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Selection of the best coefficient of performance prediction by artificial neural network model considering uncertainty

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ABSTRACT

Four artificial neural networks (ANNs) with several configurations predicted the coefficient of performance (COP) of absorption heat transformer with duplex components. In this work, uncertainty analysis is applied to these ANNs models using Monte Carlo method with the aim to select the most appropriate ANN model in the sense that its structure when uncertainty was added, it gives a prediction of the COP close to the experimental value of the same. Experimental conditions of absorption heat transformer with duplex components are used considering the COP ranged from 0.12 to 0.33. According to our numerical results the ANN model that considers three temperatures in the absorption cycle ($T_{inGE\text{-}AB}$, $T_{inAB\text{-}GE}$ and $T_{outGE\text{-}AB}$) and one pressure level (P_{AB}) in the input layer and four neurons in the hidden layer to predict COP was the most accurate. For this ANN model, normal probabilistic distribution of the COP is observed and the mean of the probabilistic distribution is close to the experimental COP.

Keywords: Standard deviation; Monte Carlo method; Water purification system; Heat transformer

1. Introduction

The present research introduces the estimation of uncertainty in the coefficient of performance (COP; the dependent variable) of the absorption heat transformer with duplex compounds; it is based on Monte Carlo method since the addition of uncertainty in the operation variables (independent variable). Martínez-Martínez et al. [1] presented four artificial neural networks (ANNs) model, who have predicted the COP for an absorption heat transformer with the aim of water purification. The models by Martínez-Martínez et al. [1] consider from three to six input operation variables to predict COP; these models meet strict statistics criterion, the error analysis and the use of matrix correlation. For the authors, the ANNs with four, five and six neurons in the input layer were successfully trained and validated ($r^2 > 0.99$) for the COP prediction and complying with strict residual analysis. However, in the study, the standard deviation in the input operation variables was not considered.

An option to estimate the standard deviation is the Monte Carlo method. Rees [2] and Anderson [3] describe it as a mathematical approach that involves repeated calculation

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of quantity, varying each time the input data randomly with their stated prediction limits. About the Monte Carlo method, Colorado et al. [4] used this numerical approach to estimate the standard deviation in an empirical model called "the hybrid model" to predict the heat load on an evaporator applied to a vapor compression refrigeration system using R134a as working fluid. According to the authors, the numerical estimation of standard deviation depends on characteristics of the model, and the limits of instrumentation can influence the accuracy of prediction. Boyaval [5] demonstrates the efficiency of the reduced-basis control-variate Monte Carlo method for uncertainty propagation in a representative uncertainty quantification framework aimed to compute expectations of a scalar output of the random partial differential equation solution for many values of the control parameters. Wang et al. [6] compared the numerical results of a random collocation method and a modified random collocation method with the traditional Monte Carlo method simulations in two heat transfer numerical cases: (i) a thermal plate and (ii) a sandwich structure, both to demonstrate the effectiveness and accuracy of the methods.

In this research, we added several levels of uncertainty in the input operation variables considers in the artificial neural models by Martínez-Martínez [1]. The aim of this work is selected the most appropriate ANN model in the sense that its structure when uncertainty is added gives a prediction of the COP close to the experimental value of the same. Moreover, the probabilistic distribution of COP prediction is observed.

The novelties between studies done by Martínez-Martínez et al. [1] and Colorado et al. [4] and the present work are:

- Martínez-Martínez et al. [1] propose a residual analysis as criteria to select the best ANN model. In this research, we added several levels of uncertainty in the independent variables, the most appropriate ANN model will be the one that propagates minor error, and it agrees with the experimental value.
- Colorado et al. [4] present numerical information about Monte Carlo method and error propagation applied in an empirical model. However, the probabilistic distribution is not shown. In this work, Monte Carlo method is used to demonstrate the probabilistic distribution of the COP, it is predicted by ANN model.

Remaining of the paper is organized as follows: section 2 shows the experimental equipment and instrumentation analyses. Section 3 presents the ANNs models used to predict the COP. Section 4 shows the uncertainty analysis and the Monte Carlo method. Section 5 contains the main results, consisting of the probabilistic distribution of COP calculated and the selection of the most appropriate ANN model. Finally, section 6 includes the concluding remarks.

2. Experimental data: heat transformer and instrumentation

In the Research Center of Engineering and Applied Science in Cuernavaca, Morelos, Mexico, has located the test facility. This experimental rig consists of a water purification system integrated to a heat transformer composed to two duplex units generator–condenser and absorber–evaporator. In the system, the lithium bromide water mixture was selected as the working mixture and water as working fluid. The thermal load design of heat transformer was 2 kW and built with stainless steel 316L (equipment and piping) to withstand corrosion effect. The approximate dimensions for heat transformer are 2.3 m × 2 m × 2 m and the entire system was covered with insulation foam. For more details of the operation and experimental data refer Morales et al. [7].

For the heat transformer, the COP is defined as the ratio of the useful heat delivered in the absorber $\dot{Q}_{AB}$ divided by the waste heat load supplied to the generator $\dot{Q}_{GE}$ plus evaporator $\dot{Q}_{EV}$, the COP is given according to the following relation:

$$COP_{exp} = \frac{\dot{Q}_{AB}}{\dot{Q}_{GE} + \dot{Q}_{EV}} \tag{1}$$

The range of experimental coefficient of performance (COP_{exp}) reported by Morales et al. [7] was from 0.1 to 0.36.

The COP by Eq. (1) was calculated by following experimental information, appropriate thermophysical properties of the lithium–bromide solution and working fluid, and thermodynamics balances.

About instrumentation, the system was equipped with temperature, pressure and flow meter sensors on each side of the components in the absorption heat transformer cycle.

The uncertainty measurement information is described in Table 1; however, it is valid only for the exclusive operating conditions investigated during the experimental campaign. A total of 16 operation variables considering flow, temperature and pressure were registered by digital data acquisition system. The refraction index is used for the concentration of lithium–bromide solution determination; sufficient samples of the solution are taken during the operation of heat transformer. Nevertheless, only six operational variables were considered to predict COP_{sim} according to the analysis of the matrix correlation presented by Martínez-Martínez et al. [1]. Table 2 shows the range of experimental input operational variables and the COP_{exp} in the absorption heat transformer.

3. Artificial neural network models to predict the coefficient of performance

ANN is a model recommended by many authors for being fast and accurate to predict the dependent variable, these capacities are necessary for the real-time estimation in the process.

In this research, the ANNs models by Martínez-Martínez et al. [1] were used. The aim of ANN models where the COP prediction in a heat transformer with duplex compounds.

Four models were analyzed considering the hyperbolic-tangent transfer function in one hidden layer and a linear function in the output layer. The ANN models are enlisted in the following sections.

3.1. ANN model with six neurons in the input layer

Eq. (2) presents the model considering T_{inGE}, $T_{inGE\text{-}AB}$, $T_{inAB\text{-}GE}$, $T_{outGE\text{-}AB}$, T_{inEV} and P_{AB} as independent variables in

Table 1
Instrumentation equipment in the heat transformer

Variable	Instrument	Accuracy
Flow	Flow meter	±5% in full scale Range: 6.50×10^{-8} to 2.83×10^{-7} m^3/s Maximum pressure: 1,378,951.81 Pa Material: stainless steel
Flow	Flow meter	±3% in full scale Maximum flow: 5.30×10^{-4} m^3/s Maximum pressure: 1,378,951.81 Pa Material: stainless steel
Pressure	Manovacuometer	±0.5% in full scale Range: 0–101,592 Pa Type: Bourdon Material: stainless steel Localization: AB-EV
Pressure	Manovacuometer	±2% in full scale Range: 0–101,592 Pa Type: Bourdon Material: stainless steel Localization: GE-CO
Pressure	Pressure transducer	±0.25% in full scale Range: 0–101,592 Pa Output voltage: 0.5–5.5 V Feed: 9-30 VDC Localization: AB-CO and GE-CO
Temperature	PT100	±0.5 K for each measurement
Temperature	RTD	±0.5 K for each measurement

RTD – Resistance temperature detector.

the input layer, four neurons in the hidden layer in order to predict the coefficient of performance (COP_{sim}).

$$COP_{sim} = 2\left[\frac{0.6578}{1+e^{\varphi_1}} - \frac{0.4038}{1+e^{\varphi_2}} - \frac{0.2596}{1+e^{\varphi_3}} - \frac{0.0655}{1+e^{\varphi_4}}\right] - \left(0.6578 - 0.4038 - 0.2596 - 0.0655\right) + 0.1822 \quad (2)$$

where

$$\varphi_1 = -2\begin{pmatrix} -8.4816T_{inGE} + 6.2576T_{inGE\text{-}AB} \\ +2.7682T_{inAB\text{-}GE} - 4.6990T_{outGE\text{-}AB} \\ -7.1527T_{inEV} + 1.0384P_{AB} + 8.1859 \end{pmatrix}$$

$$\varphi_2 = -2\begin{pmatrix} -14.2334T_{inGE} + 12.4429T_{inGE\text{-}AB} \\ +8.7167T_{inAB\text{-}GE} - 13.6168T_{outGE\text{-}AB} \\ -7.5706T_{inEV} + 2.7109P_{AB} + 10.5137 \end{pmatrix}$$

Table 2
Input variables used in the artificial neural networks models by Martínez-Martínez et al. [1] to predict the COP_{exp} in the absorption heat transformer

Input operation variables	Operation range	Localization
T_{inGE}	338.76–358.83 K	Input temperature of the generator that comes from waste heat source
$T_{inGE\text{-}AB}$	329.4–351.24 K	Input temperature of the solution to the generator that comes from absorber
$T_{inAB\text{-}GE}$	323.85–351.97 K	Input temperature of the solution to the absorber that comes from generator
$T_{outGE\text{-}AB}$	330.48–354.53 K	Outlet temperature of the solution to the absorber that comes from generator
T_{inEV}	339.76–359.18 K	Input temperature of the evaporator that comes from waste heat source
P_{AB}	338.639–1,219.10 Pa	Absorber pressure
COP_{exp}	0.10–0.36	The coefficient of performance

$$\varphi_3 = -2\begin{pmatrix} -6.5909T_{inGE} + 3.1316T_{inGE\text{-}AB} \\ +0.6686T_{inAB\text{-}GE} - 2.5646T_{outGE\text{-}AB} \\ -16.6884T_{inEV} + 3.7124P_{AB} + 13.7787 \end{pmatrix}$$

and

$$\varphi_4 = -2\begin{pmatrix} -2.5631T_{inGE} + 8.8186T_{inGE\text{-}AB} \\ -1.5703T_{inAB\text{-}GE} - 16.2158T_{outGE\text{-}AB} \\ +10.8729T_{inEV} - 1.7951P_{AB} + 3.1875 \end{pmatrix}$$

3.2. ANN model with five neurons in the input layer

Eq. (3) presents the model considering T_{inEV}, T_{inGE}, $T_{inGE\text{-}AB}$, $T_{inAB\text{-}GE}$ and $T_{outGE\text{-}AB}$ as input variables to calculate COP_{sim}.

$$COP_{sim} = 2\left[\frac{-0.8681}{1+e^{\varphi_1}} + \frac{0.1671}{1+e^{\varphi_2}} - \frac{0.0876}{1+e^{\varphi_3}} - \frac{0.7073}{1+e^{\varphi_4}}\right] - \left(-0.8681 + 0.1671 - 0.0876 - 0.7073\right) + 0.2047 \quad (3)$$

where

$$\varphi_1 = -2\begin{pmatrix} 5.3875T_{\text{inGE}} - 5.0064T_{\text{inGE-AB}} - \\ 2.6359T_{\text{inAB-GE}} + 6.1035T_{\text{outGE-AB}} \\ +8.1112T_{\text{inEV}} - 8.6445 \end{pmatrix}$$

$$\varphi_2 = -2\begin{pmatrix} 34.0036T_{\text{inGE}} - 6.5302T_{\text{inGE-AB}} \\ -32.6466T_{\text{inAB-GE}} + 19.2428T_{\text{outGE-AB}} \\ +30.2894T_{\text{inEV}} - 33.2274 \end{pmatrix}$$

$$\varphi_3 = -2\begin{pmatrix} 0.2642T_{\text{inGE}} + 5.8341T_{\text{inGE-AB}} \\ +4.8368T_{\text{inAB-GE}} - 21.4879T_{\text{outGE-AB}} \\ +11.6259T_{\text{inEV}} + 1.2319 \end{pmatrix}$$

and

$$\varphi_4 = -2\begin{pmatrix} -3.7847T_{\text{inGE}} + 5.6966T_{\text{inGE-AB}} \\ +2.8396T_{\text{inAB-GE}} - 8.8353T_{\text{outGE-AB}} \\ -9.8910T_{\text{inEV}} + 9.7929 \end{pmatrix}$$

3.3. ANN model with four neurons in the input layer

Eq. (4) considers $T_{\text{inGE-AB}}$, $T_{\text{inAB-GE}}$, $T_{\text{outGE-AB}}$ and P_{AB} as variables in the input layer of ANN.

$$\text{COP}_{\text{sim}} = 2\left[\frac{0.5677}{1+e^{\varphi_1}} + \frac{3.0483}{1+e^{\varphi_2}} - \frac{2.6140}{1+e^{\varphi_3}} - \frac{3.0089}{1+e^{\varphi_4}}\right] - \left(0.5677 + 3.0483 - 2.6140 - 3.0089\right) - 1.8245 \quad (4)$$

where

$$\varphi_1 = -2\begin{pmatrix} 6.3642T_{\text{inGE-AB}} - 0.8439T_{\text{inAB-GE}} \\ +1.5006T_{\text{outGE-AB}} - 19.9528P_{\text{AB}} + 2.3051 \end{pmatrix}$$

$$\varphi_2 = -2\begin{pmatrix} 0.1968T_{\text{inGE-AB}} + 2.0206T_{\text{inAB-GE}} \\ -1.6507T_{\text{outGE-AB}} - 7.3422P_{\text{AB}} + 1.7520 \end{pmatrix}$$

$$\varphi_3 = -2\begin{pmatrix} 1.1529T_{\text{inGE-AB}} + 2.0711T_{\text{inAB-GE}} \\ -1.5560T_{\text{outGE-AB}} - 10.2042P_{\text{AB}} + 2.1190 \end{pmatrix}$$

and

$$\varphi_4 = -2\begin{pmatrix} -0.2428T_{\text{inGE-AB}} + 1.5574T_{\text{inAB-GE}} \\ -1.5029T_{\text{outGE-AB}} - 3.4281P_{\text{AB}} + 0.1387 \end{pmatrix}$$

3.4. ANN model with three neurons in the input layer

Eq. (5) predicts the COP as a function of T_{inGE}, T_{inEV} and P_{AB}.

$$\text{COP}_{\text{sim}} = 2\left[\frac{-0.1644}{1+e^{\varphi_1}} - \frac{0.1659}{1+e^{\varphi_2}} - \frac{0.1232}{1+e^{\varphi_3}} - \frac{0.0186}{1+e^{\varphi_4}}\right] - \left(-0.1644 - 0.1659 - 0.1232 - 0.0186\right) + 0.2456 \quad (5)$$

where

$$\varphi_1 = -2\begin{pmatrix} 7.2530T_{\text{inGE}} + 18.5387T_{\text{inEV}} \\ -5.6165P_{\text{AB}} - 18.1581 \end{pmatrix}$$

$$\varphi_2 = -2\begin{pmatrix} -11.2980T_{\text{inGE}} - 45.0073T_{\text{inEV}} \\ -22.2448P_{\text{AB}} + 48.3099 \end{pmatrix}$$

$$\varphi_3 = -2\begin{pmatrix} 5.4868T_{\text{inGE}} + 61.0514T_{\text{inEV}} \\ +64.6052P_{\text{AB}} - 64.4270 \end{pmatrix}$$

and

$$\varphi_4 = -2\begin{pmatrix} -10.2878T_{\text{inGE}} - 30.3397T_{\text{inEV}} \\ -22.7721P_{\text{AB}} - 33.3396 \end{pmatrix}$$

It is important to note that, the input operation variables were normalized before being incorporated into the models. Table 3 shows some experimental information used in this study on a COP_{exp} range from 0.1200 to 0.3321.

4. Uncertainty analysis and Monte Carlo method

This section describes briefly the uncertainty analysis and Monte Carlo method used in this work. Suppose that it has *n* variables X_1, X_2, …, X_n, which could be represented pressure, temperature, concentrations, etc., and also has a function based on them, that is,

$$Z = f\left(X_1, X_2, \ldots, X_n\right)$$

In general, it has values of experimental measurements (denoted as $x_i \pm \epsilon_i$) for each variable X_i, $i = 1,2,\ldots,n$. These experimental measurements present uncertainties ϵ_i due to measurement limitations (e.g., instrument precision) which propagate to the combination of variables in the function *Z*. The uncertainty analysis (also known as a propagation of

Table 3
Experimental information used in this study on a COP_{exp}

T_{inGE} (K)	$T_{\text{inGE-AB}}$ (K)	$T_{\text{inAB-GE}}$ (K)	$T_{\text{outGE-AB}}$ (K)	T_{inEV} (K)	P_{AB} (Pa)	COP_{exp}
343.3364	339.7128	338.4872	339.7274	344.232	75,719.6804	0.1200
356.4614	346.7049	344.8414	351.5564	357.1183	52,048.8143	0.1722
356.5211	344.4626	346.0458	351.1502	355.5401	58,787.7304	0.2306
357.3365	346.6257	349.2873	353.5942	355.1991	60,480.9254	0.3321

uncertainty or error propagation) consists in analyzing the effect of variables uncertainties, $x_i \pm \in_i$ on the function Z. That is, the uncertainty analysis allows to determine the uncertainty of $Z, \in$, if the uncertainties, $\in_1, \in_2, ..., \in_n$ are known. Thus, with the uncertainty analysis, it is possible to derive confidence limits to describe the region within which the true value Z may be found. That is,

$$z - \in \leq Z \leq z + \in \tag{6}$$

where

$$z \pm \in = f\left(X_1 \pm \in_1, X_2 \pm \in_2, \ldots, X_n \pm \in_n\right)$$

To derive confidence limits (6) of the true value Z is necessary to know the probability distribution of the variables $X_1, X_2, \ldots, X_n$ or assumed. In our study, setting it assumes that each uncertainty $\in_1, \in_2, ..., \in_n$ have Gaussian probability distribution with mean 0 and standard deviation σ_i, that is,

$$\in_i \sim N\left(0, \sigma_i\right) \text{ for all } i = 1, 2, \ldots, n \tag{7}$$

The uncertainty, $\in_i$, on a measure of the variable X_i (i.e., on x_i), will be quantified regarding the relative standard deviation (RSD) which is defined as:

$$\%\text{RSD} = \frac{\sigma_i}{\bar{X}_i} \times 100 \tag{8}$$

where σ_i and $\bar{X}_i$ are the standard deviation and the mean of the variable X_i, respectively. So, from Eq. (8) it obtains:

$$\sigma_i = \frac{\left(\%\text{RSD}\right)\bar{X}_i}{100} \tag{9}$$

Let $\text{RSD}_{\text{instrument}}$ be the RSD of the measuring instrument and let $\bar{X}_i$ the mean of the experimental measurements of the variable X_i, then considering in Eq. (9), RSD = $\text{RSD}_{\text{instrument}}$ and $\bar{X}_i = X_i$, it has,

$$\sigma_i \approx \frac{\left(\text{RSD}_{\text{instrument}}\right)\bar{X}_i}{100} \tag{10}$$

In this manner, our study, it is considered $X_i \approx x_i + \in_i$ with x_i the experimental measurement, $\in_i$ as in Eq. (7) with σ_i given in Eq. (10). The uncertainties, $\in_i$, it propagates on Z with the Monte Carlo method.

Remark: Note that the probabilities, $P\left(\in_i = a\right) = P\left(X_i - x_i = a\right) = P\left(X_i = x_i + a\right)$, implies that if the probability of distribution of $\in_i$ is known, then the probability of distribution of the true value of X_i is also known.

The Monte Carlo method is developed and used in this work with the aim to propagate the error of the input variables in the ANNs models, one of the advantages of this method is to be able to show the type of distribution of the dependent variable because of adding a certain level of uncertainty in the independent variables and evaluating a mathematical function. Rees [3] and Anderson [4] were pioneers in the Monte Carlo method, and it is described in the following steps:

- For each independent variable, a certain level of uncertainty was added assuming a distribution.
- For the model selected, in this study (Eqs. (2)–(5)), the Monte Carlo method is based on repeated calculations of COP_{sim}, changing input data every time by a random selection from its error probability distribution.
- The predictions are collected and the form of the distribution is observed. In this research, the mean of the distribution was named as $\overline{\text{COP}}^{M}_{\text{sim}}$.

For this research, the ANNs models described in the previous section are evaluated under uncertainty using the Monte Carlo method. The independent variables of these models (Eqs. (2)–(5)) are operation variables measured from heat transformer temperature and pressure. Fig. 1 shows a schematic diagram of the union of experimental heat transformer, Monte Carlo method and ANN model.

The uncertain limits of instrumentation in the heat transformer can influence the accuracy of the COP prediction by

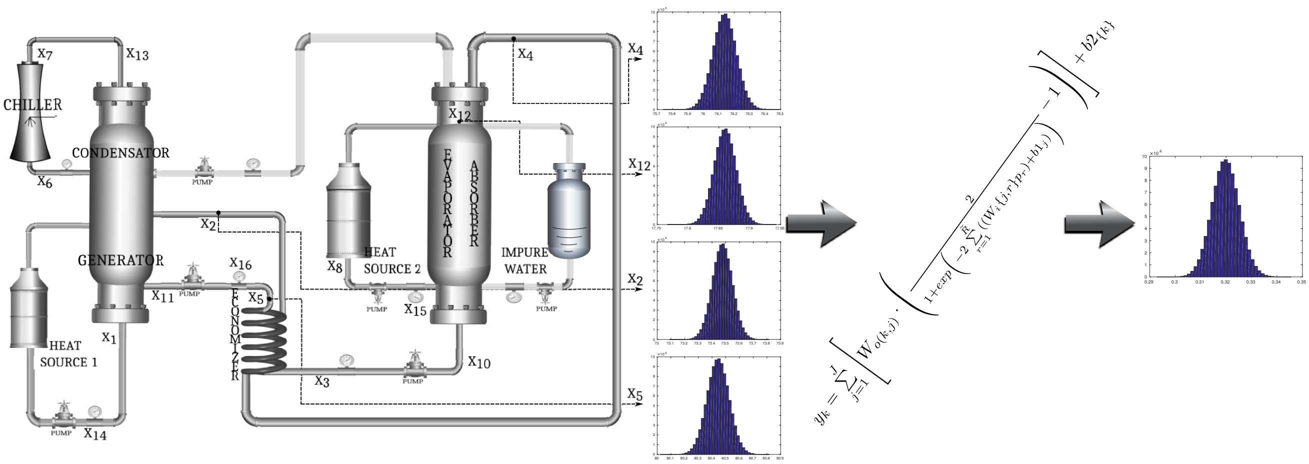

Fig. 1. Schematic representation of the system used to analyze uncertainties on artificial neural network model by the Monte Carlo method.

ANN model. This work proposed two cases for instrumental measurement errors ($RSD_{instrument}$):

Case 1: The use of high precision equipment to measure the operation variables (%RSD = 0.1).

Case 2: Increases in the uncertainty of input operation variables for a %RSD from 0.1 to 1 keep constant the COP_{exp}.

5. Numerical results

In this section, the main contributions of this work are presented. First, case 1 assuming a $\%RSD_{instrument} = 0.1$ and second the case 2 evaluating $\%RSD_{instrument}$ in the interval [0.1, 1] weep constant the COP_{exp} to 0.33.

Case 1. The use of high precision equipment to measure the operation variables: in this work, we are interested in observing changes in the form of probabilistic distribution of COP, for this reason, numerical results assuming a Monte Carlo method with 1,000,000 random number will be presented. Four levels of COP were selected with the aim of covering all the experimental conditions, COP from 0.12 to 0.33 was evaluated (Table 3).

For this case, the use of high precision equipment for temperature and pressure measures was developed. This case was added considering a %RSD = 0.1 and normal probabilistic distribution in the input operation variables for each ANN model.

The ANNs models presented in previous sections, Eqs. (2)–(5), were evaluated with Monte Carlo method with the aim of finding the best model from uncertainty propagation.

The ANN with six input variables, T_{inGE}, $T_{inGE-AB}$, $T_{inAB-GE}$, $T_{outGE-AB}$, T_{inEV} and P_{AB}, to estimate the COP under uncertainty was evaluated numerically, the results are shown in Fig. 2. Following with the numerical results, Table 4 shows the standard deviation calculated σ_{COP} for each COP_{exp} condition, it increases from 0.0001 to 0.0069 when the COP_{exp} was increased from 0.1200 to 0.3321. The discrepancy between the COP_{exp} and the mean of the probabilistic distribution were less than 4.6%.

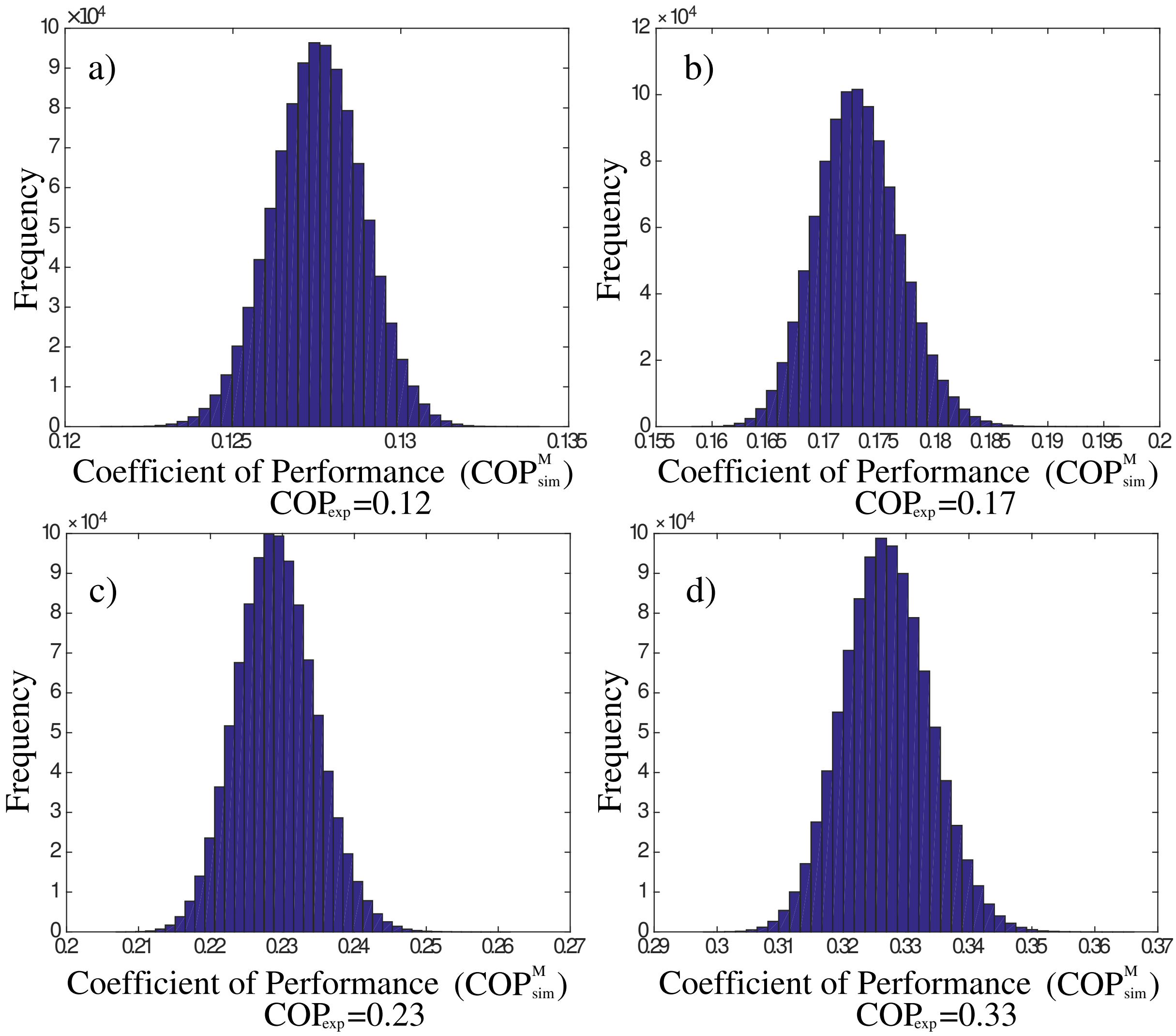

Fig. 2. Probabilistic distribution generated with Monte Carlo method and the ANN model with six input neurons for (a) COP_{exp} = 0.12, (b) COP_{exp} = 0.17, (c) COP_{exp} = 0.23 and (d) COP_{exp} = 0.33.

The ANN model considering five input operation variables, T_{inEV}, T_{inGE}, $T_{inGE\text{-}AB}$, $T_{inAB\text{-}GE}$ and $T_{outGE\text{-}AB}$, to predict the COP was tested with Monte Carlo method, the numerical results are illustrated in Fig. 3. It is interesting to note that, the probabilistic distribution for experimental conditions from COP = 0.12 to 0.33 were practically normal. The maximum absolute error between the experimental and the mean of the probabilistic distribution of the COP was calculated for each case as lesser than 6%. In this case, the standard deviation increases when the COP_{exp} was increased, as shown in Table 5.

The following ANN model considers four input variables, $T_{inGE\text{-}AB}$, $T_{inAB\text{-}GE}$, $T_{outGE\text{-}AB}$ and P_{AB}, in ANN model and

Table 4
Numerical results for Monte Carlo method, n = 1,000,000 and artificial neural network model with six neurons in the input layer

COP_{exp}	COP_{sim} Eq. (2)	$\overline{COP}_{sim}^{M}$	$\frac{\left\lvert COP_{exp} - \overline{COP}_{sim}^{M} \right\rvert}{COP_{exp}} \times 100$	σ_{COP}
0.1200	0.1254	0.1254	4.50	0.0001
0.1722	0.1662	0.1731	0.52	0.0037
0.2306	0.2389	0.2292	0.60	0.0057
0.3321	0.3098	0.3269	1.56	0.0069

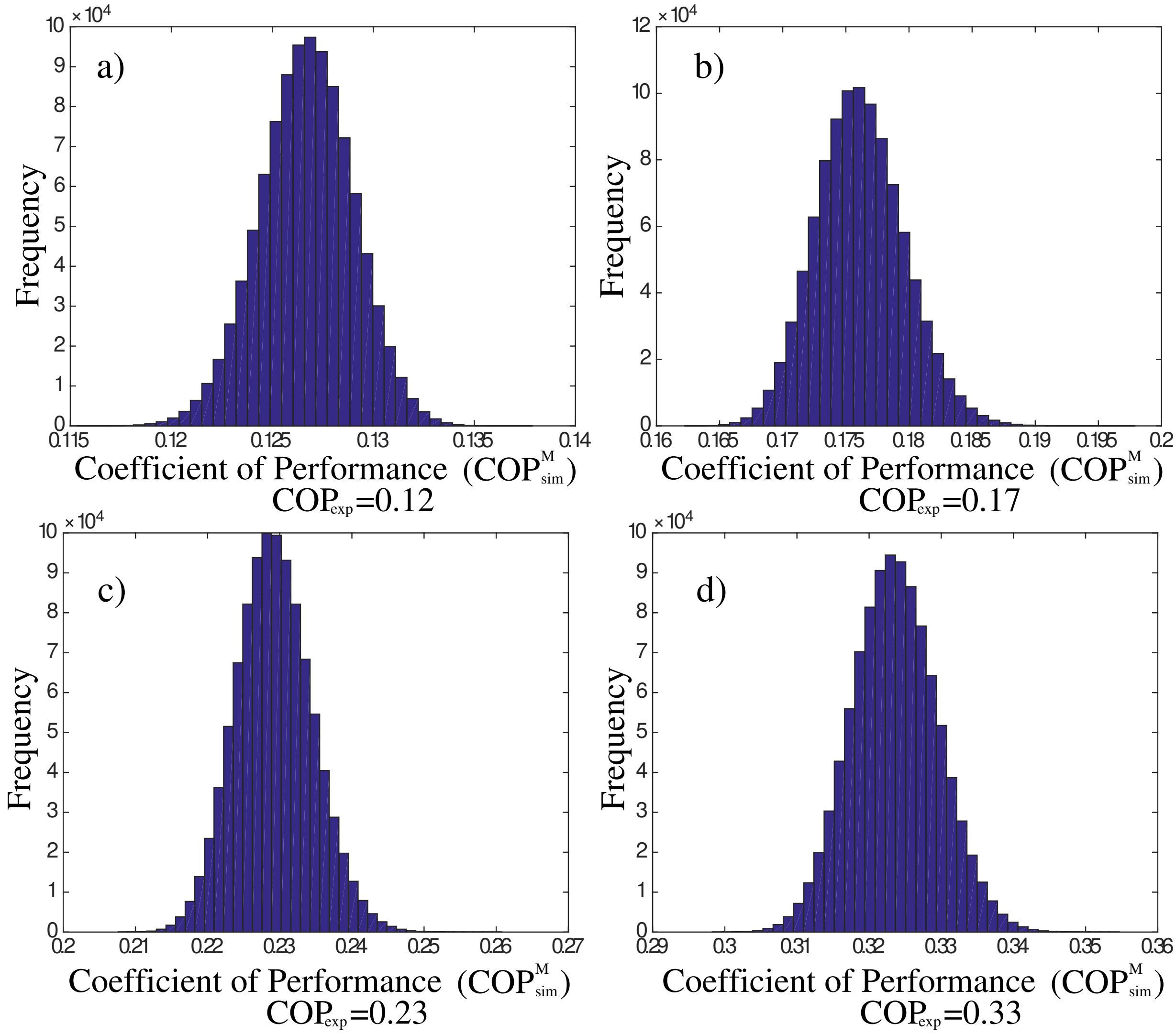

Fig. 3. Probabilistic distribution generated with Monte Carlo method and the ANN model with five input neurons for (a) COP_{exp} = 0.12, (b) COP_{exp} = 0.17, (c) COP_{exp} = 0.23 and (d) COP_{exp} = 0.33.

it is presented in Eq. (5). Fig. 4 shows the numerical results of Monte Carlo method, normal distribution was observed assuming experimental conditions to estimate the COP from 0.12 to 0.33. Table 6 showed the $\overline{COP}_{sim}^{M}$ and standard deviation σ_{COP} calculated from each probabilistic distribution for each COP_{exp}. Table 6 shows the maximum absolute error between the experimental and the mean of the probabilistic distribution of the COP was calculated for each case as lesser than 7.7%.

The model that involves three operation variables, T_{inGE}, T_{inEV} and P_{AB}, in the input layer was evaluated considering four neurons in the hidden layer to predict the COP.

Table 5
Numerical results for Monte Carlo method, n = 1,000,000 and artificial neural network model with five neurons in the input layer

COP_{exp}	COP_{sim} Eq. (2)	$\overline{COP}_{sim}^{M}$	$\frac{\left\lvert COP_{exp} - \overline{COP}_{sim}^{M} \right\rvert}{COP_{exp}} \times 100$	σ_{COP}
0.1200	0.1273	0.1272	6.00	0.0003
0.1722	0.1697	0.1761	2.26	0.0035
0.2306	0.2387	0.2293	0.56	0.0053
0.3321	0.3087	0.3236	2.56	0.0059

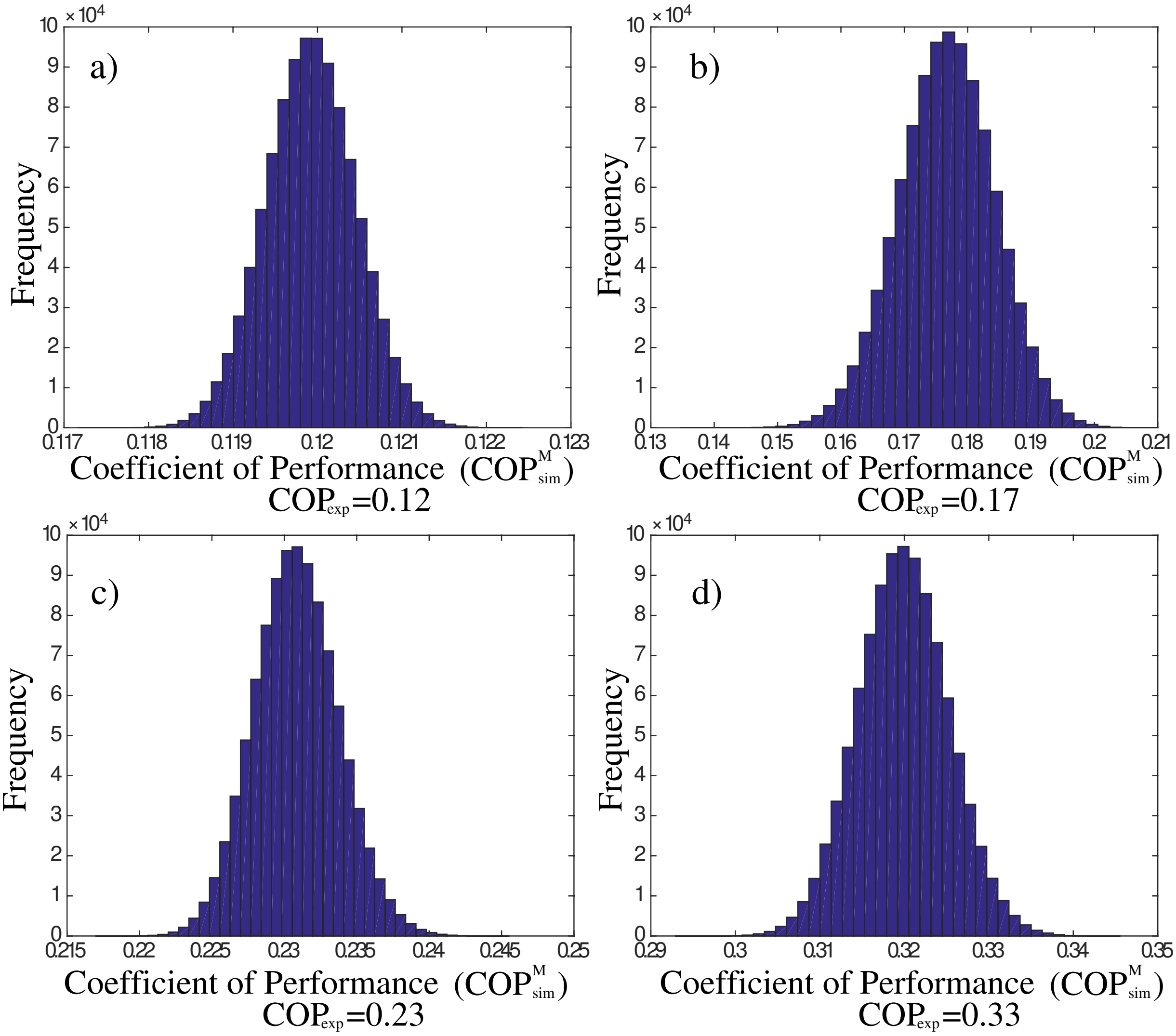

Fig. 4. Probabilistic distribution generated with Monte Carlo method and the ANN model with four input neurons for (a) COP_{exp} = 0.12, (b) COP_{exp} = 0.17, (c) COP_{exp} = 0.23 and (d) COP_{exp} = 0.33.

Table 6
Numerical results for Monte Carlo method, n = 1,000,000 and artificial neural network model with four neurons in the input layer

COP_{exp}	COP_{sim} Eq. (2)	$\overline{COP}_{sim}^{M}$	$\frac{\left\lvert COP_{exp} - \overline{COP}_{sim}^{M} \right\rvert}{COP_{exp}} \times 100$	σ_{COP}
0.1200	0.1288	0.1292	7.67	0.0007
0.1722	0.1911	0.1767	2.61	0.0076
0.2306	0.2259	0.2308	0.08	0.0029
0.3321	0.3340	0.3199	3.67	0.0054

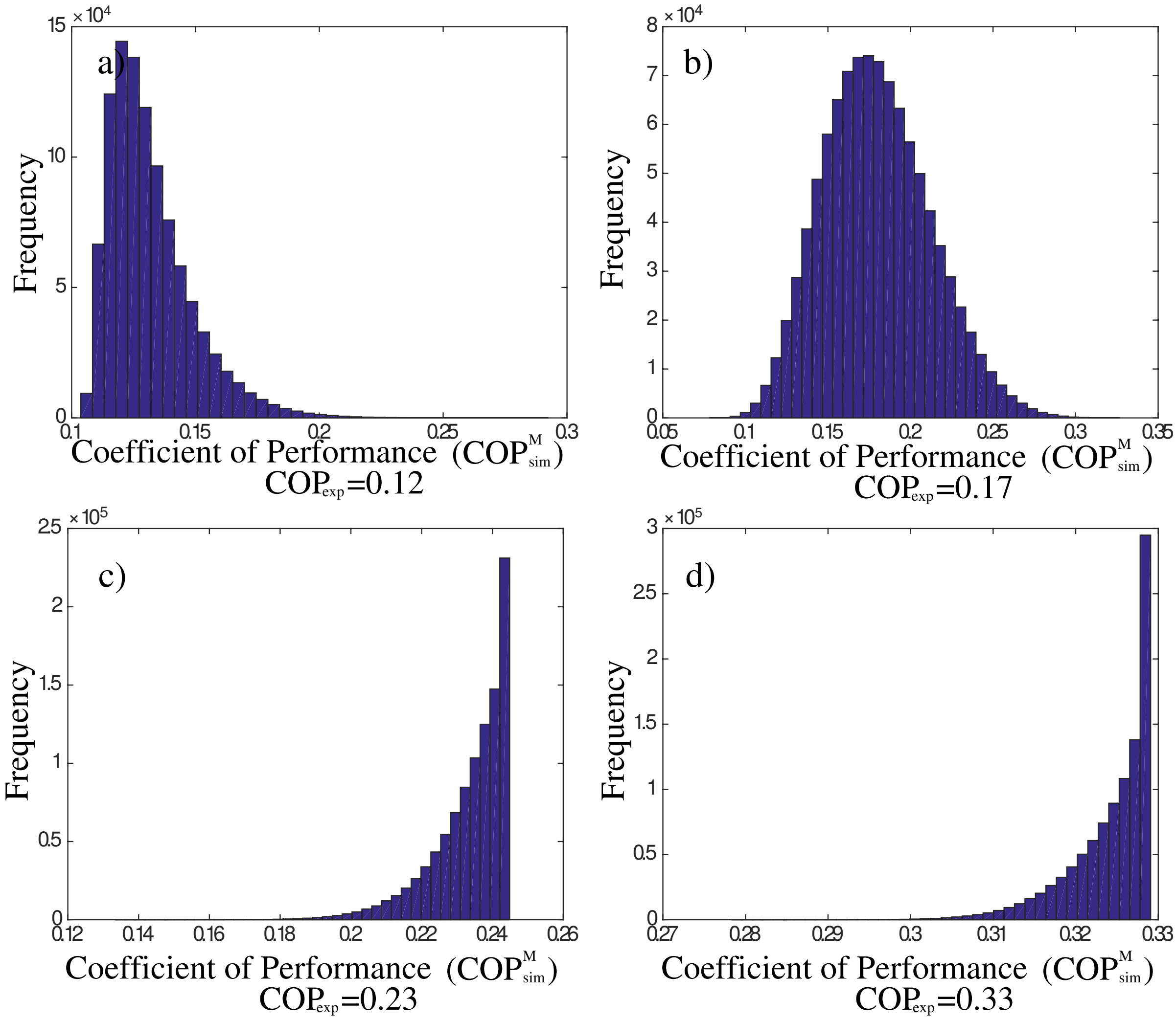

Fig. 5. Probabilistic distribution generated with Monte Carlo method and the ANN model with three input neurons for (a) COP_{exp} = 0.12, (b) COP_{exp} = 0.17, (c) COP_{exp} = 0.23 and (d) COP_{exp} = 0.33.

Fig. 5 shows the probabilistic distribution obtained with the numerical model assuming experimental conditions to estimate the COP from 0.12 to 0.33. As can be seen that, there is no normal shape in the distribution for COP_{sim} equal to 0.12, 0.23 and 0.33, this causes the mean value calculated from the distribution does not match with the prediction of ANN. Normal distribution was observed for experimental conditions of COP = 0.17. Table 7 shows that the maximum discrepancy between COP_{exp} and $\overline{COP}_{sim}^{M}$ was equal to 9.75%.

Case 2. Increases the uncertainty of input operation variables for a %RSD from 0.1 to 1 keep constant the COP_{exp} = 0.33: the %RSD was increased from 0.1 to 1 keeping fixed the COP to 0.33. Figs. 6–8 show the numerical results of probabilistic distribution with the ANN with six, five and four input operation

Table 7
Numerical results for Monte Carlo method, n = 1,000,000 and artificial neural network model with three neurons in the input layer

COP_{exp}	COP_{sim} Eq. (2)	$\overline{COP}^{M}_{sim}$	$\frac{\lvert COP_{exp} - \overline{COP}^{M}_{sim}\rvert}{COP_{exp}} \times 100$	σ_{COP}
0.1200	0.1379	0.1317	9.75	0.0168
0.1722	0.2468	0.1795	4.24	0.0324
0.2306	0.2104	0.2336	1.30	0.0107
0.3321	0.3047	0.3242	2.38	0.0050

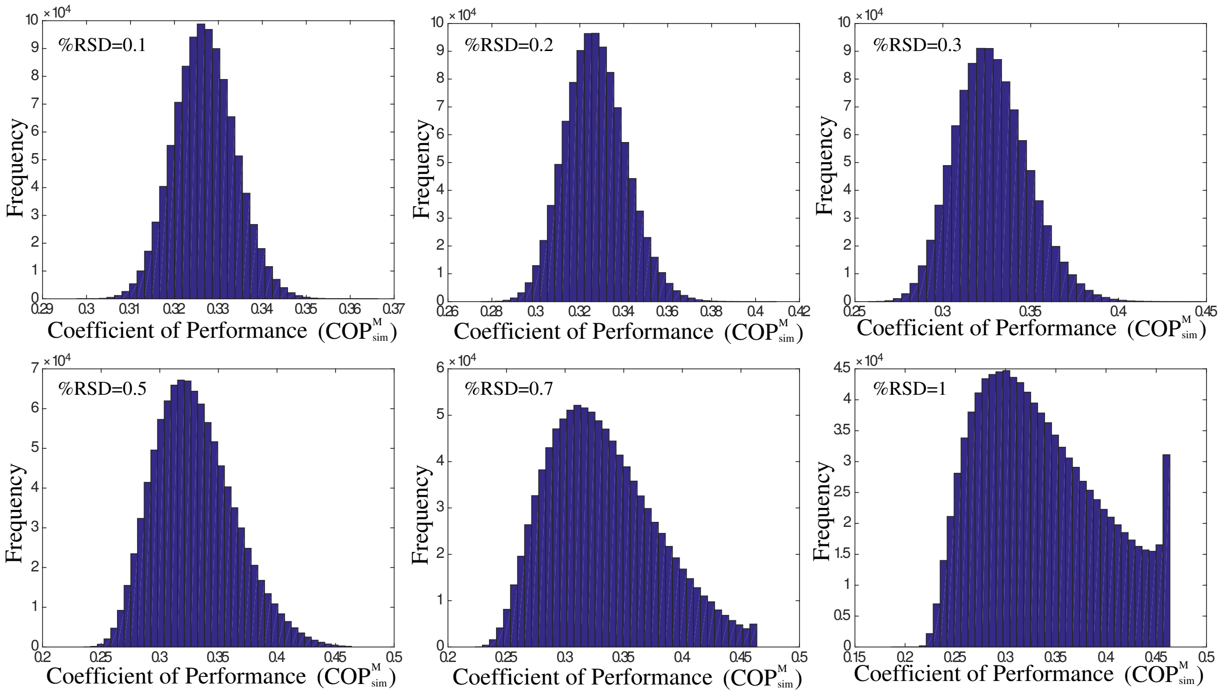

Fig. 6. Probabilistic distribution generated with Monte Carlo method and the ANN model with six input neurons keeping fixed the COP_{exp} = 0.33 and increases the %RSD.

variables, respectively. As can be seen, normal distribution was observed for the neural network model with four input operation variables considering the input layer of %RSD from 0.1% to 1%. The models with five and six neurons in the input layer did not present a normal distribution for the COP prediction considering %RSD equal to or greater than 0.3.

5.1. Analysis of numerical results

According to section 3, the following relation can be expressed:

$$COP_{exp} \approx ANN(x_1, x_2, \ldots, x_n)$$

Let x_k^i be the ith value of x_k variable, with k = 1,2,…,n, then the ith experimental coefficient of performance COP^i_{exp} associated with the variables $x_1^i, x_2^i, \ldots, x_n^i$ can be expressed as:

$$COP^i_{exp} \approx ANN\left(x_1^i, x_2^i, \ldots, x_n^i\right)$$

In the uncertainty analysis x_k^i is replaced for $x_k^i \pm \varepsilon_k$, then,

$$COP^i_{exp} \pm \varepsilon^i \approx ANN\left(x_1^i + \varepsilon_1, x_2^i + \varepsilon_2, \ldots, x_n^i + \varepsilon_n\right) \quad (11)$$

where ε^i is a random error.

In Eq. (11), the random errors ε_k are taken as 1,000,000 values, consequently several coefficients of performance values can be obtained which are denoted as COP^{M}_{sim}. Thus, Eq. (11) can be rewritten as:

$$COP^i_{exp} \pm \varepsilon \approx COP^{M}_{sim} \qquad \varepsilon = \left(\varepsilon_k^i, k = 1, \ldots, 1{,}000{,}000\right) \quad (12)$$

Taking the means E of both side of Eq. (12), it obtains:

$$COP^i_{exp} \pm E[\varepsilon] \approx E\left[COP^{M}_{sim}\right]$$

implying:

$$E[\epsilon] = E\left[COP^{M}_{sim}\right] - COP^i_{exp} \quad (13)$$

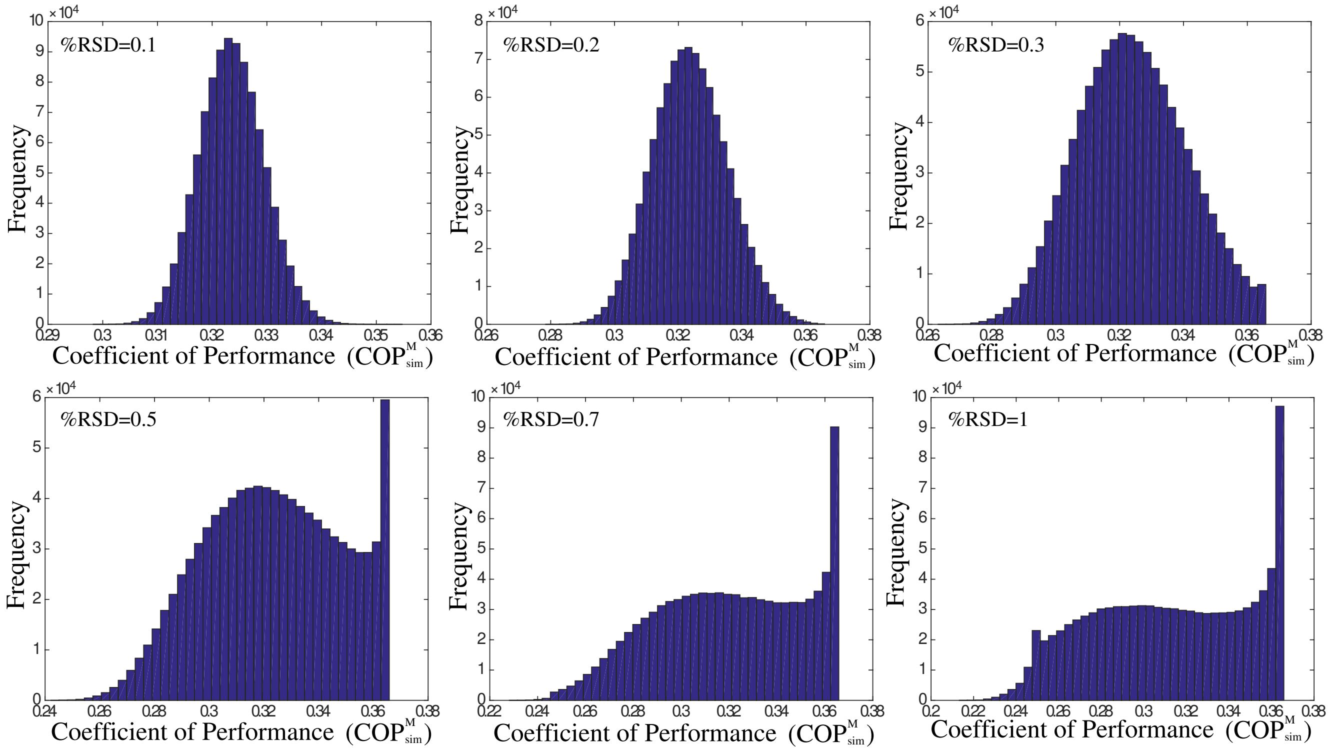

Fig. 7. Probabilistic distribution generated with Monte Carlo method and the ANN model with five input neurons keeping fixed the COP_{exp} = 0.33 and increases the %RSD.

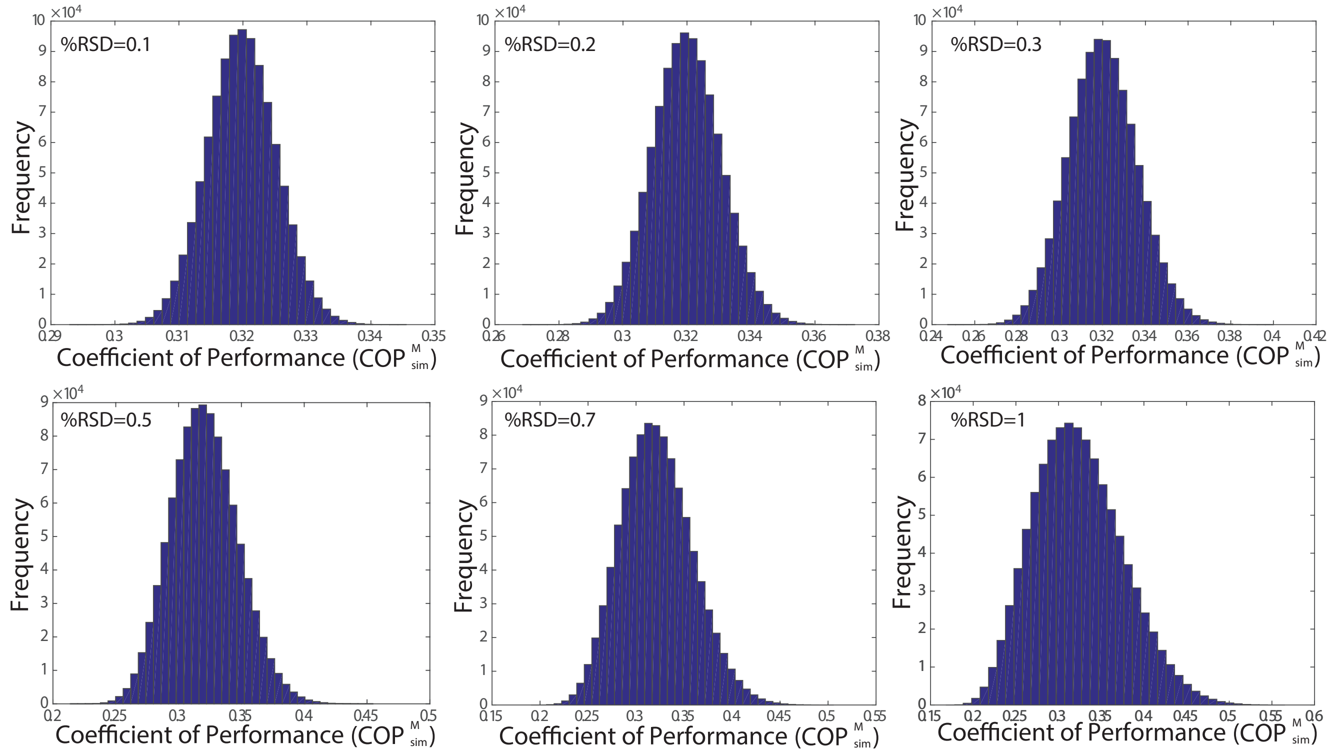

Fig. 8. Probabilistic distribution generated with Monte Carlo method and the ANN model with four input neurons keeping fixed the COP_{exp} = 0.33 and increases the %RSD.

Table 8
Numerical results of uncertainty analysis for the ANN model with four operation variables in the input layer

%RSD	COP_{exp}	$\overline{COP}_{sim}^{M}$	$\frac{\left\lvert COP_{exp} - \overline{COP}_{sim}^{M} \right\rvert}{COP_{exp}} \times 100$	σ_{COP}	$\frac{\sigma_{COP}}{\overline{COP}_{sim}^{M}} \times 100$
0.1	0.3321	0.3199	3.67	0.0054	1.69
0.2	0.3321	0.3200	3.64	0.0107	3.34
0.3	0.3321	0.3202	3.58	0.0161	5.02
0.5	0.3321	0.3208	3.40	0.0266	8.29
0.7	0.3321	0.3217	3.13	0.0370	11.50
1.0	0.3321	0.3234	2.62	0.0520	16.08

If ε letting to zero, which is the same as $E[\in]$ closer to zero, then the last equality

$$E\left[COP_{sim}^{M}\right] \approx COP_{exp}^{i} \tag{14}$$

which would imply that the $ANN\left(x_1^i + \varepsilon_1, x_2^i + \varepsilon_2, \ldots, x_n^i + \varepsilon_n\right)$ model is accurate, due to the structure of the ANN model, when the uncertainty was added, it obtains a prediction close to the COP_{exp}^{i}.

In this work, the absolute relative error $\frac{\left\lvert COP_{exp} - E\left[COP_{sim}^{M}\right]\right\rvert}{COP_{exp}} \times 100$ is calculated as a measure to show the equality (13) explained above.

In accordance with the previously explained, for the numerical results of case 1 in section 5, the ANNs with four, five and six operation variables in the input layer of architecture (see ANN models Eqs. (2)–(4)) have absolute small relative errors (Tables 4–6) satisfying the approximate (14). Therefore, these ANN models are the most appropriate for the prediction of the COP system studied.

Case 2 made the finest analysis of uncertainty because increases the uncertainty of input operation variables for a %RSD from 0.1 to 1 keep constant the COP_{exp} = 0.33. Based on the numerical results of case 2, Figs. 6–8, the ANN models with five and six input variables present highest absolute relative errors to the ANN model with four input variables. Table 8 showed the numerical results when %RSD was increased from 0.1% to 1% in the input operation variables ($T_{inGE\text{-}AB}$, $T_{inAB\text{-}GE}$, $T_{outGE\text{-}AB}$ and P_{AB}).

According to the analysis presented in the beginning of this section, it can be concluded that the ANN with four input operation variables satisfy the approximate (14) and consequently this model is the most accurate.

6. Conclusions

The main contributions of this work are:

- In this study, the ANN model with four input neurons in the input layer, $T_{inGE\text{-}AB}$, $T_{inAB\text{-}GE}$, $T_{outGE\text{-}AB}$ and P_{AB} it turned to be the most appropriate.
- The probabilistic distribution of the COP was showed and analyzed, normal distribution was observed with the ANN model with four input neurons in the input layer, $T_{inGE\text{-}AB}$, $T_{inAB\text{-}GE}$, $T_{outGE\text{-}AB}$ and P_{AB}. The other ANN models presented asymmetric distribution or excess of data in the right size when the %RSD was increased greater than 0.3.
- The ANN that considers $T_{inGE\text{-}AB}$, $T_{inAB\text{-}GE}$, $T_{outGE\text{-}AB}$ and P_{AB} in the input layer and assuming a %RSD from 0.1 to 1 presented the COP distribution as normal in the range of 0.12 to 0.33.
- The standard deviation for the ANN model with four input operation variables was calculated and shown in Table 8.

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