



## Rehabilitation of water distribution networks using particle swarm optimization

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### ABSTRACT

Urban centers are in constant growth, with increasing demand for potable water. This requires the rehabilitation of certain pipes that feed the water distribution network, allowing its expansion. In the present work, an optimization model was developed for the rehabilitation and expansion of water distribution networks (WDN), based on particle swarm optimization (PSO). The hydraulic simulator Epanet was used to calculate the velocities and nodal pressures. The decision variables (diameters) were treated as integer variables and the change in the internal roughness of the pipe due to use was considered. The problem has a mixed integer nonlinear programming (MINLP) formulation. Four case studies were used to test the applicability of the developed model. Results showed the efficiency of the proposed solution for the model and are coherent with previous published results in the literature.

*Keywords:* WDN; Rehabilitation; MINLP; PSO; Epanet

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### 1. Introduction

A water supply system should be able to transport potable water from a certain reservoir to the consumer, in a continuous way, in appropriated quantity, quality and pressure. Water should be conducted at full pipe cross-section and with a pressure greater than the atmospheric one. Water distribution networks (WDN) are part of water supply systems and consist of a piping network as well as devices such as pumps, valves, tanks, meters, among other accessories.

According to Swamee and Sharma [1], great investments are made throughout the world in order to provide or update water supply systems efficiently. Even so, a large part of the population has no access to safe, high-quality WDN. The WDN costs are between 80% and 85% of the total cost of the water supply system. Because of this, WDN design and optimization have been attracting many researchers.

Pipes, pipe junctions and hydraulic devices (tanks, valves, pumps, etc.) can be connected in several complex ways. A phenomenological model for a WDN is necessarily based on the law of conservation of mass and on the law of conservation of energy and is composed by linear and nonlinear equations. WDN optimization consists in finding optimal diameters that minimize the cost, keeping node pressure heads above the required minimum and following one or more demand patterns. The available diameters in the WDN can be arranged in a set and to each one of the diameters an integer number is associated. These integer values are decision variables in the optimization problem.

There are many studies on WDN optimization, using simultaneous or sequential, deterministic or heuristic methods. Among heuristic optimization methods, the PSO (Particle Swarm Optimization) method, introduced by Kennedy and Eberhart [2], is notable. It is a metaheuristic combinatorial optimization method belonging to a class of algorithms based on social behaviors in animal movement (birds, fishes, bees, etc.). The algorithm observes both group and individual behaviors. The term *particle* refers to each mem-

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ber of the group and the term *swarm* refers to the group as a whole. The PSO algorithm was initially introduced to work with continuous variables. In 1997, the au[3] introduced a PSO with discrete binary variables.

A particle can be considered as a vector with dimension  $M$  that represents the number of decision variables in the optimization problem. The coordinates of the particle represent the position in a given instant. Each particle has a speed that guides it to a position closer to an optimal one. For each position, the performance of each particle is evaluated. The best position occupied by a particle is named  $P_{best}$  and the best position achieved by any particle in the group is named  $G_{best}$ . The performance of each particle is evaluated in each new iteration, as well as the performance of the group, and vectors  $P_{best}$  and  $G_{best}$  are updated. The optimization process ends when the performance of  $G_{best}$  halts or when the maximum number of iterations is reached.

Some researchers used PSO in water networks optimization. Trigueros et al. [4] and Ravagnani et al. [5] proposed a model for the reuse water network optimization using PSO.

Specifically in WDN optimization, some important papers are presented below:

- Suribabu and Neelakantan [6] showed the rate of convergence of the PSO algorithm for the problems of the Two Loop network, proposed by Alperovits and Shamir [7], and the network of Hanoi [8], comparing the results with other algorithms, such as GA – genetic algorithms [9], SA – simulated annealing [10], SCE – shuffled complex evolution [11], and SFL – shuffled frog leaping [12].
- Montalvo et al. [13] optimized the Hanoi network and the New York City Water Supply Tunnels network [14], presenting a variant of the PSO algorithm with discrete variables.
- Ezzeldin et al. [15] presented another PSO variant, considering integer discrete variables, and proved the algorithm to be effective using as examples the Two Loop network [7] and the Two Source network [16].
- Qi et al. [17] used a PSO algorithm with EDA (estimation of distribution algorithm) to avoid premature convergences. This algorithm (PSO-EDA) was used for optimization of the networks in Hanoi [8] and Balerna [18].
- Surco et al. [17] presented a modification in the PSO algorithm to avoid particles being trapped in local minimum jointly with the leader particle ( $G_{best}$ ). If a particle is in the same position of  $G_{best}$ , the algorithm repositions the particle in a random way in the search space. In this way, it can find best promising solutions. Some benchmark problems were used to test the algorithm, as Two Loop network [7], Hanoi [8] network and the networks of Gomes et al. [20] and Balerna [18] and results were better or equal the literature ones.

According to Lansley and Mays [21], a water distribution network is dimensioned to serve the consumer for a long time. It is impossible to determine the number and kinds of consumers with precision. The dimensioning parameters of a WDN, such as the demands and minimum pressure heads

in the nodes, are uncertain. Another parameter to be considered is the capacity of the WDN, which is affected by pipe corrosion and accretion in the pipes. Roughness is affected mainly by the age of the pipe.

According to Suribabu and Neelakatan [22], the problems in WDN usually originate from one of three situations: when designing a new network, when modifying or expanding an already existing network or during network operation. Another consideration to be made is that the pipes in a WDN may deteriorate prematurely, losing their hydraulic capacity. Consequently, lower pressures appear in the nodes.

A network is said to need rehabilitation when the pressure heads in the demand nodes are lower than the network minimum pressure requirements. Rehabilitation consists in:

- Improving pipe roughness via cleaning or by coating it with another material.
- Adding parallel pipes or replacing old pipes with new ones for a specific use.
- Implementing new pumps and control tanks.

Gupta et al. [23] suggested that reliability and resilience criteria should be incorporated in the WDN optimization problem, either qualitatively or quantitatively. De Corte and Sörensen [24], on the other hand, suggested that the optimization problem should be treated as a multi-objective one (where other objectives such as reliability could be added to the cost minimization objective) or adding extra constraints, such as maximum flow velocity ones, to the problem.

## 2. Model for the optimization of water distribution networks

The water distribution network optimization problem can be formulated, in order to minimize installation costs, as a MINLP problem. If the network has  $M$  pipes and  $K$  nodes, its optimization consists in finding the diameters of the pipes that compose it.

In the present work, the set of available diameters to be used in a given network is  $D_{ALW} = \{D_1, D_2, \dots, D_{nd}\}$  where  $D_1 < D_2 < \dots < D_{nd}$ .  $D_1$  is named  $D_{min}$  and  $D_{nd}$  is named  $D_{max}$ . The set of costs of the respective diameters is  $Cost_1, Cost_2, \dots, Cost_{nd}$  in monetary units per meter ( $\$/m$ ).

Table 1 shows the formatting of the indexes of the available pipe diameters, as well as the respective properties of each pipe, such as diameter, unit cost and roughness, in the optimization of WDN.

The diameter ( $D_{i,j}$ ), corresponds to the piping  $j$  of particle  $i$ , is dependent of the index and is given by Eq. (1). The corresponding cost is given by Eq. (2).

$$D_{i,j} = \text{Diameter}(id_{i,j}) \quad (1)$$

$$Cost_{i,j} = \text{Cost}(id_{i,j}) \quad (2)$$

The objective function to be minimized is the total installation cost of the pipes  $C_{Ti}$  for solution  $i$ , given by Eq. (3). In this equation,  $L_j$  is the length of pipe  $j$ , and  $Cost(id_{i,j})$

Table 1  
Available diameters for the network and their properties

id no.	Diameter (mm)	Cost (\$/m)	Roughness coefficient (C or ε)*	Nominal diameter (inch or mm)
1	$D_1$	$Cost_1$	$R_1$	$DN_1$
2	$D_2$	$Cost_2$	$R_2$	$DN_2$
⋮	⋮	⋮	⋮	⋮
$nd$	$D_{nd}$	$Cost_{nd}$	$R_{nd}$	$DN_{nd}$

\*C = Hazen-Williams Coef., ε = Darcy-Weisbach Coef.

is the installation cost for pipe  $j$  per meter, whose diameter index is  $id_{ij} \in \{1, \dots, nd\}$ , with respective diameter  $D_{ij}$ .

$$Min C_{Ti} = \sum_{j=1}^M L_j Cost(id_{ij}) \tag{3}$$

In steady state, two fundamental laws of fluid mechanics must be considered:

- a) The continuity equation (law of conservation of mass) is in Eq. (4): the sum of the flow rates entering a certain node  $k$  must equal the sum of the flow rates exiting it:

$$\sum Q_{in}(k) - \sum Q_{out}(k) - Dmd(k) = 0, \tag{4}$$

where  $Q_{in}(k)$  and  $Q_{out}(k)$  represent, respectively, flow rates entering and exiting node  $k$ , and  $Dmd(k)$  is the demand in node  $k$ .

- b) Law of conservation of energy: the sum of head losses in the pipes forming a loop must equal zero. The head loss receives the same orientation as the flow. This law is represented by Eq. (5):

$$\sum h_j = 0, \quad \forall j \in loop\ set \tag{5}$$

where  $h_j$  is the head loss in pipe  $j$  belonging to a certain loop. The two laws mentioned, in electrical engineering are called the laws of Kirchhoff to solve problems of electrical circuits [25].

The head loss in pipe  $j$ ,  $h_j$ , may be calculated using the Hazen-Williams equation, Eq. (6), in the international system of units. In this equation  $Q_j$  is the flow rate in  $m^3/s$ ,  $C_j$  is the Hazen-Williams roughness coefficient,  $D_j$  is the diameter in meters and  $L_j$  is the length of pipe  $j$  in meters. The roughness coefficient (C) is a parameter related to the type of the material of manufacture of the pipe.

$$h_j = \frac{10,674 Q_j^{1.852}}{C_j^{1.852} D_j^{4.87}} L_j \tag{6}$$

Constraints c), d), and e) are minimum requirements to be imposed on a WDN:

- c) The minimum pressure heads in the nodes, adequate for consumers, are represented by inequality (7):

$$pr(k) \geq pr_{min}(k) \tag{7}$$

where  $pr(k)$  is the minimum pressure head in node  $k$ .

- d) In some WDN problems, (as in Case Study 4), the minimum and maximum flow velocities ( $v_l$ ) (in the pipes can be part of the problem constraints and can be represented by inequality (8):

$$v_{L_{min}} \leq |v_{L_j}| \leq v_{L_{max}} \tag{8}$$

where  $v_{L_{min}}$  and  $v_{L_{max}}$  are, respectively, the minimum and maximum velocities imposed to the WDN and  $v_{L_j}$  is the flow velocity in pipe  $j$ .

- e) The diameters to be used should belong to the set of available ones ( $D_{ALW}$ ), represented by Eq. (9):

$$D_j \in D_{ALW} = \{D_1, D_2, \dots, D_{nd}\} \tag{9}$$

where  $D_j$  is the diameter of pipe  $j$  and  $D_{ALW}$  is the set of available diameters to be used in the WDN.

Steady-state hydraulic variables are calculated using the software Epanet. Through this software, static and dynamic simulations of both the hydraulic behavior and the quality of the water in pressurized distribution networks can be carried out. It was developed by the United States Environmental Protection Agency (EPA), and allows the evaluation of the hydraulic variables in the nodes (pressure head) and pipes (flow velocity), as well as the pressure head in tanks at different heights and the concentration of chemical species in the network throughout the simulation period [26]. The EPA provides for free the Epanet Programmer's toolkit, specifically the dynamic-link library file *Epanet2.dll*, which allows developers to customize Epanet tools for their own needs.

### 3. PSO algorithm in the optimization of water distribution networks with rehabilitation and expansion

An algorithm based on PSO was developed in order to solve the problem of WDN optimization with rehabilitation and expansion. The use of the PSO algorithm with WDN, as proposed by Kennedy and Eberhart [2], can be carried out thusly: a particle (vector  $i$ ) is represented by  $X_i = (x_{i,1}, x_{i,2}, \dots, x_{i,j}, \dots, x_{i,M})$  where  $x_{i,j}$  is the diameter of pipe  $j$  in solution  $i$  and the  $M$  components are the decision variables of the problem to be optimized. Vector  $X_i$  is named current position of particle  $i$ . There is also the velocities vector  $V_i = (v_{i,1}, v_{i,2}, v_{i,3}, \dots, v_{i,M})$  as well as the vector of the best position ever achieved by particle  $i$  ( $Pbest$ ), represented by  $P_i = (p_{i,1}, p_{i,2}, p_{i,3}, \dots, p_{i,M})$ . The group of particles (*swarm*), comprising vectors  $X_i$ , forms matrix  $X$  of order  $N_p \times M$ , where  $N_p$  is the number of particles in the group. Each particle  $i$  moves itself within the search space with a certain velocity  $V_i$  in search

of a new time position. Matrices  $P$  and  $V$  of order  $N_p \times M$  are formed analogously.

In the context of WDN rehabilitation and expansion, two sets of pipes are considered: the existing pipes and the new pipes to be used.

If the network has  $M'$  existing pipes and  $M$  new pipes, then particle  $X_i$  will have  $(M'+M)$  components, and the group of the  $N_p$  particles will form matrix  $X$  of order  $N_p \times (M'+M)$ . Matrices  $P$  and  $V$  of order  $N_p \times (M'+M)$  are created likewise.

Afterward, position matrices  $X$ ,  $P$ , and  $V$  of the PSO algorithm should be reorganized. This process constitutes an adaptation of the PSO algorithm and is the great innovation proposed in the present work.

For the  $(M'+M)$  pipes, vectors are presented in Eq. (10).

$$X_i^A = (x_1^E, x_2^E, \dots, x_{M'}^E, x_{i,M'+1}, x_{i,M'+2}, \dots, x_{i,M'+M}) \quad (10)$$

where the components  $x_1^E, x_2^E, \dots, x_{M'}^E$  showed in Eq. (11), corresponds to  $M'$  existing pipes in the WDN, with known diameters.

$$X_s^E = (x_1^E, x_2^E, \dots, x_{M'}^E) \quad (11)$$

The new pipes in the WDN are represented by vector  $X_i$ , presented in Eq. (12).

$$X_i = (x_{i,M'+1}, x_{i,M'+2}, \dots, x_{i,M'+M}) \quad (12)$$

where  $x_{i,j}$  represents the diameter index of pipe  $j$  belonging to particle  $i$ . This representation is done analogously for vectors  $P_i^A$  and  $V_i^A$ .

The  $N_p$  particles form matrix  $X^A$ , which comprises all existing and new pipes. Consequently, matrix presents two submatrices, as shown in Eq. (13).

$$X^A = \begin{bmatrix} x_1^E x_2^E \dots x_{M'}^E & x_{1,M'+1} x_{1,M'+2} \dots x_{1,M'+M} \\ x_1^E x_2^E \dots x_{M'}^E & x_{2,M'+1} x_{2,M'+2} \dots x_{2,M'+M} \\ \vdots & \vdots \\ x_1^E x_2^E \dots x_{M'}^E & x_{i,M'+1} x_{i,M'+2} \dots x_{i,M'+M} \\ \vdots & \vdots \\ x_1^E x_2^E \dots x_{M'}^E & x_{N_p,M'+1} x_{N_p,M'+2} \dots x_{N_p,M'+M} \end{bmatrix} \quad (13)$$

Analogously, matrix  $P^A$  also presents two submatrices, as shown in Eq. (14).

$$P^A = \begin{bmatrix} x_1^E x_2^E \dots x_{M'}^E & p_{1,M'+1} p_{1,M'+2} \dots p_{1,M'+M} \\ x_1^E x_2^E \dots x_{M'}^E & p_{2,M'+1} p_{2,M'+2} \dots p_{2,M'+M} \\ \vdots & \vdots \\ x_1^E x_2^E \dots x_{M'}^E & p_{i,M'+1} p_{i,M'+2} \dots p_{i,M'+M} \\ \vdots & \vdots \\ x_1^E x_2^E \dots x_{M'}^E & p_{N_p,M'+1} p_{N_p,M'+2} \dots p_{N_p,M'+M} \end{bmatrix} \quad (14)$$

Eqs. (13) and (14) evidently show that the elements belonging to existing pipes must be constant. Consequently, the components of the velocities belonging to the existing pipes must equal zero. The velocity matrix is as shown in Eq. (15).

$$V^A = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & v_{1,M'+1} v_{1,M'+2} \dots v_{1,M'+M} \\ 0 & 0 & 0 & \dots & 0 & v_{2,M'+1} v_{2,M'+2} \dots v_{2,M'+M} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & v_{i,M'+1} v_{i,M'+2} \dots v_{i,M'+M} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & v_{N_p,M'+1} v_{N_p,M'+2} \dots v_{N_p,M'+M} \end{bmatrix} \quad (15)$$

Therefore, matrices  $X^A$ ,  $P^A$ , and  $V^A$  are formed by two submatrices, as shown in Eq. (16).

$$X^A = [X^E | X], \quad P^A = [P^E | P], \quad V^A = [V^E | V] \quad (16)$$

Submatrices  $X$ ,  $P$ , and  $V$ , representing the new pipes, are interesting for the modified PSO algorithm and are subject to optimization.

Submatrices  $X$  and  $P$  comprise the diameter indexes belonging to the set of available diameters ( $D_{ALW}$ ) for the network, which correspond to integers following Table 1, where each diameter has an index, a roughness coefficient, and an installation cost ( $\$/m$ ).

Working with every  $X^A$  and  $P^A$  elements is not necessary. The reduction of the size of the matrices also reduces the number of operations, the computation time, and the necessary memory to solve the operations.

The diameters of the existing pipes do not undergo any variations in the optimization process and are gathered in a single vector named  $X_s^E$ , whose component  $X_1^E$  corresponds to the diameter of existing pipe 1, which has its own roughness coefficient, presenting no installation cost.

The particles are initialized randomly, using the diameter indexes shown in Table 1, between limits 1 ( $D_{min}$ ) and  $nd$  ( $D_{max}$ ), being  $r$  a random number with uniform distribution in the interval  $[0, 1]$ , as shown in Eq. (17).

$$x_{i,j} = \text{Round}[(1 + r.(nd - 1)), 0] \quad (17)$$

where function *Round* approximates the value to a certain number of decimal places. When choosing zero decimal places, the diameter index is obtained, as shown in Fig. 1.

Iteration velocity  $v_{i,j}$  is an integer, according to Eq. (18). The velocity usually initializes at zero (starts at rest), but it

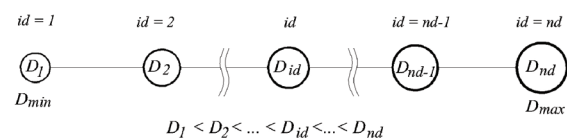


Fig. 1. Indexes of available diameters for the WDN.



can be a random value between the predetermined limits  $V_{PSOmin}$  and  $V_{PSOmax}$ . The component  $v_{i,j}$  of velocity vector  $V_i$  is described in Eq. (18), where the orientation is given by the components of vectors  $Pbest$  and  $Gbest$ .

$$v_{i,j}(t+1) = \text{Round}\left[\left(v_{i,j}(t) \cdot w + c_1 \cdot r_1 (p_{i,j} - x_{i,j}(t)) + c_2 \cdot r_2 (g_j - x_{i,j}(t))\right), 0\right] \quad (18)$$

where  $r_1$  and  $r_2$  are random numbers with uniform distribution in the interval  $[0, 1]$ ,  $c_1$  and  $c_2$  are respectively the cognitive and social acceleration coefficients,  $p_{i,j}$  is the component  $j$  of vector  $P_i$  ( $Pbest$ ),  $g_j$  is the component  $j$  of vector  $G$  ( $Gbest$ ), and  $w$  is the inertia weight [27].

The inertia weight,  $w$ , used in this algorithm is dynamic. It decreases as the iteration number,  $t$ , increases, as can be seen in Eq. (19).

$$w = w_{min} + (w_{max} - w_{min}) \lambda^{(t-1)} \quad (19)$$

where  $w_{max}$  and  $w_{min}$  are, respectively, the maximum and minimum values of inertia weight  $w$ , with  $\lambda = 0.95$  [28].

The new position of component  $x_{i,j}$  of particle  $i$  in the iteration  $t+1$  is described by Eq. (20).

$$x_{i,j}(t+1) = x_{i,j}(t) + v_{i,j}(t+1) \quad (20)$$

where  $x_{i,j}$  is an integer between the limits  $1 \leq x_{i,j} \leq nd$ . To that end, the conditions presented by Eq. (21) are considered.

$$x_{i,j} = \begin{cases} 1 & \text{if } x_{i,j} < 1 \\ nd & \text{if } x_{i,j} > nd \end{cases} \quad (21)$$

Iteration velocity values may be controlled using limits for the particle velocity ( $V_{PSOmax}$ ), which should be imposed so that the search space is not extrapolated in few iterations and so that the particles do not condense in a relatively small subspace.

$$-V_{PSOmax} \leq v_{i,j} \leq V_{PSOmax} \quad (22)$$

Particles move throughout the search space. For each movement of the particles (iteration), position is updated and evaluated according to the objective function and suffers a penalization with a predetermined value ( $Wpenal$ ), for each node that does not fulfill the specified pressure requirement or when the velocities are out of the established values. This is done so that the penalized particle makes the solution unfeasible. In this process, the hydraulic simulator *Epanet* is used in order to obtain the velocity in the pipes and the pressure head in the nodes. Variable  $AV_i$  is used to estimate the number of pressure or velocity violations for the solution  $i$ .

If a particle (vector  $X_i^A$ ) present  $AV_i$  violations, the total penalization for the particle will be  $Wpenal \times AV_i$  and this value will be added to the value of the objective function, as shown in Eq. (23), named penalized function.

$$C_{TPi} = C_{Ti} + (Wpenal \cdot AV_i) \quad (23)$$

Each favorable position (minimum) is named  $Pbest$  (personal best), which is the best personal evaluation of particle  $X_i^A$ , that is, particle  $i$  obtained the best placement. This position is stored in vector  $P_i$ . Therefore, all particles in the

group have their  $Pbest$ , whose evaluation is represented by column vector  $F$ , displayed in Eq. (24):

$$F = [F_1, F_2, \dots, F_i, \dots, F_{Np}]^T \quad (24)$$

A particular viable vector is that where every component is attributed to the maximum index  $nd$  (maximum available diameter). This vector is named  $G_{max} = (nd, nd, \dots, nd)$ . The objective function for the vector is given by  $C_{Tmax}$ , which represents the maximum value the objective function achieves relative to the costs of pipe installation. If this vector has pressure violations, the network becomes unfeasible and the network will need an increase in the height of the tank or the implementation of a pumping system.

For each new position (iteration), the best  $P$  is also evaluated. This situation generates the vector  $Gbest$ , with  $G = (x_1^E, x_2^E, \dots, x_{M'}^E, g_{M'+1}, g_{M'+2}, \dots, g_{M'+M})$ , which is, for now, the best result found by the process. Its evaluation in the objective function is a scalar, named  $C_{TG}$ . The process of particle movements (iterations) ends when  $C_{TG}$  shows no variation in the results of subsequent iterations or when a maximum total number of iterations ( $tmax$ ) is reached.

The PSO algorithm follows next, with the adaptations and modifications for WDN optimization with rehabilitation and expansion:

1. Initialize  $X_i, V_i, P_i, \forall i \in \{1, \dots, N_p\}$  and  $\forall j \in \{M+1, \dots, M+M'\}$ :  $x_{i,j}$  is initialized according to Eq. (17),  $v_{i,j} = 0$  (the particles of the set are assumed to start at rest), the components of  $P_i$  ( $p_{i,j} = 1$ ) must be initialized with performance  $F_i = C_{Tmax}$ , vector  $Gbest$  ( $g_j = nd$ ) and its respective performance  $C_{TG} = C_{Tmax}$ .
2. For each particle  $X_i^A$  ( $i = 1$  to  $N_p$ ), calculate the number of violations ( $AV_i$ ) and the value of the penalized objective function,  $C_{TPi}$ , according to Eq. (23).
3. Compare the performance  $C_{TPi}$  for each particle  $i$ :
  - 3.1 with the performance of  $Pbest_i$ . If better, update  $F_i$  and the components of vector ( $P_i \leftarrow X_i$ ).
  - 3.2 with the performance of  $Gbest$ . If better, update  $C_{TG}$  and the components of vector  $Gbest$ .
4. Verify the criteria of the number of iterations. If the maximum number of iterations is achieved, end. Otherwise, update the new position of the  $N_p$  particles and return to step 2.

#### 4. Case studies

In order to verify the effectiveness of the developed WDN optimization model with rehabilitation and expansion, four case studies were carried out, using networks from the literature. A computer with an Intel Core i5 1.6 GHz CPU was used in all case studies.

##### 4.1. Two Reservoirs network

The Two Reservoirs network was studied by Gessler [29], Simpson et al. [30], and Cunha and Ribeiro [31]. It is presented in Fig. 2, with corresponding data in Table 2.

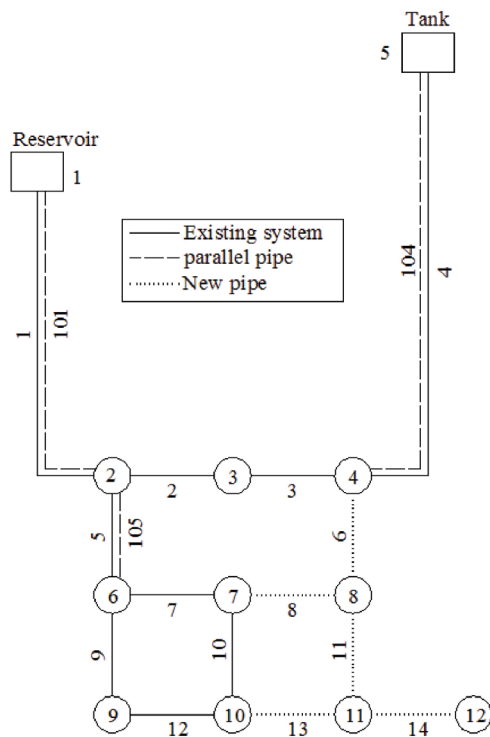


Fig. 2. Layout of the Two Reservoirs network.

- This network requires expansion and rehabilitation, with the following characteristics:
- Installation of 5 new pipes, from a set of available ones, with a Hazen-Williams dimensionless roughness coefficient  $C = 120$ .
- Some pipes can be rehabilitated with parallel pipes or maintenance (cleaning).
- The network should be efficient for the three demand patterns and their respective minimum pressure heads.

The network has nine existing pipes ( $M' = 9$ ) and eight new ones with diameters to be optimized ( $M = 8$ ), thus forming the two submatrices, as presented in Fig. 3. Table 2 presents pipe data and node elevations of the Two Reservoirs network. Table 3 presents the available diameters for new pipes as well as their respective costs.

Since parallel pipes can be used, a pipe with diameter close to zero has to be created, as shown in Table 3 b). The network must handle the three demand patterns and the pressure heads in the nodes must be greater than or equal to the minimum ones, according to their demands, as shown in Table 4.

In the present work, the following parameters were considered in the PSO algorithm for this network: dynamic inertia weight  $w \in [0.5, 0.9]$ ,  $c_1 = c_2 = 2.0$ , penalty  $W_{penal} = \$ 450,000$ , number of particles  $N_p = 51$ ,  $V_{PSOmax} = 1$ , maximum number of iterations  $tmax = 50$ . Considering every new pipe as having the maximum diameter,  $509 \text{ mm}$ ,  $C_{Tmax} = \$ 5,524,707.90$ . Table 5 presents the obtained results for this WDN. The optimal value was found to be  $\$ 1,750,103.24$ . Computational time was 1.0 sec.

Table 2  
Pipe data and node elevations of the Two Reservoirs network

a) Pipe data				b) Node elevations	
Pipe	Length (m)	Diameter (mm)	Roughness coefficient C (H-W)	Node	Elevation (m)
1	4828	356	75	1	365.76
2	1609	254	80	2	320.04
3	1609	254	80	3	326.14
4	6437	254	80	4	332.23
5	1609	254	80	5	371.86
7	1609	203	100	6	298.70
9	1609	254	80	7	295.66
10	1609	102	100	8	292.61
12	1609	203	100	9	289.56
6	1609	?	120	10	289.56
8	1609	?	120	11	292.61
11	1609	?	120	12	289.56
13	1609	?	120		
14	1609	?	120		
101	4828	?	120		
104	6437	?	120		
105	1609	?	120		

[TANKS]							
:ID	Elevation	InitLevel	MinLevel	MaxLevel	Diameter	MinVol	
[PIPES]							
:ID	Node1	Node2	Length	Diameter	Roughness	MinorLoss	Status
1	1	2	4828	356	75	0	Open
4	5	4	6437	254	80	0	Open
5	2	6	1609	254	80	0	Open
2	2	3	1609	254	80	0	Open
3	3	4	1609	254	80	0	Open
7	6	7	1609	203	100	0	Open
9	6	9	1609	254	80	0	Open
10	7	10	1609	102	100	0	Open
12	9	10	1609	203	100	0	Open
6	4	8	1609	509	120	0	Open
8	7	8	1609	509	120	0	Open
11	8	11	1609	509	120	0	Open
13	10	11	1609	509	120	0	Open
14	11	12	1609	509	120	0	Open
101	1	2	4828	509	120	0	Open
104	5	4	6437	509	120	0	Open
105	2	6	1609	509	120	0	Open
[PUMPS]							
:ID	Node1	Node2	Parameters				

Existing pipes

New pipes

Fig. 3. Part of the Epanet Two Reservoirs network file.

Table 3 Available diameters and their costs for the Two Reservoirs network

a) Available diameters and their costs			b) Format of the costs for the proposed algorithm			
D (mm)	Cost of new pipe (\$/m)	Cost of cleaning existing pipe (\$/m)	id No.	D (mm)	Cost of new pipe (\$/m)	Roughness coefficient C (H-W)
152	49.54	47.57	1	0.01	0	120
203	63.32	51.51	2	152	49.54	120
254	94.82	55.12	3	203	63.32	120
305	132.87	58.07	4	254	94.82	120
356	170.93	60.70	5	305	132.87	120
407	194.88	63.00	6	356	170.93	120
458	232.94	–	7	407	194.88	120
509	264.10	–	8	458	232.94	120
			nd = 9	509	264.10	120

In the solution of Cunha and Ribeiro [31], a diameter of 102 mm was used, however this diameter is unavailable for new pipes according to Table 3a), which shows a minimum diameter to be used of 152 mm. The optimal solution found using the developed algorithm is the global minimum, according to Simpson et al. [30]. Table 6 shows node pressure heads for each demand pattern, according to the diameters obtained in the optimization process.

The PSO velocity parameter ( $V_{PSOmax}$ ) is very responsive in the search for the optimal solution. For example, after

changing  $V_{PSOmax}$  from 1 to 2, the algorithm found another minimum value that met the demands and the pressure heads, eliminating pipe 13, with an optimized value of \$ 1,721,076.88. One of the constraints of the problem, however, is that new pipes cannot be removed.

For pipes 1 and 4, there is the option of carrying out internal cleaning, which improves the roughness coefficient. A simulation was run for each option, changing the value of dimensionless roughness coefficient C (Hazen-Williams) to 120 (cleaned pipes), similar to that of new pipes. Table 7

Table 4  
Demand patterns and their respective pressure heads for the Two Reservoirs network

Node	Demand Pattern 1		Demand Pattern 2		Demand Pattern 3	
	Demand (L/s)	Minimum pressure head (m)	Demand (L/s)	Minimum pressure head (m)	Demand (L/s)	Minimum pressure head (m)
2	12.62	28.18	12.62	14.09	12.62	14.09
3	12.62	17.61	12.62	14.09	12.62	14.09
4	0.00	17.61	0.00	14.09	0.00	14.09
6	18.93	35.22	18.93	14.09	18.93	14.09
7	18.93	35.22	82.03	10.57	18.93	14.09
8	18.93	35.22	18.93	14.09	18.93	14.09
9	12.62	35.22	12.62	14.09	12.62	14.09
10	18.93	35.22	18.93	14.09	18.93	14.09
11	18.93	35.22	18.93	14.09	18.93	14.09
12	12.62	35.22	12.62	14.09	50.48	10.57

Source: Simpson et al. [30].

Table 5  
Optimal costs and pipe diameters (mm)

Pipe	Gessler [29]	Simpson et al. [30]	Cunha and Ribeiro [31]	This Work
1	356	356	356	356
4	254	254	254	254
5	254	254	254	254
2	254	254	254	254
3	254	254	254	254
7	203	203	203	203
9	254	254	254	254
10	102	102	102	102
12	203	203	203	203
6	305	305	305	305
8	203	203	203	203
11	305	203	254	203
13	203	152	102*	152
14	203	254	203	254
101	0	0	0	0
104	356	356	356	356
105	0	0	0	0
Cost \$ (millions)	1.833	1.75	1.727	1.75

\*unavailable diameter.

presents the optimized values with pipe cleaning. The total cost is the sum of the costs of the new pipes and the costs of cleaning existing pipes, showed in Eq. (25), where  $Cost_{Clean}(D_j)$  is the cost of cleaning in \$/m for diameter  $D$  of pipe  $j$ .

According to Table 7, even if sections 1 or 4 are cleaned, a pipe has to be installed parallel to pipe 4. Cleaning is not recommended, as optimized values are greater than those of the other options.

#### 4.2. New York City Water Supply Tunnels network

The New York City Water Supply Tunnels benchmark problem, presented in Fig. 4, is based on the rehabilitation problem proposed by Schaake and Lai [14].

This problem was studied by many researchers, as Murphy et al. [32], Van Dijk et al. [33], Geem [34] and Zhou et al. [35], among others. Due to the expansion of the city and consequent increase in consumption demand, problems with insufficient pressure heads for a good performance of the network started to appear.

The New York City Water Supply Tunnels network has 21 pipes. The pressure heads were analyzed using Epanet, with the new demand pattern imposed on the network. Nodes with pressures lower than those required were present, as shown in Fig. 5.

In order to solve the problem, two options exist:

1. Replacing pipes with larger-diameter ones.
2. Adding parallel pipes to the existing ones.

Table 8 presents data on the 21 existing pipes ( $M' = 21$ ). These pipes, when doubled, will total 42 ( $M = 21$ ). For the 21 parallel pipes (pipes 22 to 42), diameters and roughness coefficients may present different values, resulting from the optimization process. Table 9 presents data for the network nodes. There is a single demand pattern.

The system has 15 diameter types to be used in the WDN, as shown in Table 10. This table also presents the format of the available diameter indexes and their respective costs and roughness coefficients, comprising 16 available diameters (including the diameter near 0, with zero cost).

Fig. 6a shows the network with every possible extension, comprising 42 pipes. From these, 21 are existing ones



Table 6  
Pressure heads in the optimal solution for the Two Reservoirs network

Node	Demand pattern 1		Demand pattern 2		Demand pattern 3	
	Minimum pressure head (m)	Actual pressure head (m)	Minimum pressure head (m)	Actual pressure head (m)	Minimum pressure head (m)	Actual pressure head (m)
2	28.18	36.33	14.09	25.05	14.09	30.56
3	17.61	30.51	14.09	19.42	14.09	24.60
4	17.61	26.90	14.09	16.26	14.09	20.54
6	35.22	46.92	14.09	18.75	14.09	34.42
7	35.22	50.09	10.57	12.78	14.09	37.61
8	35.22	59.31	14.09	41.44	14.09	48.05
9	35.22	51.92	14.09	24.12	14.09	34.70
10	35.22	49.83	14.09	22.41	14.09	26.73
11	35.22	47.57	14.09	24.91	14.09	18.26
12	35.22	50.03	14.09	27.37	10.57	13.70

Table 7  
Optimized costs considering internal cleaning of pipes 1, 4 or 5

Existing pipe	Cost of cleaning (\$)	Optimized value (\$)	Total cost (\$)
$j' = 1$	293,059.60	1,750,103.24	2,043,162.84
$j' = 4$	354,807.44	1,750,103.24	2,104,910.68
$j' = 5$	88,688.08	1,711,052.81	1,799,740.89
$j' = 1$ and 4	647,867.04	1,750,103.24	2,397,970.28
$j' = 1, 4$ and 5	736,555.12	1,390,576.53	2,127,131.65

with known diameters. Assuming every new pipe to have a diameter of 5181.6 mm ( $D_{max}$ ), the maximum cost of the network would be \$ 294,103,203.50, which represents an initial comparison value for the minimization process, whose result is shown in Fig. 6b.

The following PSO algorithm parameters were considered for this network: dynamic inertia weight  $w \in [0.5, 0.9]$ ,  $c_1 = c_2 = 2.0$ , penalty  $W_{penal} = \$ 14,000,000$ , number of particles  $N_p = 200$ ,  $V_{PSO_{max}} = 1$ , and maximum number of iterations  $t_{max} = 60$ . Table 11 presents the results obtained for this network using the developed algorithm. The optimal value was found to be \$ 38,637,704.57. Computational time was 12 s. Table 11 also presents the comparison with solutions obtained by other authors.

Table 12 shows pressure heads in the demand nodes, always greater than minimum specifications, for the New York City Water Supply Tunnels network.

In this work, the problem was resolved considering the roughness coefficient  $C = 100$  for the existing pipes and  $C = 130$  for the new pipes. In this case, using the proposed modified PSO approach, the total cost for this problem was \$ 34.48 millions, with the duplication of 6 pipes.

#### 4.3. Suribabu and Neelakantan's Example Network 1

Example Network 1, studied by Suribabu and Neelakantan [22], is fed by gravity by a tank with relative height of

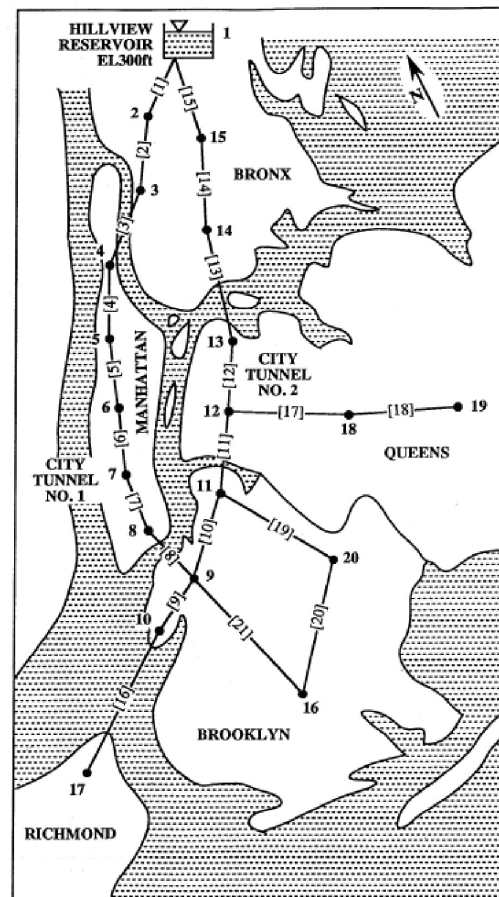


Fig. 4. New York City Water Supply Tunnels network (Source: Murphy et al. [32]).

185 m and comprises 17 nodes and 26 pipes. Nodes and pipes data are presented in Table 13. Demands and roughness coefficients  $C$  (Hazen-Williams) correspond to 15 years of operation.

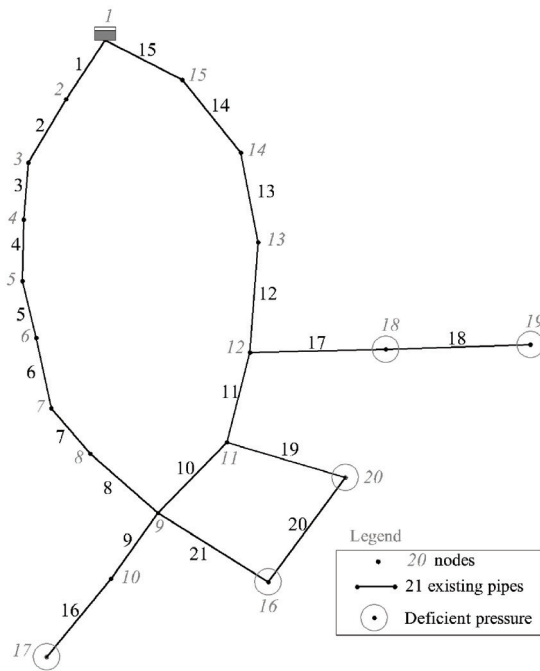


Fig. 5. New York City Water Supply Tunnels network with deficient pressure.

Table 8  
Pipe data for the New York City Water Supply Tunnels network

Pipe	Existing diameter		Length (m)
	(in)	(mm)	
1	180	4572.0	3535.68
2	180	4572.0	6035.04
3	180	4572.0	2225.04
4	180	4572.0	2529.84
5	180	4572.0	2621.28
6	180	4572.0	5821.68
7	132	3352.8	2926.08
8	132	3352.8	3810.00
9	180	4572.0	2926.08
10	204	5181.6	3413.76
11	204	5181.6	4419.60
12	204	5181.6	3718.56
13	204	5181.6	7345.68
14	204	5181.6	6431.28
15	204	5181.6	4724.40
16	72	1828.8	8046.72
17	72	1828.8	9509.76
18	60	1524.0	7315.20
19	60	1524.0	4389.12
20	60	1524.0	11704.32
21	72	1828.8	8046.72

Table 9  
Node data for the New York City Water Supply Tunnels network

Node	Elevation (m)	Demand (L/s)	Minimum pressure head (m)
1	91.44	Reservoir	–
2	0	2616.47	77.724
3	0	2616.47	77.724
4	0	2497.54	77.724
5	0	2497.54	77.724
6	0	2497.54	77.724
7	0	2497.54	77.724
8	0	2497.54	77.724
9	0	4813.86	77.724
10	0	28.32	77.724
11	0	4813.86	77.724
12	0	3315.90	77.724
13	0	3315.90	77.724
14	0	2616.47	77.724
15	0	2616.47	77.724
16	0	4813.86	79.248
17	0	1628.22	83.149
18	0	3315.90	77.724
19	0	3315.90	77.724
20	0	4813.86	77.724

Table 10  
Data on the available diameters for the New York City Water Supply Tunnels network

id No.	D (mm)	Cost (\$/m)	Roughness coefficient C (H-W)	Nominal diameter (in)
1	0.01	0.00	100	0
2	914.4	306.76	100	36
3	1219.2	439.63	100	48
4	1524.0	577.43	100	60
5	1828.8	725.07	100	72
6	2133.6	875.98	100	84
7	2438.4	1036.75	100	96
8	2743.2	1197.51	100	108
9	3048.0	1368.11	100	120
10	3352.8	1538.71	100	132
11	3657.6	1712.60	100	144
12	3962.4	1893.04	100	156
13	4267.2	2073.49	100	168
14	4572.0	2260.50	100	180
15	4876.8	2447.51	100	192
nd = 16	5181.6	2637.80	100	204

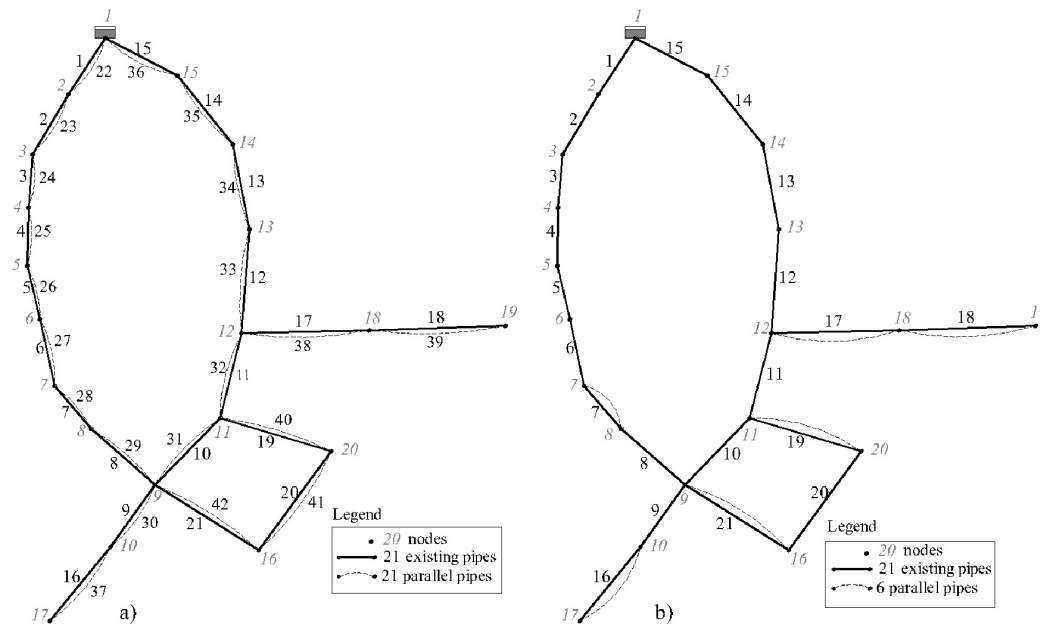


Fig. 6. (a) New York City Water Supply Tunnels network with every possible parallel pipe extension and (b) Network with optimized extensions.

Table 11  
Solutions for the New York City Water Supply Tunnels network

Pipe	Murphy et al. [32] (GA)	Van Dijk et al. [33] (GA)	Geem [34] (PSHS)	Zhou et al. [35] (STA)	This work PSO
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0
6	0	0	0	0	0
7	0	144	144	144	144
8	0	0	0	0	0
9	0	0	0	0	0
10	0	0	0	0	0
11	0	0	0	0	0
12	0	0	0	0	0
13	0	0	0	0	0
14	0	0	0	0	0
15	120	0	0	0	0
16	84	96	96	96	96
17	96	96	96	96	96
18	84	84	84	84	84
19	72	72	72	72	72
20	0	0	0	0	0
21	72	72	72	72	72
Cost \$ (millions)	38.8	38.64	38.64	38.64	38.64

Table 12  
Pressure heads for the New York City Water Supply Tunnels network

Node	Minimum pressure head (m)	Actual pressure head (m)	Node	Minimum pressure head (m)	Actual pressure head (m)
1	0	0	11	77.72	83.47
2	77.72	89.67	12	77.72	83.86
3	77.72	87.22	13	77.72	84.77
4	77.72	86.50	14	77.72	87.04
5	77.72	85.86	15	77.72	89.41
6	77.72	85.37	16	79.25	79.27
7	77.72	84.59	17	83.15	83.17
8	77.72	84.33	18	77.72	79.61
9	77.72	83.45	19	77.72	77.74
10	77.72	83.44	20	77.72	79.47

After 15 years of operation, due to the increase in population and city growth, the demands on the nodes have increased and the network has become inefficient, presenting pressure heads on the nodes below 15 m. In some cases, even negative pressure heads were found. The network requires rehabilitation in order to meet the minimum pressure requirement of 15 m. The pipes available for use in the network are presented in Table 14.

There are two pipeline rehabilitation options. The first one is the replacement of existing pipes by new ones. The second option is installing a parallel pipe or keeping the existing one as it is. The options and the layout are shown in Fig. 7.

For option 2, the following parameters were considered for the developed PSO algorithm: dynamic inertia weight  $w \in [0.5, 0.9]$ ,  $c_1 = c_2 = 2.0$ , penalty  $W_{penal} = \$ 8,000,000$ , number of particles  $N_p = 200$ ,  $V_{PSOmax} = 1$ , maximum number of iterations  $tmax = 100$ . Assuming every new pipe to have a diameter of 609.6 mm, the maximum cost of the network would be  $C_{Tmax} = \$ 152,370,400.00$ . Table 15 shows the results

Table 13  
Data for the nodes and the pipes of Example Network 1

Pipes data				Nodes data		
Pipe/Node No.	Length (m)	Diameter (mm)	Roughness coefficient C (H-W)	Elevation (m)	Previous demand (m <sup>3</sup> /h)	New demand (m <sup>3</sup> /h)
1	1,030	406.4	100	185	Tank	
2	890	254.0	98	152	50	70
3	1,015	152.4	98	145	65	80
4	243	406.4	100	125	45	90
5	570	101.6	90	155	50	50
6	350	203.2	90	133	100	130
7	422	101.6	90	128	60	65
8	450	355.6	90	128	90	70
9	320	101.6	90	126	20	40
10	580	152.4	90	149	55	70
11	750	254.0	95	152	80	90
12	750	254.0	90	124	85	90
13	500	101.6	90	122	50	60
14	378	101.6	90	139	100	120
15	570	101.6	90	129	100	130
16	560	152.4	95	123	60	50
17	700	101.6	98	121	70	80
18	610	101.6	98			
19	631	101.6	95			
20	875	254.0	95			
21	890	101.6	95			
22	808	101.6	90			
23	826	152.5	90			
24	810	203.2	95			
25	585	152.4	90			
26	631	101.6	95			

obtained for this network. The optimal value was found to be \$ 9,660,515.00. Processing time was 3.5 s.

For option 1 (Table 15), using the developed algorithm and considering the same PSO parameters as option 2, the optimal value was found to be \$ 26,692,885.00. Processing time was 2.0 s.

The most economical decision is option 2 (Fig. 8), which consists of adding some parallel pipes, according to Table 15. The results using the developed algorithm are 16.98% more profitable for option 1 and 56.71% for option 2 when compared with the solutions of Suribabu and Neelakantan [22]. Table 16, shows nodal pressures for option 1 and option 2 solutions, with pressures greater than 15 m.

4.4. WDN of the city of Esperança Nova

Esperança Nova is a small city in the South of Brazil with approximately 1,875 inhabitants and with 138.56 km<sup>2</sup> of area. The WDN is fed by a tank with relative height of 14.0 m and provides 131 nodes and 166 pipes. The existing

Table 14 Available pipe diameters and their respective details

id No.	D (mm)	Roughness coefficient C (H-W)	Cost (\$/m)
1	0.01	130	0.00
2	101.6	130	765.00
3	152.4	130	1,150.00
4	203.2	130	1,665.00
5	254.0	130	2,250.00
6	304.8	130	2,910.00
7	355.6	130	3,640.00
8	406.4	130	4,460.00
9	457.2	130	5,430.00
10	508.0	130	6,875.00
11	558.8	130	7,980.00
nd = 12	609.6	130	9,100.00

network has been operating for more than 20 years. The minimum pressure in the WDN is 10 m. By using Epanet it is possible to detect that there are 14 nodes with pressure below the minimum requirements. The available diameters are presented in Table17. The maximum velocity allowed for the WDN is 3.00 m/s.

Fig. 9 presents the WDN layout. To solve the rehabilitation problem using the proposed algorithm it was inserted pipes in parallel, totalizing 332 pipes (M' + M). The parameters used to search the minimum value to the rehabilitation are: dynamic inertia weight  $w \in [0.5, 0.9]$ ,  $c_1 = c_2 = 2.0$ , penalty  $W_{penal} = \text{US\$ } 2,000$ , number of particles  $N_p = 300$ ,  $V_{PSOmax} = 5$ , maximum number of iterations  $t_{max} = 50$ . Assuming every new pipe to have a diameter of 100 mm, the maximum total cost is  $C_{Tmax} = \text{US\$ } 292,395.13$ .

Results show that it is necessary to double two pipes, as showed in Fig. 10, with a total cost of US\$ 1,637.71. This value corresponds to 0.89 % of the initial investment (US\$ 183,780.79). The elapsed time to solve the problem was 16 s and the optimal value was found in the 38th iteration ( $t = 38$ ). Before the rehabilitation node 42 presented a pressure of 6.13 m. After the rehabilitation the pressure in the node is 10.04 m.

For all cases studied in the present paper, the tuning procedure was the same for all PSO parameters ( $w, c_1, c_2, W_{penal}, N_p$  and  $V_{PSOmax}$ ). Eq. (19) was used to calculate  $w$ , which is a dynamic value that starts at 0.9 and ends at 0.5 during the optimization process. The parameters  $c_1$  and  $c_2$  can be initialized with  $c_1 = c_2 = 2$ .  $V_{PSOmax}$  is an integer between 1 and  $nd$ .  $N_p$  is an integer that can be lower than or equal to the reference value,  $N_{pRef}$  calculated by Eq. (26).  $W_{penal}$  can also be lower than or equal to the reference value,  $W_{penalRef}$  calculated by Eq. (27).

In the optimization process using the PSO algorithm, there are different sets of PSO parameters that lead to the same optimal solution. The first parameter to be searched is  $W_{penal}$  and a good initial choice is the reference value  $W_{penalRef}$ . This value can be decreased in each new attempt, being constant the other parameters ( $w_{min}, w_{max}, c_1, c_2, N_p, V_{PSOmax}$ ).

5. Conclusions

In the present work, a model for the design and optimization of WDN using rehabilitation and expansion was

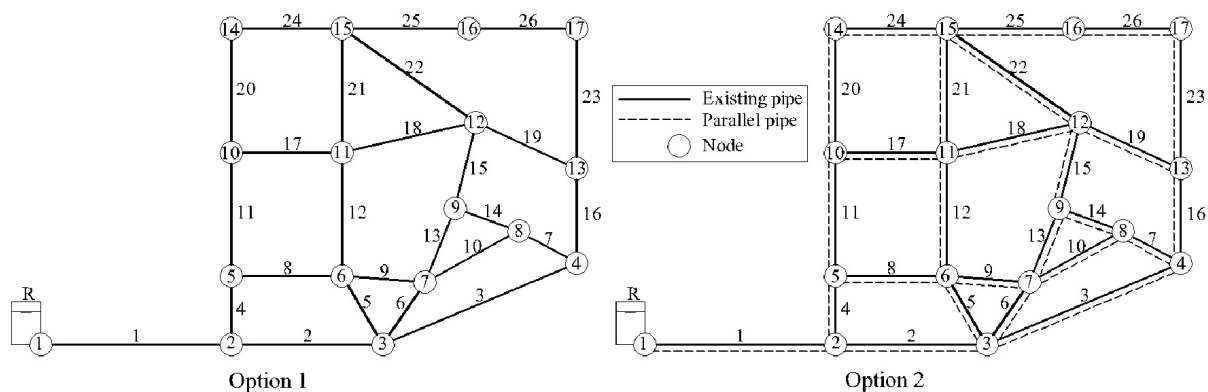


Fig. 7. Example Network 1.



Table 15  
Results of the optimized diameters for solutions of options 1 and 2

Pipe	Solution of option 1		Solution of option 2	
	Suribabu and Neelakantan [22]	This Work	Suribabu and Neelakantan [22]	This Work
	D (mm)	D (mm)	D (mm)	D (mm)
1	508.00	457.20	355.6	406.4
2	203.20	101.60	–	–
3	152.40	101.60	254	–
4	457.20	457.20	304.8	254
5	101.60	101.60	–	–
6	101.60	101.60	–	–
7	101.60	152.40	–	–
8	457.20	406.40	–	–
9	203.20	254.00	152.4	203.2
10	152.40	203.20	–	–
11	254.00	203.20	203.2	152.4
12	406.40	355.60	–	–
13	101.60	101.60	–	254
14	101.60	101.60	–	–
15	101.60	101.60	–	203.2
16	101.60	101.60	–	–
17	101.60	101.60	–	–
18	152.40	152.40	–	–
19	152.40	101.60	304.8*	203.2
20	203.20	203.20	304.8	–
21	254.00	203.20	–	–
22	101.60	101.60	–	–
23	101.60	101.60	–	–
24	101.60	101.60	203.2	–
25	203.20	152.40	–	–
26	152.40	152.40	203.2	–
Cost (\$ × 10 <sup>3</sup> )	31,226.64	26,692.89	15,138.56	9,660.52

\*Replace with 304.8 mm.

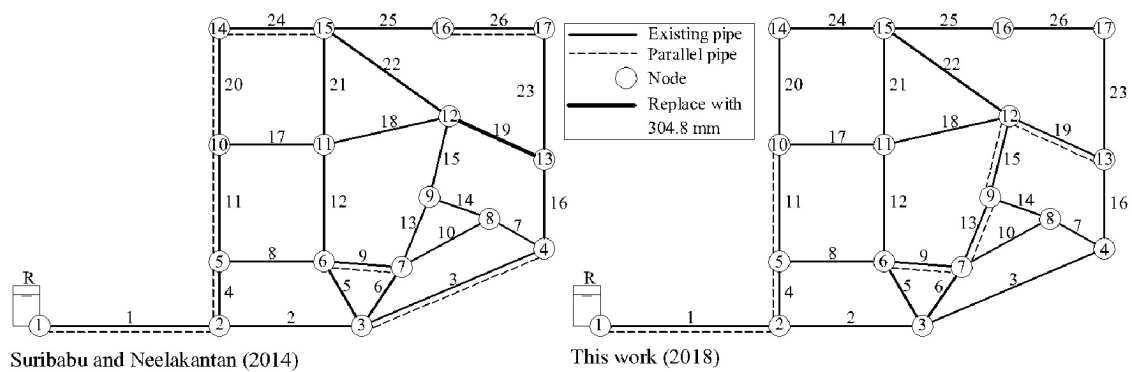


Fig. 8. Solution for option 2.

Table 16  
Nodal pressures for solutions of options 1 and 2

Node	Option 1	Option 2
	Pressure (m)	Pressure (m)
1	Tank	Tank
2	24.03	25.23
3	15.69	21.59
4	26.39	24.99
5	19.22	19.68
6	37.35	38.11
7	38.40	35.54
8	30.60	26.61
9	30.41	33.91
10	15.10	19.60
11	15.28	15.17
12	29.57	29.04
13	18.42	27.93
14	18.57	17.18
15	23.69	16.31
16	17.87	15.05
17	15.58	15.24

Table 17  
Esperança Nova existing WDN piping data

id No.	Diameter (mm)	Cost (US\$/m)	Roughness coefficient C (H-W)
1	0.01	0.00	140
2	32.00	13.31	140
3	50.00	14.30	140
4	75.00	17.82	140
nd = 5	100.00	22.24	140

presented. The model is formulated as an MINLP problem. An algorithm based on PSO with integers variables was proposed for the solution of the developed model. A real case study and three cases from the literature were used in order to test its applicability.

The proposed PSO algorithm focuses on the sub-matrix relative to the new pipes, which is subject to optimization, while the sub-matrix relative to the existing pipes is considered to be constant throughout the process with no cost. Thus, the number of operations and the processing time are minimized.

The proposed algorithm has showed to be efficient in the optimization of the four studied networks, always

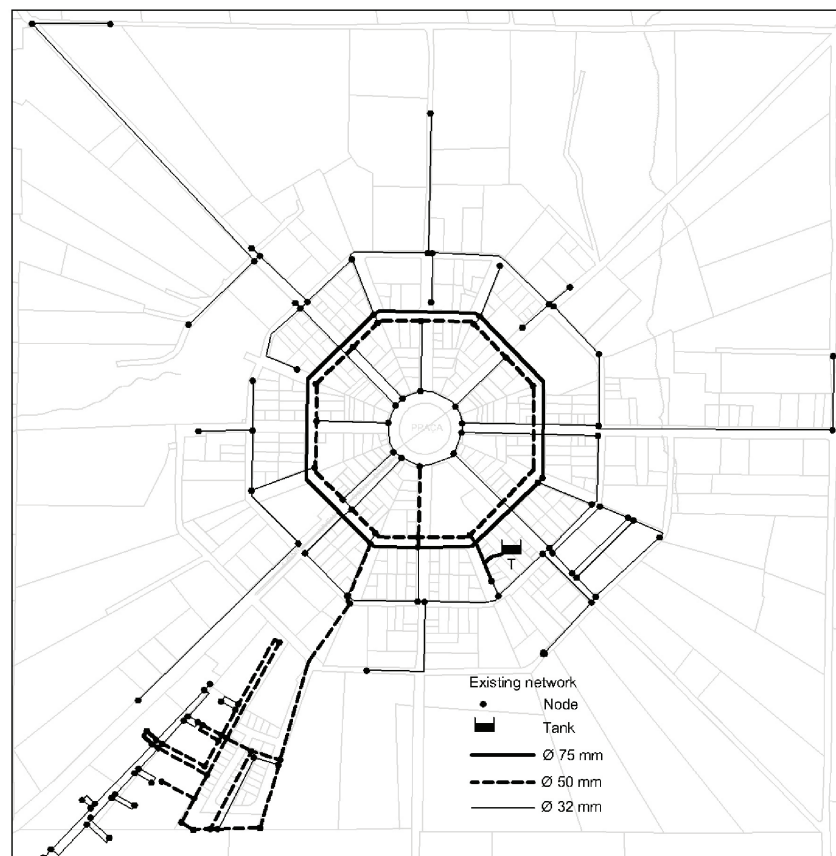


Fig. 9. Esperança Nova WDN layout.

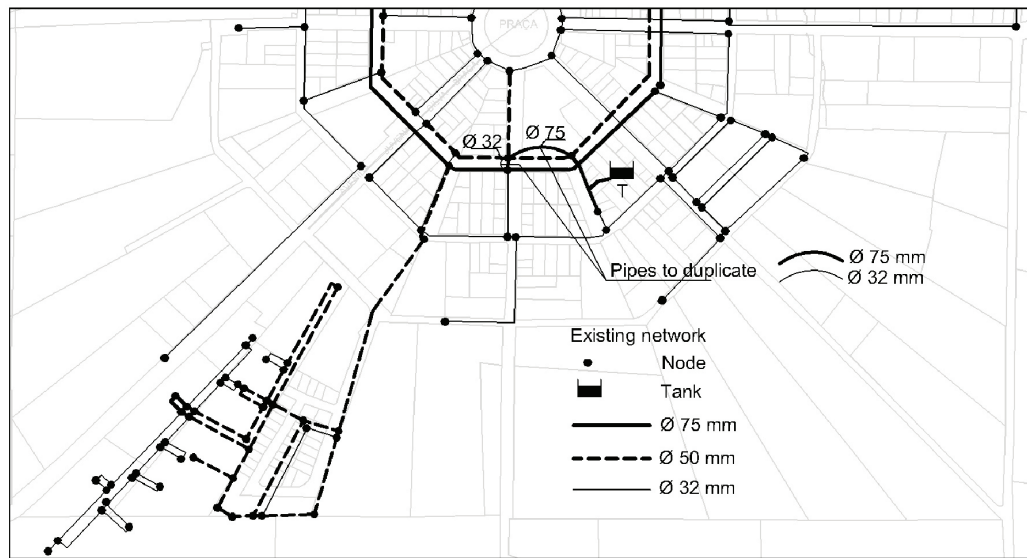


Fig. 10. Esperança Nova optimal WDN for rehabilitation.

Table 18  
Parameters  $N_p$ ,  $W_{penal}$  and  $V_{PSOmax}$  used in the current paper

WDN	$M$	$nd$	$C_{Tmax}$	$K$	$N_{p Ref}$	$W_{penal Ref}$	$N_p$	$W_{penal}$	$V_{PSOmax}$
Two Reservoirs	17	9	5,524,708	10	51	552,471	51	450,000	1
New York City	42	16	294,103,203	20	224	14,705,160	200	14,000,000	1
Example Network 1	52	12	152,370,400	17	208	8,962,965	200	8,000,000	1
Esperança Nova	332	5	292,395	132	553	2,215	300	2,000	5

showing the best or the same values, when compared with other works. For the case of the Esperança Nova WDN the algorithm showed efficiency and solved the rehabilitation problem with the duplication of two pipes with an investment of 0.89% of the total cost for the implementation of the original piping. The algorithm is efficient for small scale as well as for large-scale problems.

The maximum particle velocity,  $V_{PSOmax}$ , is one of the sensitive parameters of the standard PSO algorithm for the search of optimal solutions. In the algorithm proposed in the present work, finding the minimum network cost is not a complex task. The possible values for  $V_{PSOmax}$  are the integers inferior to the number of available diameters for the design of the WDN.

**Nomenclature**

- $AV$  — Amplitude of violations;
- $c_1, c_2$  — Cognitive and social acceleration coefficients;
- $C_j$  — Hazen-Williams roughness coefficient for pipe  $j$ ;
- $C_{TG}$  — Value of the objective function for particle  $G_{best}$ ;

- $C_{Ti}$  — Value of the objective function for particle  $i$ ;
- $C_{Tmax}$  — WDN maximum total cost;
- $D_{ALW}$  — Set of diameters available for the network;
- $D_j$  — Diameter of pipe  $j$ ;
- $D_{max}$  — Maximum allowed diameter;
- $D_{min}$  — Minimum allowed diameter;
- $d$  — Node demand;
- $F$  — Column vector whose components indicate the best evaluation achieved by each particle;
- $C_{TPi}$  — Value of the penalized objective function for particle  $i$ ;
- $G$  — Global best solution vector;
- $G_{max}$  — Vector where all pipes have diameters  $D_{max}$ ;
- $h_j$  — Head loss in pipe  $j$ ;
- $i$  — Particle  $i$ ;
- $id$  — Diameter index corresponding to the set  $D_{ALW}$ ;
- $j$  — Pipe  $j$ ;
- $k$  — Node  $k$ ;

$K$	— Total number of nodes;
$L_j$	— Length of pipe $j$ ;
$M$	— Total number of new pipes;
$M'$	— Total number of existing pipes in the network;
$nd$	— Total number of available diameters;
$N_p$	— Total number of particles in the swarm;
$P_i^A$	— Vector best position of particle $i$ ;
$pr$	— Node pressure head;
$PSHS$	— Particle-swarm harmony search algorithm;
$Q$	— Flow rate;
$r, r_1, r_2$	— Random numbers with uniform distribution in $[0,1]$ ;
$t$	— Number of iterations;
$tmax$	— Maximum number of iterations;
$V_i^A$	— Velocity vector of particle $i$ ;
$v_i$	— Flow velocity inside the pipe;
$V_{PSOmax}$	— Maximum velocity of the particle;
$WDN$	— Water distribution network;
$w_{max}, w_{min}$	— Maximum and minimum inertia weights;
$Wpenal$	— Penalty value;
$w$	— Inertia weight;
$X, V, P$	— Matrices $X, V,$ and $P$ of particles relative to the new pipes in the network;
$X^A, V^A, P^A$	— Matrices of position, velocity, and best position of particles including existing and new pipes;
$X^E, V^E, P^E$	— Matrices corresponding to existing pipes;
$X_i^A$	— Current position vector of particle $i$ ;
$x_{ij}$	— Diameter index of pipe $j$ for particle $i$ ;
$x_j^E$	— Diameter of existing pipe $j$ ;
$X_s^E, V_s^E, P_s^E$	— Vectors of position, velocity, and best position of particles relative to existing pipes.

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