from 0.425 to 0.6 mm in order to be homogeneous. The column was packed using a soil placement method with free-failing soil passing through a sequence of sieves. With this method, a good homogeneity of sand packing can be achieved. The soil column had a mean bulk density of 1.593 ± 0.015 g/cm^3 according to γ-ray measurements and was 100 cm high. All experiments were carried out at a constant temperature of 21°C ± 1°C. Measurements were also obtained for saturated moisture content which was $\theta_s = 0.385$ cm^3/cm^3 and the residual moisture content, which is equal to the initial moisture content, was $\theta_r = 0.006$ cm^3/cm^3. The experimental setup is shown in Fig. 1.

Water was applied at the initial time $t = 0$ to the one end of the sample ($x = 0$) under zero constant-head in order to obtain boundary condition $\theta_0 = \theta_s$. A fine plastic screen was used at

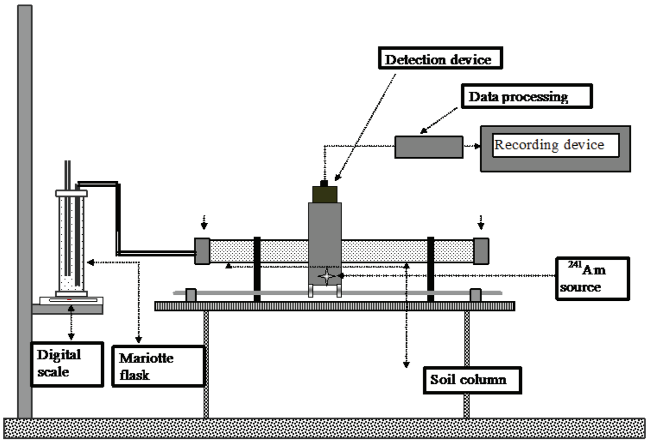

Fig. 1. Horizontal soil column.

both ends of the column, in order to prevent the soil from dispersing during the experiment. The water pressure entering the column was controlled by a Marriott burette, which was connected to the column by means of a transparent plastic tube. Continuous monitoring of the water entering the column was possible since the Marriott burette was placed on an electronic digital scale. Water content, as a function of time, was measured by scanning the column with γ-rays [21,22] at positions 11.3, 33.3 and 52.3 cm from the beginning of soil column [7]. Wetting front distance with time was also observed visually based on obvious color differences at the interface of wet and dry soil [17]. Saturated water content was measured volumetrically after the end of the horizontal absorption by continuing the wetting process in a vertical position until saturation.

4. Results and discussion

According to the visual method, cumulative infiltration was calculated from the volume of the absorbed water and the cross-section of the column at the end of visually determined profile lengths at specific times. Consequently from the calculated infiltration, the time that profile arrived at various positions and Eq. (7), sorptivity was calculated. Finally, the transformed profile length was calculated through Boltzmann transformation. The results of this procedure are presented in Table 1.

For the calculation of the constants in Eqs. (9) and (13) in order to describe analytically the transformed profile, an optimization process was used using conjugate directions. The optimization process was carried out using the value of sorptivity, the transformed profile length from visual inspection and the initial and final moisture content. The transformed profile using Eqs. (9) and (13) and the experimental points from γ-ray measurements are shown in Fig. 2. Both equations gave almost identical results compared with experimental data. Eq. (9) had a relative mean square error (RMSE) 2.50E–02 and a correlation coefficient of 0.908837 while Eq. (13) had 2.56E–02 and 0.908533, correspondingly. Using the obtained constants in Eqs. (11) and (15), $D(\theta)$ was determined analytically for the visual process (Fig. 3).

Table 1
Values obtained through the visual experimental process

t (min)	I (cm)	S (cm/min$^{0.5}$)	X_{profile} (cm)	λ (cm/min$^{0.5}$)
0.00	0.0000		0	
2.20	6.1290	4.1322	24	16.1808
7.00	11.0163	4.1638	42	15.8745
9.40	12.6869	4.1380	49	15.9820
16.30	16.7589	4.1510	65	16.0998
20.00	18.5561	4.1493	72	16.0997
Average		4.1468		16.0474

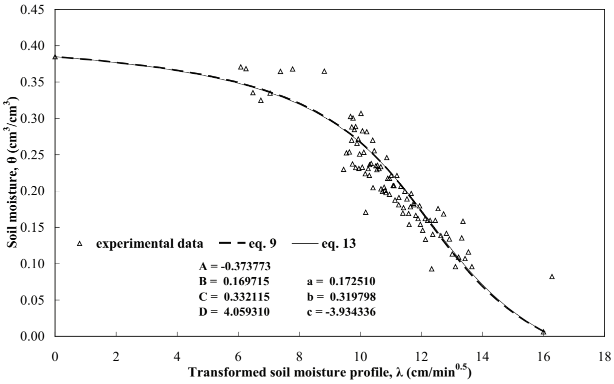

Fig. 2. Experimental points obtained with visual method and fitted profile.

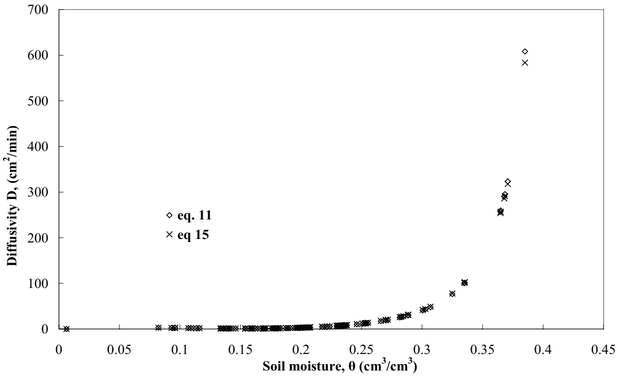

Fig. 3. Diffusivity.

5. Conclusions

The original Bruce and Klute method [5] requires knowledge of the soil moisture profile during horizontal absorption of the whole sample at different times. This was feasible, because moisture content was measured volumetrically after sectioning the soil column. On the contrary nowadays since there are limited moisture measuring devices, it is preferable to measure a whole profile in specific positions as described by Whisler et al. [7]. In this article, this method is compared with a visual method that was developed by our laboratory during a horizontal experiment.

The advantages of the described visual method is that there is no need for continuous soil moisture since the transformed wetting profile is recreated from visual distance measurements, water volume measurements, initial and final soil moisture and optimization. Consequently, there is no need for expensive equipment.

The obtained diffusion coefficients from the two equations were again very close as expected. The diffusivity was determined as 609 cm^2/min with Eq. (11), while was 583 cm^2/min with Eq. (15).

The results show that there is a very satisfactory agreement between recreated profile and experimental data for both fitting equations. The RMSE and correlation coefficient gave almost identical results compared the results of both equations with experimental data. Eq. (9) is harder to optimize compared with Eq. (13), which is expectable since it uses more constants. Nevertheless, both gave extremely close results for the transformed profile.

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