

# Research on government supervision and enterprise water pollution control based on the principal–agent model

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Received 24 February 2018; Accepted 2 May 2018

#### ABSTRACT

The conflict of interests between the government and the enterprise plays a critical role in the effect of water pollution control. In order to resolve that conflict, corresponding government policy should be made. However, the current policies are all limited to the pollution tax or the incentive mechanism, respectively. This paper presents a principal–agent model taking the both policies into consideration. The optimal ration of water pollution control amount under the given policy is investigated. The relationships among the optimal ration of water pollution control amount and the awards proportion for water pollution control and the pollution tax rate are presented. An example is given to illustrate the idea of the proposed model and the effectiveness of the designed algorithm, which shows that incentive policy and pollution tax both have positive influence on enterprise's reduction in water pollution. Further, it proves that the government applied the combined policy of water pollution control incentive policy and pollution tax charging is better than applying any of them separately. The enterprise's optimal strategy of water pollution control adaptive to the government supervision can be obtained from the proposed model.

Keywords: Water pollution control; Government supervision; Asymmetric information; Principal–agent model

# 1. Introduction

With the rapid development of social economy, the water pollution problem has been given more attention from all sectors of the country. It has been the main environment problem increasingly. Government has taken measures to control and manage the water pollution, including improving wastewater treatment, industrial pollution control, municipal sanitation, and solid waste services and management. Anyhow, most of those measures do not have obvious effect. The difficulty lies in the fact that the enterprise always changes its strategy of water pollution control so as to get adapted to the government supervision. For example, the enterprise will decrease pollution control or choose illegal emission when the government fails to supervise, while the enterprise will increase pollution control and choose legal emission when the government heavily supervises. The differences and divergences of interests lead to a game relationship between the government and the enterprise.

To improve the water pollution control effect, the domestic and foreign scholars discussed the game problems between the government and the enterprise. The government always has different policy options such as pollution tax and subsidies. Harford [1] analyzed firm behavior under imperfectly enforceable pollution standards and taxes. Zhao et al. [2] proposed a model of transfer tax in which the transfer tax rate can enhance cooperation on

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Presented at the 3rd International Conference on Recent Advancements in Chemical, Environmental and Energy Engineering, 15–16 February, Chennai, India, 2018

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pollution reduction among the regions, meanwhile, Zhao et al. [3] presented a bi-level programming framework for a harmonizing model with transfer tax; the proposed model not only solves the problem of conflicts over water pollution across regional boundaries but also utilizes the resources of the lake basin more efficiently, Chen and Lu [4] discussed practical problems of government pollution taxation under the concept of environment rent philosophy. Besides charging the pollution tax, it should establish the trade market for pollution emission right and reward the enterprise by reducing pollution discharge. Krause and Hermannöwna [5] presented the "pro-green" incentive that both the central government and the public are placing pressure on China's urban leaders to mitigate externalities. Gregory [6] established and applied the economic incentives for the control of agricultural nonpoint source water pollution to predict changes in farmer decision-making and effluent production in response to policy alternatives. Shan et al. [7], Gong et al. [8], and other scholars discussed the optimize problem of pollution control with different targets including economic target and social target. In the aspect of regional coalition of water pollution control, Xue et al [9,10] constructed an interprovincial cooperative air pollution control game model and obtained the optimal reduction and reduction cost for each province, Cao [11] gave a regional sewage090005s fuzzy comprehensive evaluation model to ensure the effectiveness of the pollution control policy and long-lasting.

Though there are a few previous researches focusing on strategies the governments make in the process of water pollution control, the study is limited to the pollution tax or the incentive mechanism. This paper will present a principalagent model combining both of the policies. The optimal ration of water pollution control amount under the given policy is investigated.

# 2. The principal-agent model based on asymmetric information

Here, we assume that the government and the enterprise both pursue their own interests. Based on the above assumption, we will discuss the principal–agent relationship between the government and the enterprise in this section, which includes two cases: the government and the enterprise choosing only two actions and choosing a continuum of actions.

#### 2.1. Two actions

The simplest case is that the government and the enterprise only have two possible actions to choose. The two actions of government are applying or not applying the reward and punishment system, and the two strategies of the enterprise are choosing a high level or a low level of water pollution control effort. The government applies or not applies the reward and punishment system means that the government heavily or weakly supervises the enterprises' level of water pollution control. Heavy supervision means the government will reward the enterprise with a high level of water pollution control effort and afford incentive payment to it. Conversely, the government will punish the enterprise with a low level of water pollution control effort and get compensation from it. Weak supervision means that the government does not apply the reward and punishment system and there will be no incentive payment or compensation whether the enterprise chooses a high level or a low level of water pollution control effort.

The notations that will be used hereafter are listed as follows:

 $S_1$ : the output of the enterprise with a high level of water pollution control effort,

 $S_1$ : the output of the enterprise with a low level of water pollution control effort,

 $P_1$ : the incentive payment that the government affords to the enterprise with a high level of water pollution control effort when the government applies the reward and punishment system,

 $P_1$ : the compensation that the government gets from the enterprise with a low level of water pollution control effort when the government applies the reward and punishment system,

 $T_1$ : the total benefits that the enterprise with a high level of water pollution control effort brings to the government which mainly consist of social benefits and economical benefits,

 $T_1$ : the total benefits that the enterprise with a low level of water pollution control effort brings to the government which mainly consist of social benefits and economical benefits,

 $I_1$ : the pay that the enterprise gets with a high level of water pollution control effort, and

 $I_1$ : the pay that the enterprise gets with a low level of water pollution control effort.

Commonly, we can assume that the correlation between output, pay, and effort degree is positive, that is, a high output and a high pay of the agent are associated with high-effort degree. Thus,  $S_1 > S'_1$ ,  $I_1 > I'_1$ . Moreover, the value of the incentive payment and compensation will be zero when the government does not apply the reward and punishment system, which is called weak supervision.

Based on asymmetric information, the government completely knows the effort levels of the enterprise, and the output with the levels as well. At the same time, the enterprise also knows its effort levels and the corresponding pay it can get with its water pollution control effort level. Simply, we firstly discussed that the government and the enterprise have only two strategies to choose, that is, the enterprise can choose a high level of water pollution control effort or a low level of water pollution control effort, and the government can choose to heavily or weakly supervise the enterprise. Therefore, there are four kinds

Table 1

The utilities of the government and the enterprise with different choices

	I(+,+)	I(+,-)	I(-,+)	I(-,-)
The utilities of the government	$T_1 - P_1$	$T_1$	$T_{1}' + P_{1}'$	$T_1'$
The utilities of the enterprise	$S_1 - I_1 + P_1$	$S_{_{1}} - I_{_{1}}$	$S_{1}' - I_{1}' - P_{1}'$	$S_{1}^{'} - I_{1}^{'}$

of possible outcomes, which are shown in Table 1. Where the first sign of *I* represents whether the enterprise chooses a high level of water pollution control effort or chooses a low level of water pollution control effort. Plus sign means to choose a high level of water pollution control effort, whereas minus sign means to choose a low level of water pollution control effort. In a similar way, the second sign of *I* represents whether the government chooses to apply or not to apply the reward and punishment system. Plus sign means to apply the reward and punishment system, whereas minus sign means not to apply the reward and punishment system. For example, I(+,-) represents that the enterprise chooses a high level of water pollution control effort (the first sign is plus), the government chooses not to apply the reward and punishment system (the second sign is minus).

Table 1 shows that for the enterprise, when the government chooses weak supervision, the utility of the enterprise with a low level of water pollution control effort  $(S_1 - I_1)$  is greater than that with a high level of water pollution control effort  $(S'_1 - I'_1)$ . That is, when the government chooses weak supervision, choosing a low level of water pollution control effort is a strictly Nash equilibrium for the enterprise.

When the government chooses heavy supervision, the utilities of the enterprise with a high and low level of water pollution control effort are  $S_1 - I_1 + P_1$  and  $S'_1 - I'_1 - P'_1$ , respectively. In this case, whether the enterprise chooses a high or a low level of water pollution control effort is up to the rewards and punishment strength by the government. If the values of the incentive payment ( $P_1$ ) and compensation ( $P'_1$ ) for the enterprise are big enough, the utility of the enterprise with a high level of water pollution control effort ( $S_1 - I_1 + P_1$ ) will be greater than that of the enterprise with a low level of water pollution control effort ( $S'_1 - I'_1 - P'_1$ ).

In conclusion, based on asymmetric information, when the government chooses weak supervision, choosing a low level of water pollution control effort is a strictly Nash equilibrium for the enterprise. Conversely, when the government chooses heavy supervision and the rewards and punishment strengths are big enough, choosing a high level of water pollution control effort is a strictly Nash equilibrium for the enterprise.

#### 2.2. Continuum of actions

When there are more than two actions, the government will design the rewards and punishment contract with the aim of maximizing its expected revenue, which can be formulated as follows:

$$E(R_g) = E[Y(x) - S(x)] \tag{1}$$

where  $R_g$  means the revenue of the government and Y(x) is pollution tax that the enterprise pay for the government, which includes economic and social compensation for pollution discharge. The specific definition of Y(x) is as follows:

$$Y(x) = t_2(1 - x)$$
(2)

Here, *x* means the ration of water pollution control amount, (1 - x) means the ratio of the remaining pollution, and  $t_2$  means the pollution tax rate.

At the same time,

$$S = t_1 x \tag{3}$$

where  $t_1$  means the awards proportion of the government offers to the enterprise for their water pollution control. *S* means the incentive payment that the government affords to the enterprise when its ration of water pollution control amount is *x*. Thus,

$$E(R_{o}) = E[Y(x) - S(x)] = E[t_{2}(1 - x) - t_{1}x]$$
(4)

Based on the given rewards and punishment contract above, the enterprise's expected revenue can be described as follows:

$$E(R_{e}) = E[S - C(x) - Y(x)]$$
(5)

where  $R_e$  means the revenue of the enterprise. For simplicity, we assume that the water pollution control cost of the enterprise is expressed as  $C(x) = \frac{1}{2}x^2$ , which is decided by its ration of water pollution control amount. Then,

$$E(R_{e}) = E[S - C(x) - Y(x)] = t_{1}x - \frac{1}{2}x^{2} - t_{2}(1 - x)$$
(6)

In the water pollution control process, the enterprise's task is deciding the optimal ration of water pollution control amount x to maximize its expected revenue, thus, the first-order optimal condition of the enterprise's expected revenue can be established as follows:

$$x \in \arg\max_{x}(t_{1}x - \frac{1}{2}x^{2} - t_{2}(1 - x))$$
(7)

Based on the above analysis, the government's incentive contract is shown as follows:

$$\max_{t_1, t_2} [t_2(1-x) - t_1 x]$$
s.t. 
$$\arg_x [t_1 x - \frac{1}{2} x^2 - t_2(1-x)]$$

$$t_1 x - \frac{1}{2} x^2 - t_2(1-x) \ge 0$$
(8)

where constraint (c) denotes the participation constraint of the enterprise, which means that if the expected benefits of water pollution control is nonnegative, and it would participate in the water pollution control, otherwise the water pollution control would not be realized. The constraint (b) states the incentive compatibility constraint of the enterprise, which means the expected revenue of the enterprise with a ration of water pollution control amount x is greater than any other ration. So the meaning of the Eq. (8) is how to design a contract to maximize the benefit of the government under the participation constraint and incentive compatibility constraint of the water pollution control enterprise.

Because the enterprises will choose the optimal ration of water pollution control amount based on the principle of maximizing profits, the optimization condition of first order can be formed as follows:

$$\frac{\partial E(R_e)}{\partial x} = -x^* + t_2 + t_1 = 0 \tag{9}$$

Equivalently,

$$x^* = t_1 + t_2 \tag{10}$$

#### **Proposition 1**

It can be seen from the Eq. (10) that  $\frac{\partial x^*}{\partial t_1} > 0$ , then there is a positive correlation between the optimal ration of water pollution control amount ( $x^*$ ) and the awards proportion of the government offers to the enterprise for their water pollution control ( $t_1$ ), which means that water pollution control incentive policy has positive influence on enterprise's reduction incentives.

### **Proposition 2**

It can be seen from the Eq. (10) that  $\frac{\partial x^*}{\partial t_2} > 0$ , then there is a positive correlation between the optimal ration of water pollution control amount ( $x^*$ ) and the pollution tax rate ( $t_2$ ), which means that charging pollution tax has positive influence on enterprise's reduction incentives.

Substituting Eq. (10) into the formulae (a) and (c) in the Eq. (8), we have the following:

$$\min_{\substack{ts.t_1,t_2\\ s.t.}} \left( t_1^2 + t_2^2 + 2t_1t_2 - t_2 \right) \quad (a)$$
s.t.
$$t_1^2 + t_2^2 + 2t_1t_2 - t_2 \ge 0 \quad (b)$$
(11)

#### Theorem 1

The Eq. (11) is a convex programming problem.

# Proof

Let the objective function of the Eq. (11) be  $f(t_1,t_2)$ , where  $f(t_1,t_2) = t_1^2 + t_2^2 + 2t_1t_2 - t_2$ . Because the Hesse matrix of  $f(t_1,t_2)$  is  $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ , which is a positive definite matrix, so  $f(t_1,t_2)$  is a convex function, Similarly, we could get the conclusion that the feasible region of the Eq. (11) is a convex set. Because the objective function of the Eq. (11) is a convex function, and its feasible region is a convex set, so Eq. (11) is a convex programming problem.

#### Which ends the proof

#### 3. Solving of the principal-agent model

In this section, a sequential quadratic programming (SQP) method [12] will be applied to solve the Eq. (11) which is a convex planning problem with nonlinear constraints.

Let  $f(t) = f(t_1, t_2) = t_1^2 + t_2^2 + 2t_1t_2 - t_2$  then the second-order Taylor expansion of f(t) in  $t^k$  is as follows:

$$f(t_{1}^{k}, t_{2}^{k}) + \left((t_{1} - t_{1}^{k})\frac{\partial}{\partial t_{1}} + (t_{2} - t_{2}^{k})\frac{\partial}{\partial t_{2}}\right)f(t_{1}^{k}, t_{2}^{k}) + \frac{1}{2!}\left((t_{1} - t_{1}^{k})\frac{\partial}{\partial t_{1}} + (t_{2} - t_{2}^{k})\frac{\partial}{\partial t_{2}}\right)^{2}f(t_{1}^{k}, t_{2}^{k})$$
(12)

where,

$$\begin{aligned} f(t_1^k, t_2^k) &= (t_1^k)^2 + (t_2^k)^2 + 2t_1^k t_2^k - t_2^k, \\ \frac{\partial f(t_1^k, t_2^k)}{\partial t_1} &= 2t_1^k + 2t_2^k, \quad \frac{\partial f(t_1^k, t_2^k)}{\partial t_2} = 2t_2^k + 2t_1^k - 1, \\ \frac{\partial f^2(t_1^k, t_2^k)}{\partial t_1^2} &= 2, \quad \frac{\partial f^2(t_1^k, t_2^k)}{\partial t_2^2} = 2, \quad \frac{\partial f^2(t_1^k, t_2^k)}{\partial t_1 \partial t_2} = \frac{\partial f^2(t_1^k, t_2^k)}{\partial t_2 \partial t_1} = 2 \end{aligned}$$

Thus, the second-order Taylor expansion of f(t) in  $t^k$  can be obtained as follows:

$$f(t_{1}^{k}, t_{2}^{k}) + \left((t_{1} - t_{1}^{k})\frac{\partial}{\partial t_{1}} + (t_{2} - t_{2}^{k})\frac{\partial}{\partial t_{2}}\right)f(t_{1}^{k}, t_{2}^{k}) + \frac{1}{2!}\left((t_{1} - t_{1}^{k})\frac{\partial}{\partial t_{1}} + (t_{2} - t_{2}^{k})\frac{\partial}{\partial t_{2}}\right)^{2}f(t_{1}^{k}, t_{2}^{k}) = (t_{1})^{2} + (t_{2})^{2} + 2t_{1}t_{2} - t_{2} - (t_{1}^{k})^{2} - (t_{2}^{k})^{2}$$
<sup>(13)</sup>

Similarly, the first-order Taylor expansion of f(t) in  $t^k$  is as follows:

$$f_{3}(x_{1}^{k}, x_{2}^{k}) + (x_{1} - x_{1}^{k}) \frac{\partial f_{3}(x_{1}^{k}, x_{2}^{k})}{\partial x_{1}} + (x_{2} - x_{2}^{k}) \frac{\partial f_{3}(x_{1}^{k}, x_{2}^{k})}{\partial x_{2}}$$

$$f(t_{1}^{k}, t_{2}^{k}) + \left((t_{1} - t_{1}^{k}) \frac{\partial}{\partial t_{1}} + (t_{2} - t_{2}^{k}) \frac{\partial}{\partial t_{2}}\right) f(t_{1}^{k}, t_{2}^{k})$$

$$= 2(t_{1}^{k} + t_{2}^{k})t_{1} + (2t_{1}^{k} + 2t_{2}^{k} - 1)t_{2} - (t_{1}^{k} + t_{2}^{k})^{2}$$
(14)

Let

$$\begin{split} &A_1(t^k) = A_1(t_1^k, t_2^k) = -t_2 - (t_1^k)^2 - (t_2^k)^2, \\ &M_1(t^k) = M_1(t_1^k, t_2^k) = 2(t_1^k + t_2^k), \\ &M_2(t^k) = M_2(t_1^k, t_2^k) = (2t_1^k + 2t_2^k - 1), \\ &M_3(t^k) = M_3(t_1^k, t_2^k) = -(t_1^k + t_2^k)^2 \end{split}$$

The subproblem of the Eq. (11) can be turned into as follows:

$$\min (t_1)^2 + (t_2)^2 + 2t_1t_2 - t_2 + A_1(t^k)$$
  
s.t.  $M_1(t^k)t_1 + M_2(t^k)t_2 + M_3(t^k) \ge 0$  (15)

In the following, we provide the detailed procedure of the SQP method for finding an optimal solution to the Eq. (15). The procedure of the algorithm is shown as follows:

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Algorithm:

Step 1: Let

 $F(t^k) = (t_1)^2 + (t_2)^2 + 2t_1t_2 - t_2 + A_1(t^k)$ 

At the same time, let the initial feasible solution and the objective function value of the Eq. (15) be  $t^0$  and  $F(t^0)$ , respectively, then the sequential quadratic subprogramming problem can be described as follows:

$$(SQP_0)\min(t_1)^2 + (t_2)^2 + 2t_1t_2 - t_2 + A_1(t^0)$$
  
s.t.  $M_1(t^0)t_1 + M_2(t^0)t_2 + M_3(t^0) \ge 0$  (16)

Then  $t^1$  (the optimal solution) and  $F(t^1)$  (the optimal objective function value) can be obtained by solving the Eq. (16) using an interior point method [13]. If  $|F(t^1) - F(t^0)| < \varepsilon$ ,  $\varepsilon \le 10^{-6}$ , then  $t^* = t^1$ ,  $F(t^*) = F(t^1)$ , where  $t^*$  and  $F(t^*)$  are the optimal solution and the optimal objective function value of the Eq. (16). Otherwise, turn Step 2.

Step 2:

When k = m ( $m \ge 1$ ,  $m \in z^+$ ), let the optimal solution and the optimal objective function value of the *m*th iteration subproblem of the model (20) be  $t^m$  and  $F(t^m)$ , respectively, the procedures of solving SQP can be described as follows:

$$(SQP_0)\min(t_1)^2 + (t_2)^2 + 2t_1t_2 - t_2 + A_1(t^m)$$
  
s.t.  $M_1(t^m)t_1 + M_2(t^m)t_2 + M_3(t^m) \ge 0$  (17)

Then  $t^{m+1}$  (the optimal solution) and  $F(t^{m+1})$  (the optimal objective function value) of the Eq. (17) can be obtained by solving Eq. (17) using an interior point method.

Step 3:

If  $|F(t^{m+1}) - F(t^m)| \le \varepsilon$ ,  $\varepsilon \le 10^{-6}$ , the optimal solution  $t^*$  and the optimal objective function value  $F(t^*)$  of the Eq. (16) can be obtained as  $t^* = t^{m+1}$ ,  $F(t^*) = F(t^{m+1})$ . Otherwise, let k = m + 1 and turn Step 2.

#### 4. Results and discussion based on numerical examples

In this section, an example is given to express the idea of the proposed model. In this example, we use the method of SQP to solve the problem and two relationships are discussed as follows:

(1) Relationship between the optimal ration of water pollution control amount and the awards proportion for water pollution control When the awards proportion of the government offers to the enterprise for their water pollution control ( $t_1$ ) is set to 0.05, 0.1, ..., 0.95 in the value interval of [0.05, 0.95], the pollution tax rate ( $t_2$ ) is set to 0.2, the optimal ration of water pollution control amount ( $x^*$ ) can be obtained as Table 2 shows by solving the Eq. (8). Accordingly, Fig. 1 shows the effect of the awards proportion that the government offers to the enterprise for their water pollution control ( $t_1$ ) on the optimal ration of water pollution control amount ( $x^*$ ) for enterprise when  $t_2 = 0.2$ ,  $t_1 = 0.05$ , ..., 0.95.

Table 2 and Fig. 1 show that when the pollution tax rate  $(t_2)$  is set to 0.2, the optimal ration of water pollution control amount  $(x^*)$  increases with increasing  $t_{1'}$  which means that the enterprise will promote the ration of water pollution control amount when the awards proportion of the government offers to the enterprise increase. It can be concluded that water pollution control incentive policy has positive influence on enterprise's reduction incentives.

(2) Relationship between the optimal ration of water pollution control amount and the pollution tax rate

When the pollution tax rate  $(t_2)$  is set to 0.05, 0.1, ..., 0.95 in the value interval of [0.05, 0.95], the awards proportion of the government offers to the enterprise for their water pollution control  $(t_1)$  is set to 0.3, the optimal ration of water pollution control amount  $(x^*)$  can be obtained as Table 3 shows by solving the Eq. (8). Accordingly, Fig. 2 shows the effect of the pollution tax rate  $(t_2)$  on the optimal ration of water pollution control amount  $(x^*)$  for enterprise.

Table 3 and Fig. 2 show that when the awards proportion of the government offers to the enterprise for their water pollution control ( $t_1$ ) is given by 0.3, the optimal ration of



Fig. 1. The effect of  $t_1$  on the optimal ration of water pollution control amount ( $x^*$ ) for enterprise when  $t_2 = 0.2$ ,  $t_1 = 0.05$ , ..., 0.95.

Table 2

The optimal ration of water pollution control amount for enterprise when  $t_2 = 0.2$ ,  $t_1 = 0.1$ , ..., 0.95

$t_1$	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
<i>x</i> *	0.1884	0.2501	0.3186	0.3278	0.3495	0.3721	0.4012	0.4288	0.4493	0.5
$t_1$	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	
<i>x</i> *	0.5226	0.5384	0.5562	0.5722	0.5996	0.6218	0.645	0.7023	0.7186	

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$t_2$	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
<i>x</i> *	0.3252	0.4021	0.4586	0.4758	0.4965	0.5234	0.5497	0.5764	0.5985	0.6534
$t_2$	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	
<i>x</i> *	0.6712	0.6834	0.7012	0.7242	0.7506	0.7724	0.7958	0.8493	0.8645	

Table 3 The optimal ration of water pollution control amount for enterprise when  $t_1 = 0.3$ ,  $t_2 = 0.05$ , ..., 0.95



Fig. 2. The effect of  $t_2$  on the optimal ration of water pollution control amount ( $x^*$ ) for enterprise when  $t_1 = 0.3$ ,  $t_2 = 0.05$ , ..., 0.95.

water pollution control amount ( $x^*$ ) increases with increasing  $t_{2'}$  which means the enterprise will promote the ration of water pollution control amount when the pollution tax rate ( $t_2$ ) increase. It can be concluded that charging pollution tax has positive influence on enterprise's reduction incentives.

# 5. Conclusion

This paper presents a model to describe the principal-agent relationship between the government and the enterprises in the water pollution control process. First, based on the asymmetric information theory, a simple game model is established to analyze the government's and the enterprise's strategy choosing. Then, a principal-agent model is established and the method of SQP is proposed to solve it. Results and discussions based on a numerical example are given to illustrate the relationships between the optimal ration of water pollution control amount, the awards proportion for water pollution control, and the pollution tax rate. The following conclusions can be drawn from this study:

Water pollution control incentive policy and charging pollution tax both have positive influence on enterprise's reduction. Further, the governments applied the combined policy of water pollution control incentive policy and charging pollution tax is better than apply any of them separately. The enterprise's optimal strategy of water pollution control adapted to the government supervision can be obtained from the proposed model.

#### Acknowledgments

The authors acknowledge the Fundamental Research Funds for the Central Universities (Grant no.: 2015-yb-006), Educational Commission of Hubei Province of China (Grant no.: B2016403), the Research Fund of Wuhan Huaxia University of Technology (Grant no.: 15027), and Humanity and Social Science Youth Foundation of Ministry of Education of China (Grant no.: 14YJCZH165).

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