



## Study on mathematical model of condenser in seawater desalination device

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### ABSTRACT

This paper aims to design the mathematical model of the condenser in seawater desalination system. And the working process of the condenser is deduced by formula, and the modeling is set up through the analysis of the structure of the condenser. Moreover, the model is verified, which carried on a further basic theoretical study on the application of heat pump-type seawater desalination device.

*Keywords:* Seawater desalination; Condenser; Modeling

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### 1. Introduction

Due to its energy consumption, desalinating sea water is generally more costly than freshwater from rivers or groundwater, water recycling, and water conservation. However, these alternatives are not always available, and depletion of reserves is a critical problem worldwide. Currently, approximately 1% of the world's population is dependent on desalinated water to meet daily needs, but the UN expects that 14% of the world's population will encounter water scarcity by 2025 [1]. As the contradiction of freshwater resources shortage is prominent, it has become an international consensus to make water from sea, and the means is seawater desalination. After decades of research and development [2], seawater desalination technology has been matured, so far, regardless of the current technology or economy; in some areas, it can turn seawater into freshwater in a large scale. Until 2012, the whole world desalination unit capacity per day of more than 100 tons in total production capacity has been over  $120 \times 10^6$  t/d [3], thus, solving the regional water supply has become a reality. According to the statistics from International Desalination Association, at present in the world, 15,233 seawater desalination units are in the operation

status totally, which are distributed in 125 countries and solve the problem of 100 million people's living water, as well as a large amount of industrial water [4]. In this paper, a new seawater desalination device is presented, and the working principle of the system is expounded, the corresponding experimental platform is set up, which also analyzed and calculated the performance of the system theoretically.

The feature of this device is to take the evaporation condenser as the heat source of seawater desalination device for heating water [5], while the evaporation condenser is made up of condenser and water shower, seawater spray to form water film in the condenser coil and fins, which can fully absorb the latent heat released by the refrigerant of the tube, so that the temperature of seawater can rise, the driving force of heat transfer is the temperature difference between the condensing temperature and water film; on the other hand, through the heat exchange of water film and air flow, it can realize salt water separation [6], the vaporization process needs to absorb great amount of latent heat of vaporization, which further promote the condensation process of the refrigerant in the tube, so as to release latent heat; the comprehensive result of two aspects is to reduce the condensing temperature, which can make the temperature and

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humidity improved through the evaporation condenser, so as to make it close to the saturated moist air. The saturated wet air obtained by the evaporation condenser can be condensed into freshwater by the evaporator (surface cooler) [7]. At this time, the refrigerant in the evaporator tube and the wet air outside the tube are undergoing phase change heat transfer, which is the same as the evaporation condenser that is an efficient heat transfer component for phase change heat transfer. In this paper, based on research on the mathematical modeling of the evaporation condenser, the mathematical model of the condenser in the heat pump desalination system is proposed.

## 2. Mathematical model of evaporation condenser

### 2.1. Change of air state in evaporation condenser

The changing process of air state in evaporation condenser is shown in Fig. 1. Point 1 indicates the state of the inlet of air, Point 2 indicates the state of the outlet of air, Point m indicates the average state of inlet and outlet of air, and Point w indicates the saturated air state of water film surface. The imported air is changed along 1–W line.

### 2.2. Derivation of mass equation and energy equation

The condensing heat of the refrigerant in the evaporation condenser transfers to water film through the condensing coil. Then water film absorbs the heat and evaporates, and the vaporized water vapor is absorbed and carried away by air, so that the air baking increases [8]. As the dry ball temperature of air has little influence on the system, the main factor that can affect the heat transferring air and water film is the enthalpy of the imported air. Therefore, when the dry ball temperature of the imported air has little change, the enthalpy of the air is determined mainly by the wet bulb temperature, so the wet bulb temperature of the inlet air has a great influence on the performance of system.

The structure and flow status of evaporation condenser: when water and air are counterflowing, water and refrigerants are downstreaming. Before establishing a mathematical model, the following assumptions are made:

- (1) The air is a complete gas.
- (2) The change of the wet air with full pressure  $p$  can be ignored, which is considered to be equal to the atmospheric pressure  $p_0$ .
- (3) The air uniformly flows through the evaporation condenser coil, and the flow of the air mass is the same on the cross section of the circulation.
- (4) The spray water can spray evenly, the spray volume of each spray section is the same, and the condenser coil is wet evenly.
- (5) The heat ratio of water, steam, refrigerant, and air in the considering temperature range is a constant, the latent heat  $r$ , the convection heat transfer coefficient  $a$ , the convection mass transfer coefficient  $\beta_x$  can be regarded as constant on the evaporation surface.
- (6) Neglecting the heat and humidity loss of the air and the outside, as well as the heat loss around the condenser.

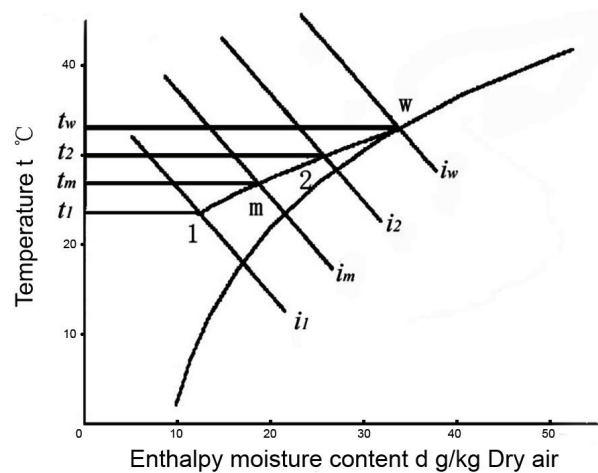


Fig. 1. The changing process of air state.

Assuming the flow rate of spray water is  $m_w$ (kg/s), the dry air flow is kg/s, the length between water and air is  $X$ , its width is  $B$ , its height is  $Y$ , and the unit volume vapor liquid contacting area is  $f_e$ .

#### 2.2.1. Mass conservation equation of air

The mass of water vapor entering the microelement with air in unit time is:  $m_a \frac{dx}{X} w$ , the mass of water vapor discharged from the microelement with air in unit time is:  $m_a \frac{dx}{X} (w + \frac{\partial w}{\partial y} dy)$ , and the evaporation volume of microelement in unit time is  $\beta_x f_e B dx dy (w_s - w)$ . According to the law of conservation of mass, we can get Eq. (1):

$$\frac{m_a}{X} \frac{\partial w}{\partial y} = \beta_x f_e B (w_s - w) \tag{1}$$

#### 2.2.2. Energy conservation equation of air

The enthalpy value of flowing into the infinitesimal body in unit time is  $m_a \frac{dx}{X} h_a$ , the enthalpy of outflow of microelement in unit time is  $m_a \frac{dx}{X} (h_a + \frac{\partial h_a}{\partial y} dy)$ , the convection heat of air and water in unit time is  $a_w f_e B dx dy (t_w - t_p)$ , and the required heat for evaporation of water in unit time is  $\beta_x f_e B dx dy (w_s - w) r$ . According to the law of conservation of energy, we can get Eq. (2):

$$\frac{m_a}{X} \frac{\partial h_a}{\partial y} = a_w f_e B (t_w - t_p) + \beta (w_s - w) r \tag{2}$$

#### 2.2.3. Conservation equation of water quality

The quality of the water flowing into the microelement in unit time is  $\frac{dx}{X} m_w$ , the mass of the water coming out of

the microelement in unit time is  $\frac{dx}{X}(m_w + \frac{\partial m_w}{\partial y} dy)$ , and the evaporation volume of the microelement in unit time is  $\beta_x f_e B dx dy (w_s - w)$ . According to the law of mass conservation, we can get Eq. (3):

$$\frac{1}{X} \frac{\partial m_w}{\partial y} = -\beta_x f_e B (w_s - w) \quad (3)$$

#### 2.2.4. Energy conservation equation of water

The water flows into the element in unit time is  $m_w c_w t_w \frac{dx}{X}$ , the enthalpy of water coming out of the microelement in unit time is  $\frac{dx}{X}(m_w c_w t_w + \frac{\partial(m_w c_w t_w)}{\partial y} dy)$ , the convection heat between air and water in unit time is  $a_w f_e B dx dy (t_w - t_p)$ , the convection heat between water and water vapor in unit of time is  $\beta_x f_e B dx dy (w_s - w)(c_w - c_{pw})$ , the required heat for evaporation of water in unit time is  $\beta_x f_e B dx dy (w_s - w)r$ , and the conversion heat between water and refrigerant in unit time is  $k_z f_e B dx dy (t_f - t_w)$ . According to the law of conservation of energy, we can get Eq. (4):

$$\begin{aligned} \frac{c_w}{X} \frac{\partial(m_w t_w)}{\partial y} &= k_z f_e B dy (t_f - t_w) - a_w f_e B dy (t_w - t_p) \\ &\quad - \beta_x f_e B t_w dy (w_s - w)(c_w - c_{pw}) \\ &\quad - \beta_x f_e B dy (w_s - w)r \end{aligned} \quad (4)$$

#### 2.2.5. Energy conservation equation of refrigerant

The enthalpy of refrigerant flowing into the microelement in unit time is  $m_f h_f \frac{dy}{Y}$ , the enthalpy of refrigerant flowing out of microelement in unit time is  $m_f \frac{dy}{Y}(h_f + \frac{\partial h_f}{\partial x} dx)$ , and the convection heat between refrigerant and water in unit time is  $k_z f_e B dx dy (t_f - t_w)$ . According to the law of conservation of energy, we can get Eq. (5):

$$\frac{m_f}{Y} \frac{\partial h_f}{\partial x} = -k_z f_e B dy (t_f - t_w) \quad (5)$$

#### 2.3. Mass equation and energy equation simplified

Assuming  $F_e = f_e BXY$ , the differential equations can be arranged in the following forms (Eqs. (6)–(11)):

$$m_a \frac{\partial w}{\partial \eta} = \beta_x F_e (w_s - w) \quad (6)$$

$$m_a \frac{\partial h_a}{\partial \eta} = a_w F_e (t_w - t_a) + \beta_x F_e (w_s - w)r \quad (7)$$

$$\frac{\partial m_w}{\partial \eta} = -\beta_x F_e (w_s - w) \quad (8)$$

$$\begin{aligned} c_w \frac{\partial(m_w t_w)}{\partial \eta} &= k_z F_e (t_f - t_w) - a_w F_e (t_w - t_a) \\ &\quad - \beta_x F_e (w_s - w)r - \beta_x F_e t_w (w_s - w)(c_w - c_{pw}) \end{aligned} \quad (9)$$

$$m_f \frac{\partial h_f}{\partial \xi} = -k_z F_e (t_f - t_w) \quad (10)$$

$$\frac{\partial h_w}{\partial t_w} = c_w, \quad \frac{\partial h_a}{\partial t_p} = c_p, \quad \frac{\partial h_f}{\partial t_f} = c_f \quad (11)$$

As for Eq. (9), considering the difference between sensible heat and latent heat of water, the heat needed for evaporation can be ignored. Assuming the specific heat of wet air is  $c_p$ , we can get Eq. (12):

$$\begin{aligned} m_a \frac{\partial h_a}{\partial \eta} &= \beta_x F_e \left[ \frac{a_w}{c_p \beta_x} (h_s - h_a) + r \left( 1 - \frac{a_w}{c_p \beta_x} \right) (w_s - w) \right] \\ c_w \frac{\partial(m_w t_w)}{\partial \eta} &= k_z F_e (t_f - t_w) \\ &\quad - \beta_x F_e \left[ \frac{a_w}{c_p \beta_x} (h_s - h_a) - r \left( 1 - \frac{a_w}{c_p \beta_x} \right) (w_s - w) \right] \end{aligned} \quad (12)$$

#### 2.4. Mass transfer coefficient $\beta_x$

It is necessary to know the value of mass transfer coefficient  $\beta_x$  in solving the differential equations. As for evaporation condenser, the mass transfer coefficient can be defined according to the following formula. It indicates mass transfer volume exist in per unit volume of vapor–liquid contact area in a unit moisture content difference. It is a function of air flow density  $g_a$  and water density  $g_w$ . It usually has the following form:

$$\beta_x = A g_a^m g_w^n \quad (13)$$

Among  $g_a = \frac{m_a}{A_a} = \frac{m_a}{BY}$ , it represents the fair mass flow on the unit area of circulation  $A_a$ , while  $g_w = \frac{m_w}{A_w} = \frac{m_w}{BX}$  can indicate the mass flow of water on the unit spraying surface  $A_w$ . As for the evaporation condenser  $A$  with certain structure, both  $m$  and  $n$  is constant and the value of them can be obtained by experiment.

### 3. Results and discussion

Considering the advantage of Fortran language in formula expression, this program uses Fortran language to simulate the expression of the model. Table 1 shows the experimental values and simulation results of the effects of the water-cooled condenser system parameters on freshwater production. The data in the table show that the experimental results are in good agreement with the calculated results.

Table 1  
Comparison of experimental and calculated values of the effects of parameter changes on freshwater production

Seawater inlet temperatures (°C)	Freshwater production (kg/h)	
	Experiment values	Calculated values
20	1.321	1.347
25	1.453	1.441
30	1.514	1.484
35	1.575	1.531
40	1.654	1.598
45	1.701	1.698
50	1.774	1.781
55	1.805	1.811
60	1.889	1.923
65	1.967	1.998
70	2.053	2.156

For the seawater desalination system with water-cooled condenser, the biggest characteristic is that the water-cooled condenser is used as a heat source to heat the sea water and raise the spray temperature of seawater. Because of the system constraints, it is impossible for seawater temperature to rise indefinitely. Thus, under certain conditions of other parameters, fresh water production is certain.

4. Conclusion

By establishing the mathematical model of evaporation condenser, simplifying the model and controlling the discretizing of equation, Fortran language can be used to program, and the numerical simulation is carried out, then numerical calculation of the temperature distribution of air and water in the evaporation condenser can be done. Through numerical simulation and experimental research, we can determine the mass transfer coefficient of evaporation condenser under certain conditions and a certain refrigerant, so that we can give a qualitative value to the circulating water volume as well as the supplementary water volume. By studying the mathematical model of evaporation condenser with air and water countercurrent, it can be applied to the situation of cross flow of air and water, namely the application of evaporation condenser in seawater desalination system. In this paper, the mathematical modeling of evaporation condenser is carried out, and its heat transfer characteristics are numerically simulated. Finally, the temperature distribution of air and water in evaporation condenser is analyzed, which can provide a theoretical basis for further application.

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Symbols

- a* — Heat emission coefficient, W/m<sup>2</sup>
- B* — Atmospheric pressure, Pa
- c* — Specific heat, kJ/kg °C
- d* — Moisture, kg/kg(dry)
- F* — Heat transfer tube surface area, m<sup>2</sup>
- m* — Flow mass, kg/h
- h* — Specific enthalpy of wet air, kJ/kg
- r* — Latent heat of vaporization, kJ/kg °C
- t* — Temperature, °C
- $\epsilon$  — Heat exchanger efficiency coefficient
- $\beta$  — Physical coefficient
- $\rho$  — Density of condensate, kg/m<sup>3</sup>
- $\mu$  — Dynamic viscosity of condensate, N s/m<sup>2</sup>
- $\lambda$  — Thermal conductivity of condensate, W/m °C
- X* — Length of efficient evaporator, m
- Y* — Height of efficient evaporator, m

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