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# Geographic Information System-based watershed geomorphic mapping

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**ABSTRACT**

The evolution process of geomorphic morphology is very long, and the geomorphic problems are very complex, so the measurement of geomorphology is very difficult. This exploration aims to enrich the description of comprehensive quantitative characteristics of watershed geomorphology, strengthen the research on the relationship among the fractal characteristics of geomorphology, the fractal dimension of geomorphology and the development of geomorphology, as well as the research on the fractal geomorphology model. The existing quantitative methods of watershed geomorphology have been fully studied. Then, the characteristics of the geomorphological quantitative description methods, such as mathematical statistics, correlation analysis and geomorphological single factor index, are comprehensively applied. The single factor index slope is introduced into the fractal model as an important feature information value of geomorphology. In the three-dimensional space scanning box coordinate system, the three-dimensional cube boxes with different scales are used as statistical units to calculate the amount of ground undulation under the average slope of the surface, so as to construct the fractal three-dimensional feature information dimension Geographic Information System model to represent the degree of ground undulation. The results show that the scale-free region is limited by the altitude, so it is only suitable for the mountainous area with large slope and high-resolution digital elevation model data to obtain more statistical data. Moreover, the value of the fractal dimension is proportional to the average slope. This exploration provides some reference ideas for the study of watershed geomorphic mapping, and has a positive impact on the development of watershed geomorphic mapping.

*Keywords:* Geographic Information System; Geomorphic morphology; Watershed geomorphic mapping; Geomorphic fractal dimension value; Fractal geomorphic model

## 1. Introduction

The evolution process of geomorphology is long-term in time and broad in space. Therefore, with the development and progress of science and technology, modern geomorphology research has gradually developed from qualitative description to quantitative research [1]. In addition, it is difficult to quantitatively study geomorphology because of the variable medium, wide range of action factors, complex process and long cycle involved in geomorphology. Therefore, the fundamental causes of geomorphology evolution are often studied and its evolution process is often explained from the perspective of material composition and dynamic process [2]. The scientific quantitative study of watershed geomorphology is one of the hot issues in geomorphology [3]. Its development has experienced a process from graphic to digital description. At present, the main methods of quantitative description of watershed geomorphology are correlation analysis and mathematical statistics [4]. This method has obvious advantages, but it is far from enough to describe the cross-scale characteristics of watershed geomorphology [5].

Geomorphology is a part of geography and geology [6], which mainly studies the formation, characteristics, distribution and evolution rules of the earth surface [7]. It is of great significance to agricultural production, engineering construction, prevention and control of natural disasters, mineral exploration and environmental protection [8]. Many geomorphologists focus on the genesis of geomorphology, which mainly acts on the forces of shaping and changing the original terrain elements on the earth surface [9], including tectonic activities and earth surface movement, as well as erosion and weathering. In recent years, many scholars have done a lot of research on the causes, characteristics, relationship between erosion and sediment production and soil erosion. At present, the research on the fractal of watershed geomorphology mostly focuses on the fractal study of the water system, while the research on the relationship between fractal dimension value of geomorphology and geomorphic development process, fractal characteristics, fractal scale range and geomorphic model of the whole watershed geomorphology are still to be strengthened.

This exploration proposes that the single factor index slope is added to the fractal model as an important feature information value of geomorphology. In the coordinate system of three-dimensional space scanning box, the three-dimensional cube box with different scales is used as a statistical unit to calculate the surface fluctuation under the average slope of the surface, so as to construct the fractal three-dimensional feature information dimension Geographic Information System (GIS) model to represent the degree of ground undulation. The measurement of slope can use fractal dimension. In geomorphology, the fractal dimension may be related not only to morphology, but also to evolutionary dynamics. The formation and development of fractal theory open up a new way for the overall description of watershed geomorphology and the nonlinear study of geomorphic processes.

## 2. GIS watershed geomorphic mapping model

### *2.1. Summary of watershed geomorphology theory*

#### *2.1.1. Overview of fractal theory*

The word "Fractal" comes from the Latin word "fractus", which means "irregular and fragmented" objects. In How Long is the Coast of England, B.B. Mandelbrot discusses the measurement scale and length of the British coastline. He shows that the length of the coastline is uncertain, and the length of the British coastline measured by different measuring tools is also different.

#### *2.1.2. Overview of fractal dimension theory*

Fractal dimension is an important parameter for quantifying irregular, unsmooth and extremely complex broken fractal objects [10]. It is a generalization index used to describe the structure, shape and functional complexity of an object, which shows the roughness and complexity of a fractal [11]. In other words, the rougher and more complex the object is, the larger the fractal dimension is. There are many forms of fractal dimension definition. The common forms are:

The first one is the Hausdorff dimension ($D_f$) and the similarity dimension ($D_s$).

If the L-length line segment, the L-side square and the cube are reduced by three times, (1/3) L-length line segment, (1/9) L-area square and (1/27) L-volume cube will be obtained. This kind of relationship can be expressed by the following equation:

$$N(r) = r^{-D} \tag{1}$$

The graph change multiple is $N(r)$; the magnification of the side length is $r$; the dimension of the corresponding geometry is $D$, the line segment is one dimension, the square is two-dimensional, and the cube is three-dimensional.

Regular geometry $D$ is an integer, that is, Euclidean dimension, and irregular geometry $D$ is a fraction, that is, the fractal dimension defined above. Hausdorff, a German mathematician, put forward the concept of fractional dimension. If the specific value $N(r)$ (line length, square area, and cube volume) of the fractal object is measured, $r$ is used as the measurement (line length, and side length). The measured result $N(r)$ in this measurement unit corresponds to the following equation:

$$N(r) \propto r^{-D_f} \tag{2}$$

$D_f$ is Hausdorff dimension, and $D_f$ is fractal dimension relative to fractal dimension, so the definition of $D_f$ is as follows:

$$D_f = \lim_{r \to 0} \frac{\ln N(r)}{\ln(1/r)} \tag{3}$$

If a cube with L-side length is divided into 4 parts, small cubes with (1/4) L-side length will be obtained, $N = 64$. In this case, the small cube is completely similar to the original cube, and the ratio of the side length of each small cube to the side length of the original cube is $r = 1/4$, which is the similarity ratio between local and global. The similarity dimensions are as follows.

$$D_s = \frac{\ln N}{\ln(1/r)} \tag{4}$$

No matter how complex a fractal object is, the similarity dimension of the fractal object can be calculated as long as the similarity ratio r between the local and the global is known. Although the definitions of similarity dimension and Hausdorff dimension are different, the calculated results are the same.

The second one is the capacity dimension ($D_c$) and information dimension ($D_i$).

For complex fractal elements, a regular box with a side length $r$ can be used to cover the measurement, and the dimension can be calculated according to the number of covered boxes, which is the capacity dimension $D_c$. The capacity dimension can be defined by the following equation:

$$D_c = \lim_{r \to 0} \frac{\ln N(r)}{\ln(1/r)} \tag{5}$$

Kolmogorov derived the definition of capacity dimension, which is very close to the Hausdorff dimension. Information dimension ($D_i$) is also called the dimension of the amount of information [12]. In the capacity dimension, only the number $N(r)$ of the boxes with the required side length $r$ is considered, and the filling degree of each box is not considered. Therefore, the above fractal dimension box-counting method is improved by the information dimension, and the probability $P_i$ of the fractal entity belonging to the $i$-th box is counted. Information can be expressed as:

$$D_t = \lim_{r\to 0} \frac{\sum_{i=1}^{N} P_i \ln P_i}{\ln(r)} \quad (6)$$

According to the above equation, in the case of equal probability $P_i = 1/N(r)$, the dimension of information and capacity is equal, that is, $D_c = D_i$.

Third: correlation dimension ($D_g$).

Because the calculation of information dimension involves probability, it will be inconvenient to use, so Grass-Berger and Procacoia proposed the correlation dimension [13]. If the distance between two points in the fractal is $r$, its correlation function is $C(r)$, so the correlation dimension can be expressed by the following equation:

$$D_g = \lim_{r\to 0} \frac{\ln C(r)}{\ln(1/r)} \quad (7)$$

In the above equation:

$$C(r) = \frac{1}{N^2} \sum_{i,j=1}^{N} H\left(r - \left|x_i - x_j\right|\right) = \sum_{i=1}^{N} P_i^2 \quad (8)$$

*2.1.3. Fractal properties*

Mandelbrot gave a popular definition of the fractal. In some form, the form of the part similar to the whole is fractal [14]. So far, there is no strict mathematical definition of fractal, which is easy to understand from the physical point of view, but the current mathematical definition is not accurate. This definition reveals that fractal has an important characteristic, that is, when a part of the research object is enlarged, it is similar to the whole. In the field of earth science, there are many self similar phenomena from mountains to rocks. If the mountain range is reduced to a very small scale, it can be found that the surface of the rock is very similar to that of the restored mountain range. Mandelbrot connected this self similarity with the fractal dimension and gave the original mathematical definition of fractal, that is, the fractal is the set of Hausdorff dimension ($D_f$) which is strictly larger than topological dimension ($D_t$), that is, $D_f > D_t$, so Hausdorff dimension becomes the basic index of fractal description [15–18].

After the review of the researchers, a more extensive and appropriate definition of the fractal is given: (1) it has fine structure; (2) its irregularity can not be described by traditional geometric language in the whole and part; (3) it has some form of self similarity; (4) in general, $D_f > D_t$, that is, the Hausdorff dimension is strictly larger than the topological dimension; (5) the set can be generated by iteration and can be defined by very simple methods; (6) its size can not be measured by ordinary measures.

*2.1.4. Self similarity and scale invariance*

Self similarity refers to the similarity in process and structure between the whole and the part as well as the part and part from different spatial and temporal scales in a complex system. Or, the parts taken from the whole have the basic characteristics of the whole. However, in general, the self similarity system has a more complex form of expression, and it is not possible to overlap with the whole by enlarging the part, although the quantitative and fractal properties of the self similarity system and structure will not be changed by narrowing or enlarging. In nature, most graphs have statistical self similarity, which is called statistical fractal.

Scale invariance means that any local area of a fractal image can be enlarged or reduced by any factor, and the shape of the image is similar to the original image. For some phenomena in nature, this change in size is only a good description in a specific area. In the study of fractal objects, the first step is to analyze the adaptive range of scale invariance, which is the scale-free region of fractal objects. If it exceeds the range, the fractal of the object will be meaningless; moreover, the self similarity is closely related to the scale invariance. If it has self similar structure, it must satisfy scale invariance, and they are two important characteristics of fractal objects [19–21].

*2.2. Measuring principle of fractal dimension*

The definitions of Hausdorff dimension ($D_f$), similarity dimension ($D_s$), capacity dimension ($D_c$), information dimension ($D_i$) and correlation dimension ($D_g$) are different. In different fractal objects, the same fractal dimension with different values belongs to the concept of fractal dimension. Even the same fractal object is different in different fractal dimension values. Therefore, different fractal dimensions should be selected according to different fractal objects. After the concept of fractal dimension is defined, the value of fractal dimension should be measured. These methods are commonly used: (1) by changing the observation scale; (2) according to the measurement relationship; (3) according to the correlation function; (4) according to the distribution function; (5) according to the spectrum. This exploration mainly focuses on the fractal dimension measurement method related to watershed fractal characteristics, which is the first method.

*2.2.1. Principle of box counting method*

The box counting method is a commonly used method to calculate the fractal dimension of fractal entities. Because of its simple principle, easy operation and simple mathematical calculation, it has become one of the most widely used dimensions. The small box with side length r is used to cover the fractal entity. Because there are various levels of cracks and holes in the fractal, some small boxes are empty, and some small boxes cover part of the fractal. It is necessary to count how many small boxes are not empty, and the number of non-empty boxes is $N(r)$. Next, the size of the box r is reduced and the fractal entity is covered, so the number of non-empty boxes $N(r)$ will naturally increase. When $r \to 0$, the fractal dimension defined by the number box method is obtained.

$$D_0 = \lim_{r\to\infty} \frac{\ln N(r)}{\ln(r)} \tag{9}$$

In the actual calculation process, only a limited value of r in the scale-free area can be taken. Generally, after a series of $r$ and $N(r)$ are calculated in the reliable range, $D_0$ is calculated from the slope of log$N$–log$r$ line in double logarithmic coordinates. However, there must be a scaling relation in the above equation.

$$N(r) \infty r^{-D_0} \tag{10}$$

If the scaling relation does not exist, the concept of fractal dimension cannot be used, and the fractal dimension thus obtained is called capacity dimension.

#### 2.2.2. Accuracy of the box counting method

The equation of finding fractal dimension by box counting method has a great disadvantage. In order to ensure the accuracy of fractal dimension calculation, the definition of fractal dimension can be further improved. The method of counting boxes is changed to number the small boxes. If the $i$-th box falls into the $N_i(r)$ point, the probability of the point in the fractal falling into the $i$-th box can be known.

$$P_I(r) = \frac{N_i(r)}{N(r)} \tag{11}$$

$N(r)$ is the total number of points, and then the information quantity is used to define the equation:

$$I(r) = -\sum_{i=1}^{N(r)} P_i(r) \ln P_i(r) \tag{12}$$

Definition equation of information dimension:

$$D_i = \lim_{x\to 0} \frac{I(r)}{\ln r} \tag{13}$$

It reveals that when each box has the same weight, that is, $P_i(r) = 1/N(r)$, the amount of information $I(r) = \ln N(r)$, and the information dimension $D_i$ is equal to the capacity dimension $D_0$. The essence of box counting method is to change the degree of coarsening. The measurement of fractal dimension is usually to start from the big box and decrease the size of the small box in turn, and only the number of "non-empty" boxes is calculated. The number box counting method of fractal dimension is also called the covering method. This method is suitable for both simple fractal and complex fractal.

### 2.3. GIS modeling of watershed geomorphology

The information dimension is selected as the fractal dimension index of geomorphology. On the basis of fully combining GIS system technology, the fractal three-dimensional feature information dimension GIS model of geomorphic morphology is proposed, and the characteristics of the model are analyzed. Geomorphology can be classified into various types according to the cause of formation, among which water geomorphology is an important one, and surface water is almost everywhere. Watershed geomorphology is one of many types of water geomorphology. Natural geomorphology has the characteristics of fine structure, self similarity and scale invariance. They have fractal properties, and their spatial morphological characteristics can be expressed by fractal dimension. The digital elevation model (DEM) is the digital representation and simulation of the earth's natural surface elevation. It uses a group of ordered numerical arrays to represent the ground elevation, and can accurately record the three-dimensional positioning information. Based on GIS spatial modeling, DEM can be used to simulate watershed geomorphology.

### 2.4. Measurement method based on GIS model

Based on GIS technology, the three-dimensional feature information dimension GIS model of watershed geomorphology fractal is as follows:

$$D_{3i} = \lim_{r_3\to 0} \frac{I(r_3)}{\ln r_3} \tag{14}$$

$D_{3i}$ is the three-dimensional fractal information dimension; $r_3$ is the side length of the three-dimensional cube box, which is the integral multiple of the length of DEM grid pixel; $I(r_3)$ is the amount of geomorphic feature information.

$$I(r_3) = -\sum_{i=1}^{N(r_3)} P_{3i}(r_3) \ln P_{3i}(r_3) \tag{15}$$

$N(r_3)$ is the total number of non-empty small cube boxes; $P_{3i}$ is the feature distribution probability of the three-dimensional fractal entity in the $i$-th box;

$$P_{3i}(r_3) = \frac{c_i}{C} \tag{16}$$

$c_i$ is the feature information of the three-dimensional fractal entity contained in the $i$-th non-empty small cube box; $C$ is the total feature information of the three-dimensional fractal entity;

$$C = \sum_{i=1}^{n} c_i \tag{17}$$

The measurement steps of fractal dimension are as follows: (1) determining the scale-free area of fractal entities; (2) scanning the entities in the scale-free area with small cube boxes in turn, and counting the characteristic information of the entities contained in the non-empty scanning boxes during the scanning process; (3) calculating the relevant fractal parameter values according to the GIS model of fractal information dimension of watershed geomorphology, that is: $I(r_3)$ and $-\ln r_3$; (4) taking $-\ln r_3$ as the abscissa axis and $I(r_3)$ as the ordinate axis, and plotting the calculation results in the double logarithmic coordinate system to fit a straight line. The slope of the straight line is the information dimension value of the fractal three-dimensional characteristics of watershed geomorphology.

## 3. Geomorphological survey results

### *3.1. Measurement of information dimension of watershed geomorphology*

Fig. 1 shows the length of a coastline measured by different lengths.

Fig. 1 shows that the shorter the ruler is, the longer the coastline is, the more detailed the expression of coastline is. Therefore, traditional measurements can no longer meet the coastline mapping. Only a new measure can be found to better characterize the characteristics of the coastline. Mandelbrot has made a deep study on this issue and brought forward the theory concept of the fractal.

Fig. 2 is the geomorphological form of the small watershed:

The dark area in the map is the higher geomorphology. The lighter the color is, the lower the regional geomorphology is. The overall geomorphology elevation ranges from 3,200 to 4,200 m.

Fig. 3 shows the point distribution of three-dimensional fractal rough information dimension in small watershed.

Fig. 4 presents the distribution of three-dimensional fractal double logarithm linear regression points in small watershed.

Fig. 3 shows that the value of 1–12 rough information dimension is between 2 and 3. The 13th rough information dimension is the box scale of 1,180–1,270, and the calculated information dimension is 1.81326, which should be eliminated. Therefore, the scale-free area of the small watershed is finally determined as 100–1,070 m.

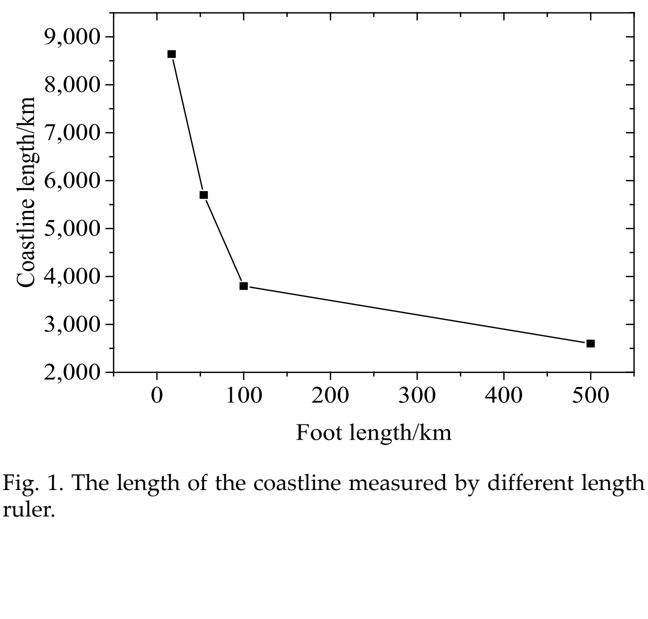


Fig. 1. The length of the coastline measured by different length ruler.

In Fig. 4, the regression relationship can be established by linear regression, and the correlation coefficient is 0.9997. The slope of the straight line is the fractal volume information dimension of small watershed geomorphology, that is, $D_{3i} = 2.6999$.

### *3.2. Measurement of geomorphic information dimension of watershed based on GIS*

Fig. 5 is the geomorphic morphology of the watershed.

The information content of three-dimensional solid surfaces measured in the scanning process of small cube boxes

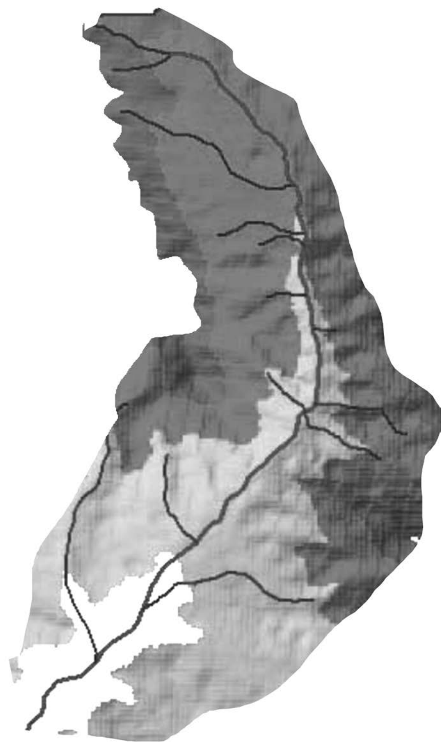

Fig. 2. Geomorphology of small watershed.

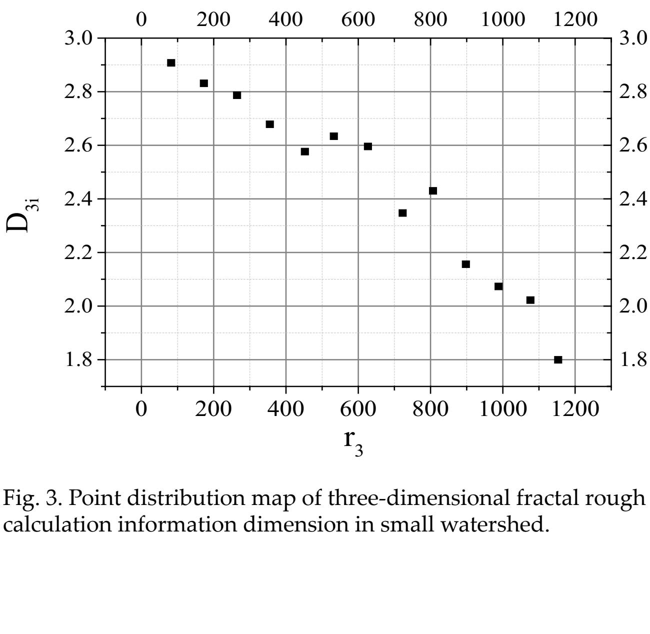


Fig. 3. Point distribution map of three-dimensional fractal rough calculation information dimension in small watershed.

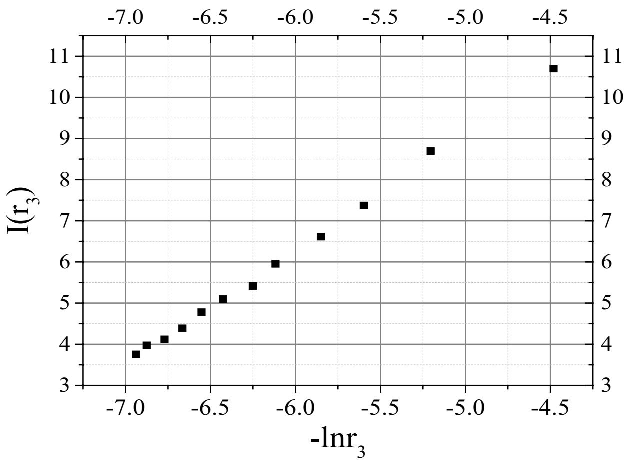


Fig. 4. Point distribution map of three-dimensional fractal double logarithm linear regression in a small watershed.

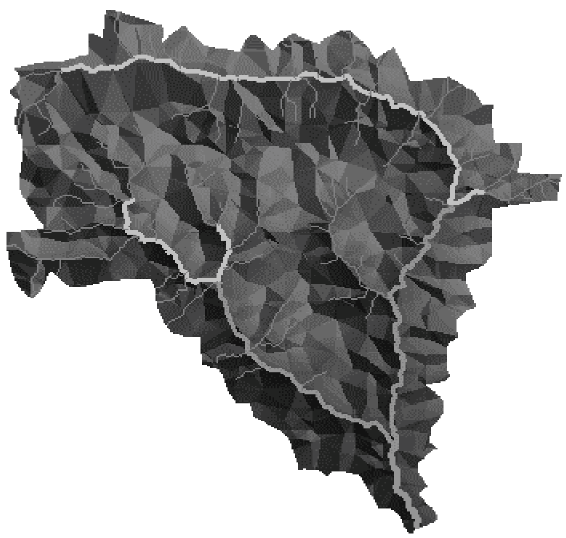

Fig. 5. Geomorphic morphology of the watershed.

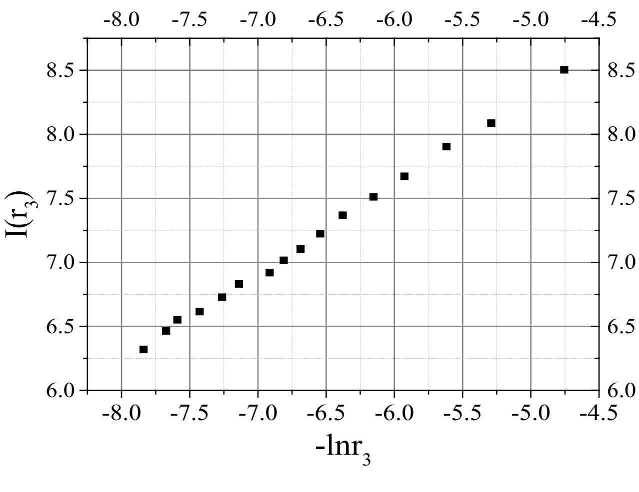


Fig. 6. Point distribution map of fractal double logarithm linear regression.

with different scales is different. When a certain number of small cube boxes with different scales are selected to complete three-dimensional scanning, the scanning statistical results of geomorphic feature information $I(r_3)$ at different scales $r_3$ can be obtained.

Fig. 6 is the point distribution map of fractal double logarithm linear regression. The scan results are used to determine the scale-free interval by linear regression. With $\ln(r_3)$ as the abscissa axis and $I(r_3)$ as the ordinate axis, the calculated results are plotted on the double logarithmic coordinate system, and the linear regression correlation coefficient is 0.9998. The slope of the straight line is the information dimension value of the three-dimensional fractal characteristics of watershed geomorphic morphology, that is, $D_3$ = 1.869. Moreover, the scale-free range of this fractal dimension is 430–4,520 m.

## 4. Conclusion

Based on DEM data, fractal GIS scanning is studied. In order to facilitate fractal calculation and GIS program, the coordinate system and length *r* are decomposed to discuss the relationship between the scanning coordinate system and the solid space. The spatial relationship characteristics of feature boxes are summarized. Meanwhile, the concept of rough calculation probability of ground undulation distribution is proposed, and the corresponding conversion equation is derived. According to the results of the scan, the linear regression is carried out and the relationship among the fractal, correlation coefficients and the scale-free interval is studied respectively. The three characteristics of the watershed geomorphic fractal dimension inverse growth, rapid growth and stability period are summarized. The theory of calculating fractal dimension of watershed geomorphology by using three growth periods and linear correlation coefficient not less than 0.9999 as the value standard of scale-free area of watershed geomorphology is put forward. There are still some shortcomings. For example, the influencing factors in geomorphology research are complex, but the watershed geomorphology is only studied from the aspects of climate and topography. In the future, the geomorphological mapping will be studied in a more comprehensive way, such as hydrology and soil. This exploration provides technical reference for the mapping of the geomorphic form of the following watershed, and verifies the feasibility and effectiveness of GIS technology.

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