*Desalination and Water Treatment*
www.deswater.com

doi: 10.5004/dwt.2018.22213

109 (2018) 344–349
March

# Parametric evaluation of the Euler–Lagrangian approach for tracer studies

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Received 13 October 2017; Accepted 8 March 2018

**ABSTRACT**

The determination of dispersion number or its coefficient is pertinent to the control of pollution. This study evaluated the importance of the parameters measured from a river in South West Nigeria, during tracer studies using the Euler–Lagrangian approach. Several measurements which included tracer concentration, width, velocity, sampling time, and sampling point interval were obtained between January and April of 2017, cutting across the raining and dry seasons for model development and sensitivity analysis. The result revealed that a 1% increase in the dispersion coefficient will result from a 2.487% increase in velocity ($t$ = 2.671, $p$ = 0.020) and 8.914% increase in the channel width ($t$ = 6.124, $p$ = 0.000), which were statistically significant at 5% and 1%, respectively. This finding is well supported by previous studies which made use of the variable distance and constant time method. Furthermore, sampling time ($t$ = 5.087, $p$ = 0.000), sampling point interval ($t$ = 6.124, $p$ = 0.000), and tracer concentration ($t$ = 2.453, $p$ = 0.030) were new variables identified and all were statistically significant and had a direct relationship with dispersion coefficient. It is recommended that the Euler–Lagrangian approach should be adopted in other rivers to verify these claims, as it could be seen as a sustainable method for conducting tracer studies.

*Keywords:* Dispersion; Dispersion coefficient; Tracer studies; Pollution; Risk assessment; Sustainable methods

## 1. Introduction

With increase in industrialization globally, pollution of rivers and streams has become a common occurrence. This has the propensity to increase anthropogenic activities within the river reach thereby impacting negatively on water bodies. To either mitigate or eliminate such situations, development of strong policies and adherence, constant monitoring, as well as applying a more sustainable approach among others are advocated. Moreover, these sustainable approaches vary from the application of tracer harvesting techniques, which will result in reduced cost and limited time [1–3] and utilization of developed mathematical models [4–6]. The latter methods are still exhibiting significant variability as all parameters responsible for predicting the dispersion coefficient are not completely known [7,8], although it has been suggested that one of the major reasons could be the inconsistency in the shear stress and inappropriate representation of the velocity across the channel sections [9]. Furthermore, apart from the aforementioned, the issues of climate change cannot be neglected, and this has made some researchers to recommend the former approach as a lasting solution. The Euler–Lagrangian approach has been proposed by Agunwamba [1], and very few studies in the literature have adopted it both in the laboratory and in the river [2–3]. This method of sampling involved the collection of tracer from the outlet through to the inlet at equally marked distances irrespective of the sampling time interval [10]. Similarly, it considered the variability in the hydrodynamic conditions of the river or stream and reduces cost and subjectivity in sampling time interval

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selection that appeared to be a limitation in the Levenspiel and Smith approach [11]. On the other hand, there is a need to evaluate the measured parameters in a bid to identify and establish how they affect the dispersion coefficient and to ascertain if this approach gives similar relationship with our a priori expectations when other methods are used. However, selecting the various explanatory variables for a model can be a difficult task, but when this is achieved, it will aid the prediction of the dependent variables, which is dispersion number or its coefficient in this case, taking less time, cost, and energy [12–16]. Most often, the use of least squares regression or multiple regression methods is engaged for this process. This involved the systematic combination of variables to produce an equation that can be used for trend analysis. Likewise, it can also be applied to show relationships between dependent and independent variables. It was revealed that the use of least square methods could help reduce the bias associated with the set of data and the curve generated [17]. This method has been widely used by several researchers for trend analysis in different environmental engineering–related problems and in other fields. For example, it was used to predict biochemical oxygen demand from degraded sewage, develop sensitivity analysis on roughness effects on dissolved oxygen and dispersion coefficient [18–21].The mathematical technique was also used to develop reaeration coefficients of rivers around the globe [22–27]. However, it is suggested that the outright use of ordinary least square estimates leads to inconsistent coefficients and relationships between the explanatory variables. This is because the assumption that the variables (independent and dependent) which has a linear relationship are error free is likely to be untrue [28]. In addition, the need to perform sensitivity analysis on model parameters as well as evaluate model performance using different statistical techniques has been mentioned previously. This is because some parameters may not contribute meaningfully and are therefore insignificant [29]. In the literature, the importance and types of sensitivity analysis used have been well reported [13–16], making it a pertinent process. Therefore, the objectives of this investigation are to carry out tracer studies making use of the Euler–Lagrangian method in River Balogun in South West Nigeria and to identify the relationship between the variables and also comment on the similarities in the relationships between the selected parameters with already established relationships in other dispersion coefficient empirical models.

## 2. Materials and methods

### 2.1. Euler–Lagrangian method of harvesting tracer data

With this method, 25 kg of common salt was premixed with 50 L of the river water sample and poured gently into the river at the inlet, and then, electrical conductivity measurements were taken from the outlet to the inlet without placing any priority on the sampling time interval. At every sampling point which was identified and marked out, tracer readings were obtained using the multileveller sampler that was fabricated. In the same vein, the tracer collected was poured into a clean container and the electrical conductivity readings were measured and recorded. The experiment was conducted between January and April of 2017 under suitable weather conditions and accessibility of the river. Also, the value of the dispersion number was generated using Eq. (1) [1,10].

$$\sigma^2 = \frac{\sum_{i=1}^{M}\left(\frac{\tau_j}{1-\xi i}\right)^2 C_i}{\sum_{i=1}^{M} C_i} - \left[\frac{\sum_{i=1}^{M}\left(\frac{\tau_i}{1-\xi_i}\right) C_i}{\sum_{i=1}^{M} C_i}\right]^2 \tag{1}$$

The relationship between the dispersion number and the normalized variance ($\sigma^2$) is given by Eq. (2) [1].

$$\partial = \frac{1}{29.2}(\sqrt{1+15\sigma^2-1} \tag{2}$$

Eq. (2) was derived from the statistical moment equation as reported by the authors Levenspiel and Smith [12] in Eq. (3), while Eqs. (4) and (5) represent the ratio of the time taken to collect tracer samples to the detention time and ratio of the sampling distance to the overall length of the river [1].

$$\partial = \frac{1}{8}(\sqrt{8\sigma^2+1}-1 \tag{3}$$

$$\text{where } \tau = \frac{t}{\theta} \text{ and} \tag{4}$$

$$\xi = \frac{x}{L} \tag{5}$$

## 3. Data analysis

The arrangement of the dataset for evaluation and sensitivity analysis was achieved using Microsoft Excel 2013. Furthermore, EViews version 8.0 was used to carry out descriptive statistics, modelling, and sensitivity analysis. Data variability was conducted using the Jarque–Bera test for normality [18]. Cumulative sum of recursive residual (CUSUM) and the cumulative sum of squares of recursive residual (CUSUMSQ) test which measures the collective deviations of the parameters in a model were used to determine the structural stability of the coefficients of the variables. The Durbin–Watson (DW) statistics measured whether or not the residual error value of a regressed model (linear or multiple) were independent and determined the extent of serial correlation, as it revealed whether the model formulated is either positive, negative, or first-order correlated and was employed in the analysis [30]. In addition, the DW values obtained were expected to be within the range of 0–4, with the value at 2 or close to indicated the absence of correlated error of residuals [31]. However, Field [32] stated that DW values between 1.5 and 2.5 are still acceptable; however, with values less than one peculiar with most time series experiments. The mathematical expression of DW statistics is given by:

$$\mathrm{DW} = \frac{\sum_{t=2}^{T}(e_t - e_{t-1})^2}{\sum_{t=1}^{T} e_t^{\,2}} \tag{6}$$

where $e_i = y_i - \overline{y_i}$ represents the observed and predicted values of the response variable for individual $i$ and $T$ in the total number of observations.

## 4. Results and discussion

The descriptive statistics of all the variables employed in this study are presented and discussed in Table 1. Specifically, the mean, median, minimum, maximum values, standard deviation, the skewness, kurtosis, Jarque–Bera values, and their corresponding probability values were also reported (Table 1). The mean of each of the variables is a pointer to the average of the corresponding variables obtained from the study. The standard deviation showed a drift of the variable from the mean; thus, revealing the explosiveness of the variables. Additionally, the skewness and kurtosis indicators revealed asymmetry and peakedness of the distribution while the normality test was conducted using the Jarque–Bera statistics to indicate the strength of the tails of the distribution. The results in Table 1 revealed that both mean and median values for all the variables were in line with normal (random) time series trend. From the descriptive statistics presented, it is suggested that only spacing (distance), velocity, width, and time follow normal distribution as revealed by the skewness statistical values. This was due to the constant value obtained at each experimental run. All the variables were positively skewed except for time. Dispersion coefficient, tracer concentration, width, and velocity were found to be leptokurtic in their distribution, while distance and time were platykurtic in their distributions.

In addition, to determine parameter relationship and to identify the presence of multicollinear dataset that existed among variables, a correlation matrix was conducted using the Pearson's correlation coefficient ($r$). Table 2 showed that there was a medium positive relationship between tracer concentration and dispersion ($r$ = 0.502) and a negatively weak relationship between tracer concentration and distance ($r$ = –0.072). Also, tracer concentration–time and tracer concentration–distance revealed a positive weak relation with $r$ = 0.221 and 0.060, respectively, and a negative weak relationship with the width ($r$ = 0.13). Dispersion–distance, dispersion–time, and dispersion–width all showed a weak positive relationships having $r$ = 0.051, 0.097, and 0.118, respectively, with the exception of dispersion–velocity

Table 1
Summary statistics for datasets obtained between January and April of 2017

| | Tracer concentration (mg/L) | Dispersion number | Spacing (m) | Time (s) | Velocity (m/s) | Width (m) |
|---|---|---|---|---|---|---|
| Mean | 47.75 | 0.02 | 181.82 | 250.91 | 0.42 | 3.41 |
| Median | 2.30 | 0.00 | 200.00 | 240.00 | 0.43 | 3.05 |
| Maximum | 309.25 | 0.11 | 400.00 | 480.00 | 0.77 | 4.71 |
| Minimum | 0.03 | 0.00 | 0.00 | 0.00 | 0.12 | 2.57 |
| Standard deviation | 99.02 | 0.04 | 121.07 | 126.80 | 0.15 | 0.62 |
| Skewness | 1.95 | 1.16 | 0.03 | –0.54 | 0.77 | 0.82 |
| Kurtosis | 5.25 | 3.20 | 1.82 | 2.86 | 4.17 | 2.99 |
| Jarque–Bera | 27.90 | 7.43 | 1.94 | 1.43 | 5.13 | 3.69 |
| Probability | 0.00 | 0.02 | 0.38 | 0.49 | 0.08 | 0.16 |
| Sum | 1,575.83 | 0.72 | 600.00 | 8,280.00 | 13.67 | 112.54 |
| Sum of squared deviations | 313,734.20 | 0.04 | 469,090.00 | 514,472.00 | 0.61 | 12.19 |
| Observations | 33 | 33 | 33 | 33 | 33 | 33 |

Note: Null hypothesis: reject when $p < 0.05$ that the variable is not normally distributed.

Table 2
Correlation statistics of all variables

| | Velocity (m/s) | Tracer concentration (mg/L) | Time (s) | Distance (m) | Dispersion number | Width (m) |
|---|---|---|---|---|---|---|
| Velocity (m/s) | 1.000 | 0.060 | 0.346 | 0.160 | –0.265 | –0.435 |
| Tracer concentration (mg/L) | 0.060 | 1.000 | 0.221 | –0.072 | 0.502 | –0.132 |
| Time (s) | 0.346 | 0.221 | 1.000 | 0.0337 | 0.097 | –0.093 |
| Distance (m) | 0.160 | –0.072 | 0.337 | 1.000 | 0.051 | –0.462 |
| Dispersion number | –0.265 | 0.502 | 0.097 | 0.051 | 1.000 | 0.118 |
| Width (m) | –0.435 | –0.132 | –0.093 | –0.462 | 0.118 | 1.000 |

Source: Authors' computation achieved with EViews 8.0 statistical software.

that revealed a negative weak relationship ($r$ = –0.265). Additionally, both distance–time and distance–velocity showed positive weak relationships with $r$ = 0.337 and 0.160, respectively, while an inverse relationship was observed between distance and width ($r$ = –0.461). Consequently, a negative weak relationship was observed between time–width and velocity–width ($r$ = –0.093 and –0.435), while a positive but weak relationship emerged between velocity and time with $r$ = 0.346.

The equation representing the variables used in the analysis is shown in Eq. (4).

$$\begin{aligned}\mathrm{LN(Dispersion)}_t = \beta_o + \beta_{1i}\sum_{i=0}^{n}\mathrm{LN(Tracerconcentration)}_t \\ + \beta_{2i}\sum_{i=0}^{n}\mathrm{LN(Spacing)}_t + \beta_{3i}\sum_{i=0}^{n}\mathrm{LN(Time)}_t \\ + \beta_{4i}\sum_{i=0}^{n}\mathrm{LN(Velocity)}_t + \beta_{5i}\sum_{i=0}^{n}\mathrm{LN(Width)}\end{aligned} \tag{7}$$

where $i$ = 1, 2, 3…$n$. Also, $\beta_0$ is contant or intercept, $\beta_{1i}$ is coefficient of tracer concentration, $\beta_{2i}$ is coefficient of spacing, $\beta_{3i}$ is coefficient of time, $\beta_{4i}$ is coefficient of velocity, and $\beta_{5i}$ is coefficient of width.

Also, the developed model can be expressed in standard form as follows:

$$D = 2.777 \times 10^{-16}(W^{3.401}V^{0.291}T^{2.776}S^{2.390}TR^{0.214}) \tag{8}$$

In Table 3, we present the dispersion coefficient model for both the dry and wet season for River Balogun. The model has tracer concentration, spacing interval between sections, time of sampling, velocity, and width of the river as the independent variables. It revealed that there is a positive relationship between the dispersion coefficient and the tracer concentration obtained during the experimental scheme. In addition, it showed that for a unit accuracy of the dispersion number to be obtained, the tracer concentration observed at the river section should be increased by 0.162 units. The relationship is supported by the $t$-ratio and the $p$-value, which had their values set at –10.321 and 0.0001 at 1% significance. This implied that we are 99% sure that this relationship stands for these variables based on the datasets generated from the experiment. Furthermore, the relationship between spacing between sampling points and dispersion coefficient were also investigated. These relationships showed positive relationship and revealed that for the dispersion coefficient to decrease by 1%, the spacing would have to decrease by 3.80%. Likewise, the corresponding $t$ and $p$ values from the table revealed also that spacing or sampling distance interval was a significant parameter to be considered for the effectiveness of our model. With $t$ = 6.124 and $p$ = 0.0001, the variable is statistical significant at 1%. In the same vein, the relationship between the sampling time interval and the dispersion coefficient were not different. It exposed that for the dispersion coefficient to be more accurate by unity, the sampling time should be increased by 2.308% thus showing a direct relationship. This findings corroborates with the findings of Agunwamba [1], which earlier identified the possible errors to be obtained from the subjectivity in sampling time interval by most researches on tracer studies found in the literature. Similarly, consultation of the $t$ and the $p$ values revealed the statistical and model relevance of sampling time interval. This implied that the various time adopted by many researches in the literature on tracer studies might have contributed to the inaccurate estimation of dispersion number or its coefficient. Therefore, care should be taken in this regard in order to reduce subjectivity in sampling time selection. This is one strong point of this method as no top priority is given to time selection. Additionally, the $t$ and $p$ values of this variable, which is set at 5.087 and 0.0001, confirmed the statistical significance of sampling time interval. Consequently, the relationship of the dispersion coefficient with the channel width were evaluated. The result showed a direct relationship between the two variables and supports the earlier findings of Seo and Cheong [33], Deng et al. [34], Kashefipour and Falconer [35], Seo and Baek [36], and Sedighnezhad and Salehi [37]. Specifically, 1% increase in the dispersion coefficient would require the width to increase by 8.914%, which was statistically significant ($t$ = 4.897, $p$ = 0.000). Lastly, we examined the relationship between the velocity of flow and dispersion coefficient. It showed that an increase in the velocity of the river flow by a unit would result in an increase in the dispersion coefficient by 2.487%. The assertion from this model agreed with the literature studies on dispersion number or coefficient determination [38,39]. However, this research like the other approach that considered variability in velocity has pointed out one of the major limitations in the Levenspiel and Smith approach that assumed that velocity is uniform along the channel. The corresponding $t$-statistics and probability values confirm the statistical relevance of this parameter. The value

Table 3
Model and parameter sensitivities using the Euler–Lagrangian appproach

| Variable | Coefficients | Standard error (SE) | $t$-Ratio | Probability | $R^2$ | Adjacent $R^2$ | Durbin–Watson statistic (DW) |
|---|---|---|---|---|---|---|---|
| C | –44.419 | 4.304 | –10.321 | 0.000 | | | |
| Tracer concentration | 0.162 | 0.066 | 2.453 | 0.030 | 0.76 | 0.72 | 2.24 |
| Spacing | 3.809 | 0.620 | 6.124 | 0.000 | | | |
| Time | 2.308 | 0.454 | 5.087 | 0.000 | | | |
| Velocity | 2.487 | 0.931 | 2.671 | 0.020 | | | |
| Width | 8.914 | 1.820 | 4.897 | 0.000 | | | |

Source: Authors' computation achieved with EViews 8.0 statistical software.
Note: C is constant.

of DW revealed the serial correlation between two datasets from a particular variable measured at different times [32,40]. Also, it is used to identify variables that are not skewed as well as to increase the confidence placed on the suitability of a time series data at normalized state. The DW statistics was employed as a dimensionless value that also controlled for multicollinearity and autocorrelation of datasets. The recommended value for a good model ranges between 1.8 and 2.2. For this model, the DW obtained was 2.24 revealing the absence of autocorrelated errors [31,32] after modeling and further strengthened the predictive capacity of the model generated. Additionally, high *F*-statistical value of 3.480 as well as the probability (*F*-statistics) of 0.035 are also a strong indication of a good model and well selected parameters. Moreover, to determine the stability of our coefficients in the model, CUSUM and CUSUMSQ tests were carried out (Figs. 1 and 2). The test helped to know the stability of our variables and the produced coefficients both in the long and short term. Specifically, the red lines showed the boundaries where it was expected that the stability of the variables do not cross. The blue line in turn represents the coefficients. The hypothesis

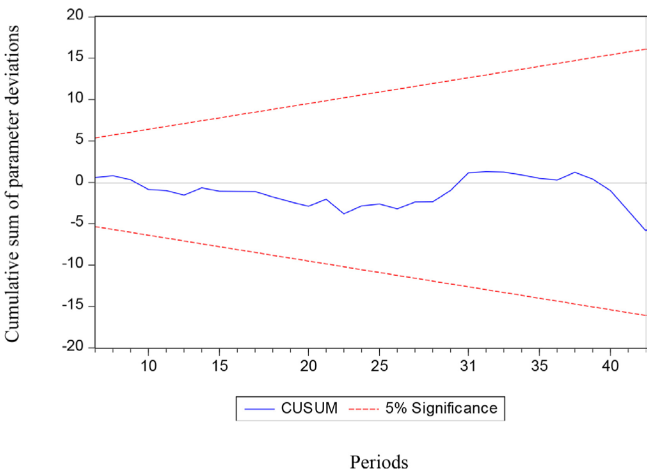


Fig. 1. CUSUM test result for log-transformed variables for wet and dry experimental schemes.

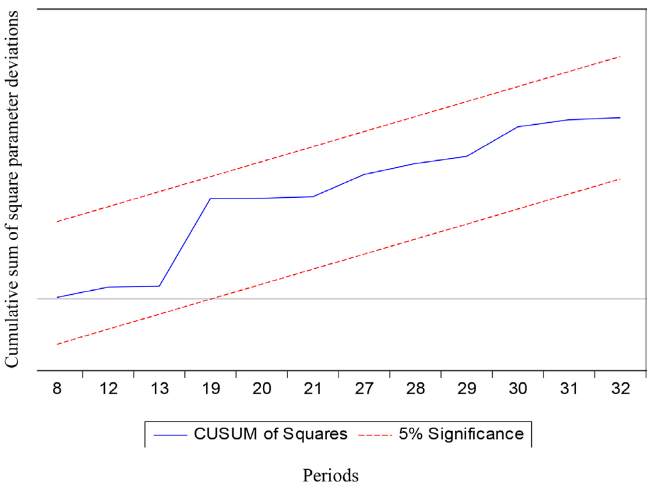


Fig. 2. CUSUM of squares test result for log-transformed combined dataset.

from the figure is that for a stable set of coefficients, the blue lines should not exceed the boundaries. Therefore, from this investigation, we see that the parameters and their coefficients were stable (Figs. 1 and 2). Furthermore, the stability of the coefficients developed was tested both for the short and long term. From the CUSUM and CUSUMSQ tests had good stability overtime as the blue lines do not exceed the boundary of the red line.

## 5. Conclusion

This study assessed the parameters required for the estimation of dispersion number or its coefficient using the Euler–Lagrangian approach for River Balogun in South West Nigeria as case study. This was used to develop a statistical model which revealed similar relationships with the variable distance and constant time method which is a widely used method in the literature, that is, dispersion coefficient had a direct relationship with width ($t = 4.90$, $p = 0.000$) and velocity ($t = 2.671$, $p = 0.020$). Additionally, tracer concentration ($t = 2.543$, $p = 0.030$), sampling distance ($t = 6.124$, $p = 0.000$), and sampling time interval ($t = 5.087$, $p = 0.000$) have revealed to be pertinent variables to be considered while carrying out tracer studies as all were statistically significant thereby showing good signs of a promising, alternative, and affordable technique. It is also suggested that the Euler–Lagrangian method should be applied to all rivers especially rivers with dispersion coefficient history in order to validate its usability and performance.

## Acknowledgment

The authors appreciate the commitment of Covenant University to the actualization of this study by providing good research environment.

## Symbols and abbreviations

| | | |
|---|---|---|
| CUSUM | — | Cumulative sum of recursive residual |
| CUSUMSQ | — | Cumulative sum of squared recursive residual |
| DW | — | Durbin–Watson statistics |
| LN (velocity) | — | Log of velocity coefficients |
| LN (width) | — | Log of width coefficients |
| LN (tracer concentration) | — | Log of tracer concentration coefficients |
| LN (spacing) | — | Log of spacing interval coefficients |
| LN (time) | — | Log of sampling time interval coefficients |
| $r$ | — | Coefficient of determination |
| $R^2$ | — | Coefficient of correlation |
| SD | — | Standard deviation |
| SE | — | Standard error |
| $t$ | — | *t*-Statistics |
| $e_i = y_i - \bar{y}_i$ | — | Difference between the observed and predicted values of the response variable for individual *i* |
| $T$ | — | Total number of observations |

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