

Water quality modelling of Sebou River estuary (Morocco) after the installation of the Kenitra town's WWTP

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ABSTRACT

Before 2020, urban wastewaters of Kenitra were discharged without prior treatment through six collectors into Sebou River estuary (60 km). This situation caused many health and environmental problems. The construction of the wastewater treatment plant (WWTP) had to convey all urban wastewaters before being put under treatment. Thus, the WWTP, situated 19.4 km from the mouth of the estuary, became the only discharge point in Sebou estuary. This study aims to model Sebou estuary water quality and simulate the fate of the urban waters discharged by the WWTP. Our study started with hydraulic modelling of the river using a 1D model (HECRAS 5.0.6), since water quality is strongly depending on hydraulic regime. HECRAS has been calibrated and validated using hydraulic and morphological database of the year 2020. The spatiotemporal evolution of hydraulic variables (water velocity, water level, etc.) was calculated by the hydraulic model and used in the water quality module to simulate two parameters: dissolved oxygen and biochemical oxygen demand (BOD₅). Two scenarios were put under examination, one is a simulation of untreated discharge (mean BOD₅ of 300 mg/L) and the second is a simulation of discharge after treatment at the WWTP (mean BOD₅ of 13 mg/L). Results demonstrated that WWTP reduces BOD₅ concentration in the river by 32% compared to the case of an untreated discharge. Also, simulations showed that BOD₅ concentration downstream of the WWTP changes according to the tide cycle. It is greater at high tide than at low tide, with a difference of 0.8 mg/L on average. Final simulations considered a discharge happening during 2 d. Results demonstrated that the total pollution evacuation occurs when there is freshwater flow of 300 m³/s (fresh flow dominance) after 8 h, while pollution is completely cleared out after 3 d, when there is a flow of 20 m³/s (tidal cycle dominance).

Keywords: Sebou estuary; Water quality; Modelling; HEC-RAS; Residential time; Biochemical oxygen demand

1. Introduction

Water is of a paramount importance for the survival and progress of human civilization, it is a vital element in human life and activity. Across the world, water is used in all daily activities either in housing, industry, agriculture, economy, or energy, which makes it a receptor element susceptible to all kinds of pollution. This phenomenon is one of

the main causes of water resources limitation. Water shortage is a limiting factor for the development of social and economic sectors of a country. A major goal is to establish policies for sustainable management and governance rules to ensure water resources sustainability [1].

The traditional water supply in Morocco suffers from scarcity and irregularity. Water-intensive activities and climate change effects are main factors behind this problem.

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As a matter of consequence, we are obliged to find other alternative of water resources which have not yet been put under exploitation, for example, estuary water [2].

Hydrobiologically speaking, estuaries or areas of fresh-water/saltwater interface are distinguished by specific hydrodynamic features.

The demographic growth with the rapid urbanization have increased the water consumption thereby polluting the rivers basins. Domestic and municipal waste, agricultural activities, run-off, industrial activities and sand mining are all factors causing rivers' pollution and leading to impacts that can be seen clearly in these very fragile ecological balance areas [3]. Also, they are contaminated by point source pollution and non-point source pollution. Besides, it is mandatory to have control over these problems and prevent them through determining the water quality variations [4].

The Sebou River is draining in the northwest of Morocco, an area nearly estimated by 40,000 km², that is 5.5% of the total area of the country, and running from its source in the central Atlas Mountains to the Atlantic Ocean, a distance of 614 km. Sebou estuary (60 km) is situated between the Lalla Aïcha storage dam and the mouth which represents the outlet of Sebou basin (Fig. 1). Its flow regime knows seasonal and numerous fluctuations following the tidal regime and the control of numerous dams [4]. The role of Lalla Aïcha Dam is to keep enough water for agricultural pumping stations and to avoid upwelling of salty waters toward these stations [5].

A significant amount of wastewater heads from Kenitra (about 17 km from the mouth) to Sebou estuary, and this amount is increasing because of the demographic growth, as well as the agricultural and industrial effluent discharges. These discharges are loaded with a variety of contaminants susceptible to make temporary concentration at levels exceeding their standards in this aquatic environment, degrading the quality of Sebou estuary waters. Before 2020, water was discharged without prior treatment through six collectors into Sebou estuary [6]. Constructing the wastewater treatment plant (WWTP) in Kenitra was aimed at conveying the wastewaters and putting them under treatment. The WWTP thus became the only discharge point for Sebou estuary. This study tends to model Sebou estuary water quality and simulate the future of the discharges from the WWTP. Water quality is influenced by the tidal hydraulic regime of the estuary which is being characterised by a filling at high tide through the bottom of the river and by an emptying at low tide [5]. The semi-diurnal tidal amplitude ranges from 0.97 to 3.11 m and the tides influence extends to 35 km from the mouth [5]. Furthermore, because the Sebou estuary is a narrow one, wind has a negligible effect on the flow [7].

A number of scholars quantitatively classified estuaries on stratification by means of dimensionless numbers, such as Estuarine Richardson number N_R [8] and estuary number N_e [9]. According to water column stratification, Sebou estuary can be classified as partially mixed [10,11].

The WWTP has a paramount importance in minimizing the inlet load of Sebou estuary and it is very significant to observe its efficiency.

Early studies on water quality of Sebou estuary have been carried out since 1966 [2,4–6,12] and showed that

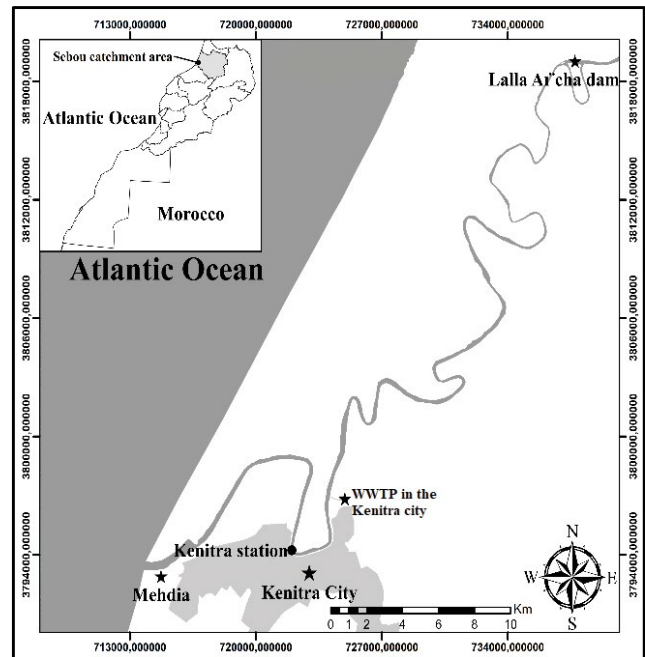


Fig. 1. Study area and situation of Kenitra's WWTP.

physico-chemical quality of Sebou River estuary does not meet OMS standards for discharges into the natural environment. All these studies recommended the construction of WWTP to treat urban wastewaters of Kenitra. But these studies did not show attention to the influence of pollution evolution by hydrodynamic and morphological conditions. Managerially speaking, the managers are seeking rapid estimation of longitudinal pollution distribution in alluvial estuarine. One-dimensional mathematical models can be the appropriate tools for usage because they are easy for application, and more adapted to management contexts. Furthermore, it is methodologically correct to begin with the most basic description of the phenomenon under study and assess the limits of this approximation before investigating on more complicated issues.

Since water quality is strongly linked to the hydraulic regime, the HECRAS software was used to model the estuary hydraulic regime. The hydraulic module has been calibrated and validated using a large hydraulic and morphological data base. The hydraulic module outputs (water velocity, water level, depth etc.) were used in the water quality module to simulate two key parameters: dissolved oxygen and biochemical oxygen demand (BOD₅). Two scenarios were tested: one is a simulation of untreated discharge and the second is a simulation of discharge after treatment at WWTP. The two simulations showed an impact of the tidal cycle and freshwater flows (coming from the upstream) on the fate of the river discharges. The results demonstrated the good impact of the WWTP on BOD₅ attenuation in the river. The simulations provided other answers such as the release dispersion and the residence time in the estuary. The current study proves the validity of recommendations found in previous studies concerning the need for the installation of a WWTP in the city of Kenitra. The WWTP is very effective

for the treatment of urban wastewaters that now meet OMS standards, except during some exceptional periods of high entrance of water during which the WWTP is bypassed.

2. Materials and methods

2.1. Processing methods in the WWTP

The BOD₅ was measured at the laboratory of RAK (Agence Autonome de Distribution d’Eau et d’Electricité de Kenitra). Endress+Hauser XE4302.2 instrument was employed for automatic sampling. BOD system was used for BOD₅ measurement. This system can measure BOD based on the manometric principle. Manometric respirometers bind the uptake of oxygen to the change in pressure caused by oxygen consumption while keeping a constant volume. It is important to mention that the BOD level of a sample relies on the amount of the organic matter available, which can mark considerable variation. The BOD measuring system is therefore calibrated according to the volumes of various samples under study. Furthermore, temperature equalisation is necessary before doing biological testing, as temperature has a major impact on biological activity. BOD measurements are performed in a thermostatically controlled cabinet at 20°C [13].

2.2. Hydrodynamic model

In this study, we used a one-dimensional approach, which is appropriate in the case of the river reach having long distance. In this study, we used HEC-RAS mathematical model, which was adopted to simulate the hydrodynamic regime, sediment transport, and water quality for many rivers [14]. Water quality is found under the influence of Sebou estuary’s hydrodynamic regime, which in turn relies highly on the river morphology.

The HEC-RAS model is based on the one-dimensional conservation equations of mass and Barré Saint-Venant momentum, which are defined as follows [14]:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_1 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial(QV)}{\partial x} + gA \left(\frac{\partial z}{\partial x} + S_f \right) = 0 \tag{2}$$

where Q stands for the discharge (m³/s), V for the velocity (m/s), A for the cross-sectional area (m²), x for the distance along the channel (m), t for the time (s), q_1 for the lateral inflow per unit length (m²/s), g for the acceleration due to gravity (m/s²), Z for the flow depth (m) and S_f for the frictional slope (Dimensionless). The frictional slope is expressed as [14]:

$$S_f = \frac{Q|Q|n^2}{2.202A^2R^{4/5}} \tag{3}$$

where n is Manning’s roughness coefficient (m^{1/3}/s), and R is the hydrodynamic radius (m).

The empirical formula established by Cowan and Chow is used to evaluate initially the Manning coefficient used in the momentum equation.

$$n = (n_0 + n_1 + n_2 + n_3 + n_4) \cdot m_5 \tag{4}$$

where n_0 is the basics, n is the value for a straight uniform and smooth channel, n_1 is the adjustment for the effect of surface irregularity, n_2 is the adjustment for the effect of variation in shape and size of the channel cross section, n_3 is the adjustment for obstruction, n_4 is the adjustment for vegetation, and m_5 is a correction factor for meandering channels.

The factor n_0 is evaluated using granulometric data taken in the examined reach from upstream to downstream. The other coefficients were evaluated using observations of the river in aerial photos, from the cross-sectional areas and accessible photos, and field visits [2].

Eqs. (1) and (2) are solved by the four-point implicit box finite difference scheme. The general forms of derived equations for a function f are [14]:

The time derivative:

$$\frac{\partial f}{\partial t} \approx \frac{\Delta f}{\Delta t} = \frac{0.5(\Delta f_{j+1} + \Delta f_j)}{\Delta t} \tag{5}$$

The spatial derivatives:

$$\frac{\partial f}{\partial x} \approx \frac{\Delta f}{\Delta x} = \frac{(f_{j+1} - f_j) + \theta(\Delta f_{j+1} - \Delta f_j)}{\Delta x} \tag{6}$$

Function value:

$$f \approx \bar{f} = 0.5(f_{j+1} + f_j) + 0.5\theta(\Delta f_j + \Delta f_{j+1}) \tag{7}$$

where θ : weighting factor. In HEC-RAS, the default value of θ is 1 [2].

Finding a solution to the equations system need to discretize spatially the section into characteristic grids and define the river geometry, the conditions of the initial flow and the upstream and downstream boundary. Then, the estuary is discretized into 203 grids with a length between 58 and 996 m, an average value of 337 m. For each grid we have specified the length, the cross section and the Manning friction factor.

2.3. Transport model

The description of contaminants transport in surface waters is generally made by the advection-dispersion equation which is a derivative of the equation of mass balance [15]. The BOD transport equation is given as:

$$\frac{\partial(AC_{BOD})}{\partial t} = -\frac{\partial(QC_{BOD})}{\partial x} + \frac{\partial}{\partial x} \left(AD_x \frac{\partial C_{BOD}}{\partial x} \right) - AK_1C_{BOD} + AR_{BOD} \tag{8}$$

where C_{BOD} is the concentration of the organic matter (kg/m³), R_{BOD} is the release of the organic matter (mg/L), K_1 is the oxidation coefficient (d⁻¹), D_x is the dispersion coefficient

(m^2/s), the key parameter that must be estimated appropriately. An estimation of this important parameter is elaborated using Fischer equation (1979) [14].

3. Results and discussion

3.1. Hydraulic calibration and validation

The upstream boundary condition is the flow releases by Lalla Aïcha Dam (Fig. 2); as stated by the Hydraulic Department of Kenitra. The downstream boundary condition represents the water level variation at the mouth (Mehdia Port), as pointed by the Hydrographic and Oceanographic Service of the Marine (Fig. 3).

The Hydrodynamic model has been put under calibration and validation. The parameter used for calibration is the river Manning's roughness. The Manning coefficient was

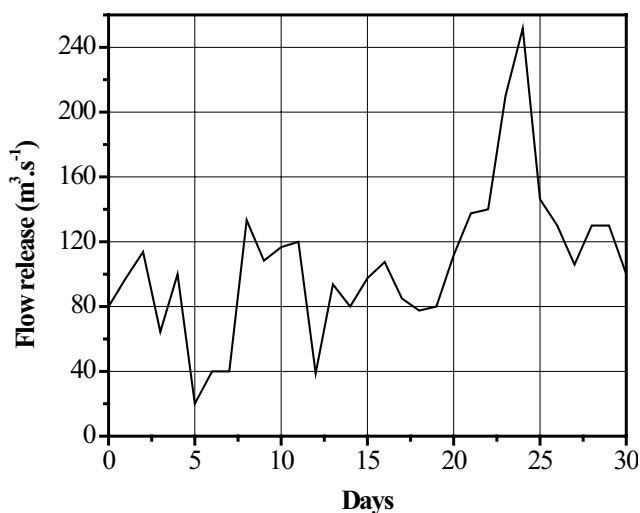


Fig. 2. Upstream boundary condition (Lalla Aïcha Dam), January 2020.

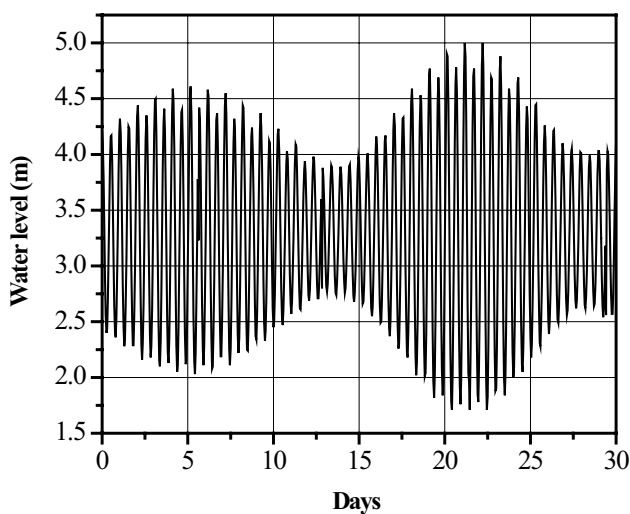


Fig. 3. Downstream boundary condition (Estuary mouth), January 2020.

modified to the same degree along the reach under study, because we assumed that the error sources involved in its evolution are corresponding for all the grids.

The calibration and validation are performed by the use of water level data at the Kenitra station, situated 17 km from the mouth (Fig. 1), because measurements of water level can be obtained there by the ANP (Agence Nationale des Ports). The data used for calibration is of the period between January 1st, 2020 and January 15th, 2020. Fig. 4 reveals good identical results between the water levels of the simulation model and those of the Kenitra station measurements.

To establish the correctness of the calibration results, a test for model validation is usually put on performance. In this test, the flow regime is modelled, for a period other than that used for the calibration, without changing Manning values. In our case, we made this validation based on the data provided between January 15th, 2020 and January 30th, 2020. A good agreement was found between calculated and observed water levels.

The statistical indicators used to evaluate the performance of the hydraulic model are: the root mean square error (RMSE), the normalized objective function (NOF), and the Nash–Sutcliffe coefficient (NSC) [2].

The coefficient (RMSE) provides a rating for the model error and shows a perfect similarity between the values that are observed and those predicted, in case of being equal to 0. For NOF, when it is less than 1, the model error is said to be negligible, and the NSC ranges from $-\infty$ to 1, a value close to 1 marks that the model is well performed.

The results of statistical calculations appear with low coefficient (RMSE), a value of the function (NOF) that is less than 1 and a value of the coefficient (NSC) that is very approximate to 1. This proves that the model is performing and that the calibration and validation comes with results (Table 1).

The HEC-RAS model outputs enable to calculate several hydraulic variables; namely, water level and velocity when evolved according to space and time.

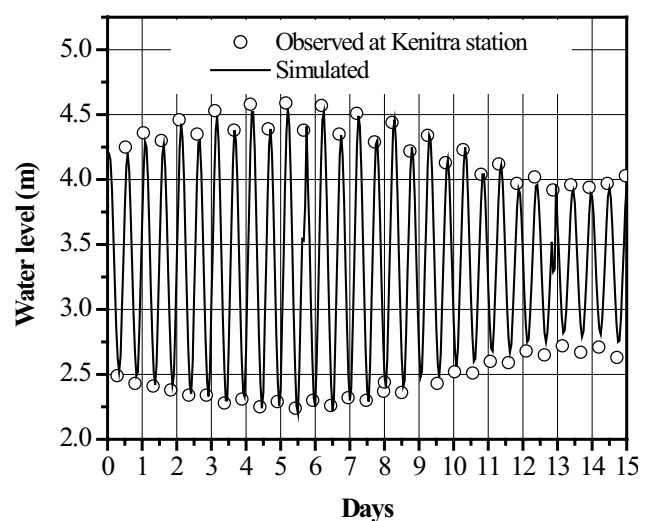


Fig. 4. Water level calibration at the Kenitra station: during January 1st 2020 to January 15th 2020.

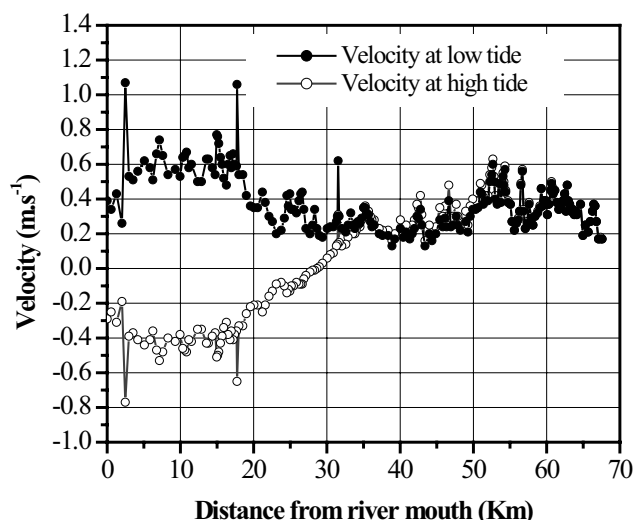


Fig. 5. Calculated velocity profile, at low tide and high tide, along the studied reach.

Table 1
Statistical indicator of model performance

Model	RMSE	NOF	NSC
Calibration	0.20	0.064	0.95
Validation	0.18	0.054	0.96

Fig. 5 represents longitudinal velocity variation at low tide and high tide. The velocity strongly fluctuates due to morphology and bottom level influence. Also, the velocity is influenced by tidal conditions. At low tide, the velocity is in the opposite direction (from downstream to upstream).

3.2. Water quality simulation results

Water quality of Sebou estuary was simulated for the period from January to August 2020. Dispersion coefficient was estimated to 150.27 m²/s on the basis of the Fischer formula.

In water quality simulations, two scenarios were put under examination; namely, a simulation where urban wastewaters are untreated and a simulation in which they are treated in the WWTP.

Concerning the first simulation, raw urban wastewaters have an inflow into the river shown in Fig. 6 and BOD₅ temporal variation shown in Fig. 7.

Fig. 8 shows longitudinal evolution of BOD₅ from downstream (river mouth, 0 km) to upstream (Lalla Aïcha Dam, 69 km) for four tidal situations: high tide, low tide, rising tide and falling tide.

BOD₅ concentration increases either during upstream or downstream, but it is higher in downstream parts of the river. At high tide, the downstream concentration of the WWTP is greater than that at low tide, this due to the effect of dilution by unpolluted continental waters. The pollution rejected displaces on the basis of the tide cycles; that is, it moves downwards when the tide cycle

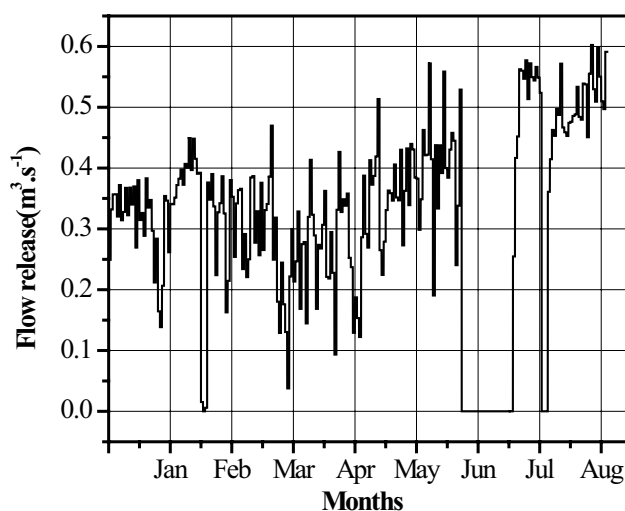


Fig. 6. Raw water flow released by the Kenitra WWTP (January 1st 2020 to August 31st 2020).

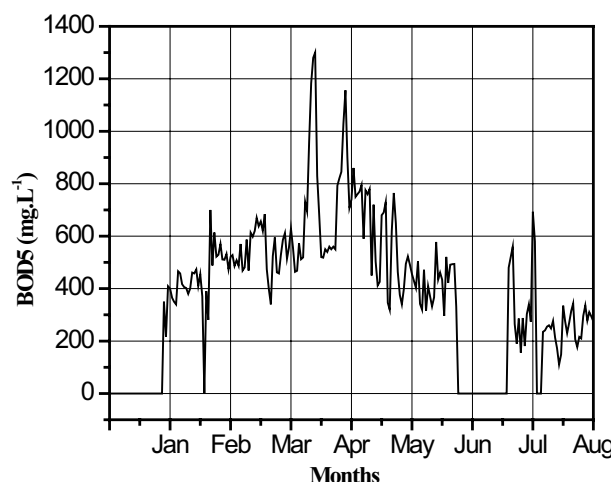


Fig. 7. BOD₅ concentration of raw water (lateral condition) (January 1st 2020 to August 31st 2020).

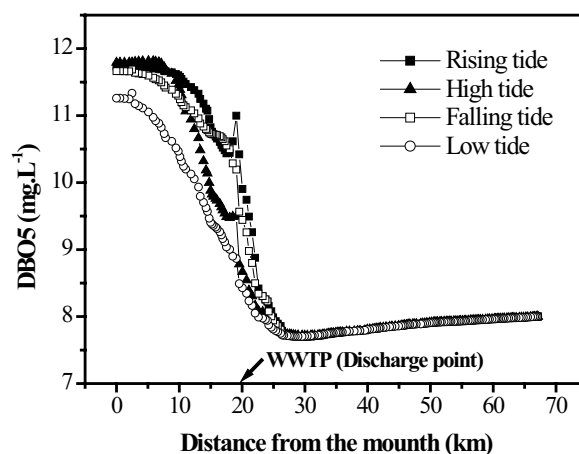


Fig. 8. Longitudinal evolution of the BOD₅ in the case of untreated urban wastewaters (April 12th 2020).

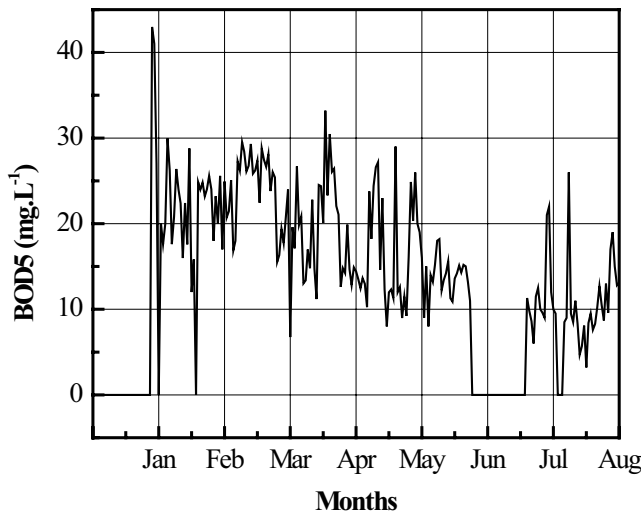


Fig. 9. BOD₅ concentration of water treated, (lateral condition) (January 1st 2020 to August 31st 2020).

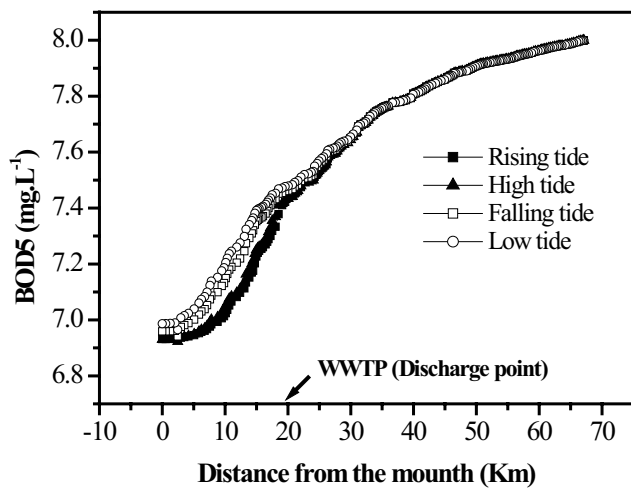


Fig. 10. Longitudinal evolution of the BOD₅ in the case of treated urban wastewaters (April 12th 2020).

is low, and moves upstream when the tide cycle is high, associated with an accumulation of the pollution in the downstream and a decrease in its amplitude by the dispersion and biochemical reactions. The increase in BOD₅ does not reach the areas located more than 30 km upstream of the mouth.

The second simulation of water quality concerns an urban wastewaters after being put under treatment by the WWTP. The simulations have been developed for the same period as for the first case (January to August 2020). Fig. 9 represents BOD₅ concentration of urban waters released into the river after treatment. (January 1st 2020 to August 31st 2020).

This figure shows that the BOD₅ concentration in the discharge is considerably reduced. On other hand, before 2020, urban wastewater was discharged into the estuary without prior treatment through six discharge points [6]. The recorded BOD₅ in the discharge points ranged from: 450

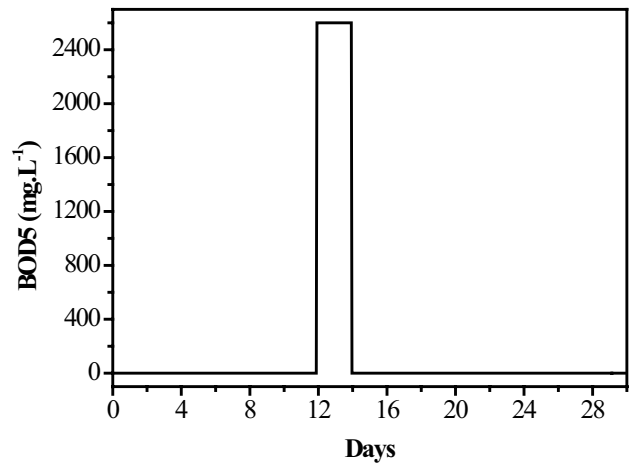


Fig. 11. BOD₅ discharge during 2 d from the WWTP.

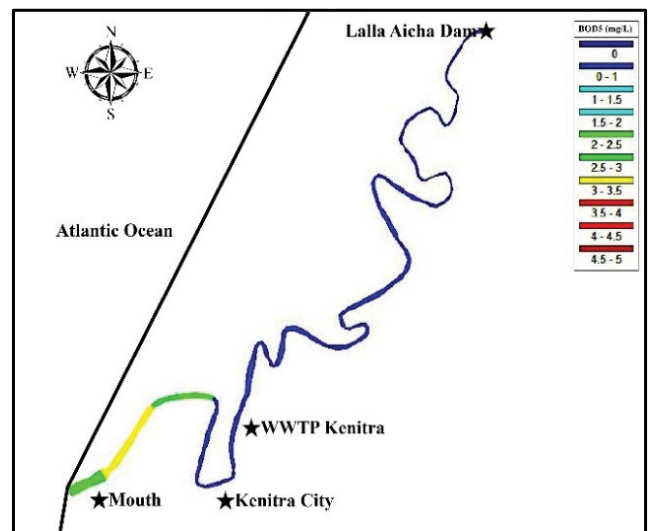


Fig. 12. BOD₅ along the estuary, 1 h after discharge end, at low tide and for Q = 300 m³/s.

to 1,900 mg/L and was not conform to the Moroccan standards. Fig. 9 shows that pollution is considerably reduced with the presence of the WWTP since that the rejected BOD₅ concentration does not exceed 20 mg/L.

Fig. 10 shows that BOD₅ decreases from upstream to downstream due to dispersion and biochemical reactions. No influence was registered on the waters quality. This confirms that the Kenitra WWTP is well performing.

3.3. Simulation of an isolated discharge

Final simulations considered a discharge happening during 2 d only, as shown in Fig. 11. These simulations was conducted at freshwater flows of 300 and 20 m³/s.

For a flow of 300 m³/s, Figs 12 and 13 reveal that after discharge end, BOD₅ discharged from WWTP is evacuated to the ocean with an important reduction of its concentration due to biochemical reactions and dispersion. The residence time is evaluated to 10 h.

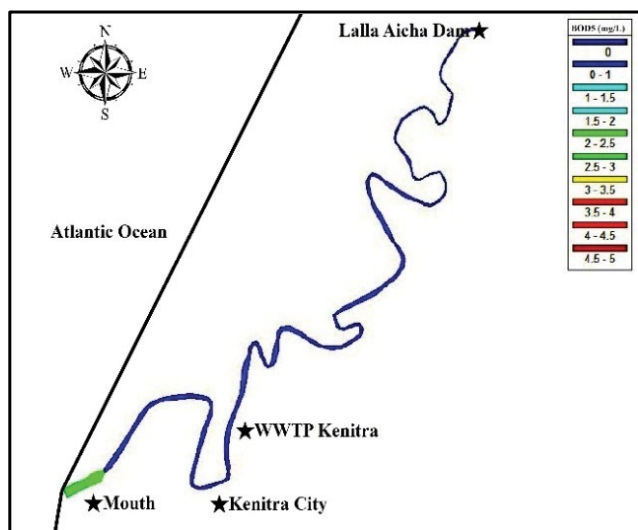


Fig. 13. BOD₅ along the estuary, 8 h after discharge end, at low tide and for $Q = 300 \text{ m}^3/\text{s}$.

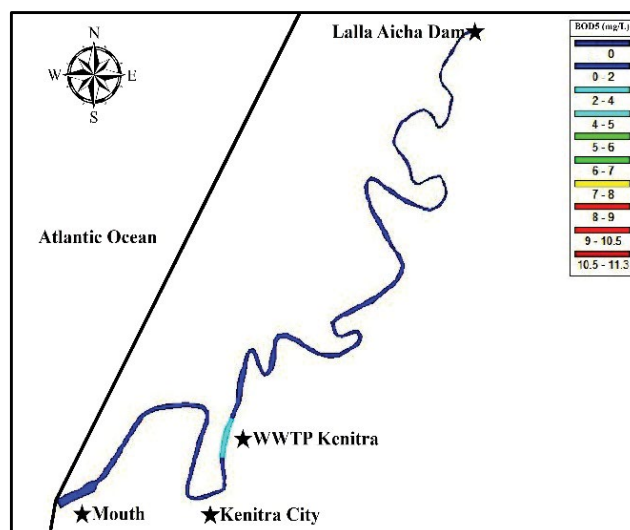


Fig. 15. BOD₅ along the estuary, 3 d and 7 h after discharge end, at low tide and for $Q = 20 \text{ m}^3/\text{s}$.

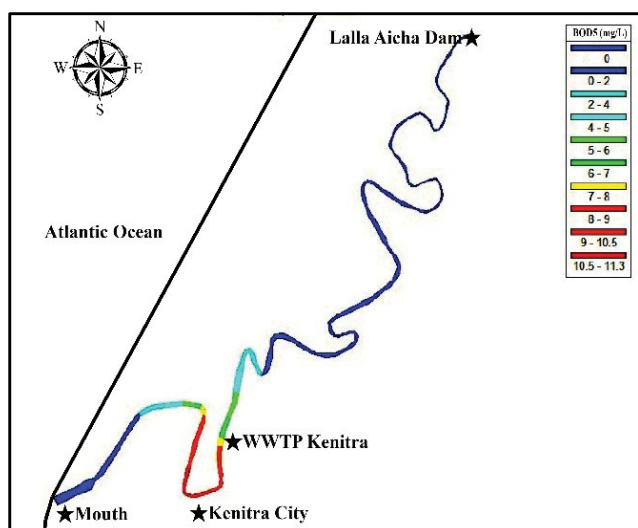


Fig. 14. BOD₅ along the estuary, 3 h after discharge end, at low tide and for $Q = 20 \text{ m}^3/\text{s}$.

For a flow of $20 \text{ m}^3/\text{s}$, Figs. 14 and 15 show that BOD₅ from WWTP remains in the estuary even after 3 d and 7 h with a concentration of approximately 4 mg/L . In this case, the residence time is evaluated to 3 d 12 h. Also, since the fresh flow is low, BOD discharges are controlled by marine tides.

Finally, we must specify that water quality simulations can be considered credible because the transport model is based on calibrated hydrodynamic model and accurate measurements of water quality in the WWTP.

4. Conclusion

urban wastewaters from Kenitra had not been discharged for a long time without prior treatment into Sebou River estuary until WWTP was installed in 2020. This

study aims to simulate the fate of the urban wastewaters discharged by the WWTP into the river, using HECRAS 5.0.6 model. Since water quality is strongly depending on hydraulic regime. HECRAS has been calibrated and validated using hydraulic and morphological database. Two scenarios were put under examination, one is a simulation of untreated discharge (mean BOD₅ of 300 mg/L) and the second is a simulation of discharge after treatment at the WWTP (mean BOD₅ of 13 mg/L). Simulation results demonstrated that WWTP reduces BOD₅ concentration in the river by 32% compared to the case of an untreated discharge. Also, simulations showed the influence of tide cycle on water quality of the estuary. BOD₅ is greater at high tide than at low tide (average difference of 0.8 mg/L). Also, simulations highlight the role of freshwater flows, coming from the upstream, on the fate of the river discharges. Important freshwater flow permits a total pollution evacuation, while low freshwater leads to a longer residence time of wastewater discharges. Thus, this paper made it possible to study the evolution of urban wastewater loaded into Sebou River estuary, taking into consideration the pollution load, the hydraulic regime of the river (strongly influenced by its morphology), the state of the tide as well as the inflows of continental freshwater. This was made possible by the use of mathematical modelling that can be used as a tool to aid decision-making for WWTP managers.

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