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Testing of an optimization model for optimal sewer system layout and wastewater treatment locations

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A B S T R A C T

Wastewater systems are one of the most crucial systems for urban infrastructure, especially in regions with large population densities. Determining the optimal (minimum cost) sewer pipe layout and the location of wastewater treatment plants (WWTPs) must take into considerations of economic, environmental, and hydraulics of pipe flows. This paper presents an optimization model for minimum cost design of sewer system layout and wastewater treatment plant locations of the combined systems. The model can be used to minimize the total costs associated with a sewer network and WWTPs by determining an optimal layout of sewer pipes and the locations of WWTPs that meet connectivity, continuity, and capacity requirements. The model is formulated as a 0–1 Integer nonlinear programming (INLP) problem solved using in the general algebraic modeling system (GAMS). The application of the model is illustrated using a simple example to demonstrate that the method allows for significant cost savings.

Keywords: Water resources; Sewer layout; Optimization models; Wastewater planning

1. Introduction

The problem of regional wastewater systems planning is determining the minimum cost sewer system layouts and locations of wastewater treatment plants (WWTPs). The primary goal in this paper is to develop an optimization model for sewer system layouts and locations of regional wastewater treatment systems minimizes total costs. The concept of iso-nodal lines (INL) to define dendritic or branching piping systems originally introduced by the study of Mays [1] are important to this work.

There are various types of collection systems from the local level, as in storm sewer systems, to the regional level, as in planning wastewater systems. In addition, the INL method can be used for any system that has a dendritic or tree-type network. Brand and Ostfeld [2], Mays et al. [3] and Cunha et al. [4] provide background work that is very important to the concepts of regional wastewater pipelines and treatment plant systems. Haghighi and Bakhshipour [5] & Karovic and Mays [6] provide background information on the optimal layout of dendritic piping systems.

2. Cost functions

The cost functions for wastewater systems, including installation, maintenance, and operating costs are usually strictly non-linear [2]. Mays et al. [3] developed cost functions for regional water/wastewater systems, including installation, operating, and maintenance costs. These functions are strictly non-linear equations and are hard to define for different regions and economies of scale. This indicates that solutions would concentrate treatment into one or very few plants rather than in many plants. The influence of the degree of economies of scale can be seen in the results in Table 1.

The results concern a case study where all data is maintained except the cost function ($C = aQ^b$). Therefore, if only

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the level of the economy of scale is changed, the solutions will be different. As the economy of scale level increases (b value is lower), the solution is obtained for a smaller number of WWTPs. The solution of the wastewater problem at the regional level is a compromise. On the one hand, the solution where each community treats its own wastewater does not take into account the important economies of scale. On other hand, the solution where there is only one WWTP implies higher costs for taking wastewater from all discharge points to the WWTP (centralized system). Neither solution is an efficient, sustainable solution. Therefore, to find the best solution, there must be a trade-off between transportation costs and savings provided through economies of scales. Haghighi and Bakhshipour [5], Karovic and Mays [6], Swamee and Sharma [7] used cost functions to determine such things as considered costs per unit length, commercial diameter, or per unit volume, such as manholes. In reality, the system will be highly complex. It is anticipated that the parameters that will be the most dominant in determining the total cost will differ as a function of the particular systems' layout, components, cost functions, and imposed loadings. Table 2 provides a summary of information about the overall cost functions associated with wastewater reuse systems that can be used in the project.

3. Iso-nodal line and connectivity model

An iso-nodal line (INL) is defined as an imaginary line connecting nodes and pipes that a total number of INLs must be equal to the total number of pipes connected to outlines of the sewer system as shown in Fig. 1. Mays and Wenzel [11] applied the concept of INL for determining the minimum cost design of storm sewer system using discrete differential dynamic programming (DDDP). Mays et al. [12] used the INL concept for simultaneously determining the minimum cost layout and design of sewer systems considering physiographic, topographic, and hydrologic conditions. Other researchers used the concept of iso-nodal lines [13,14].

Recently, Alfaisal and Mays [15] used the concept of INL in an optimization model for simultaneously determining minimum costs of layout and pipe design of storm sewer systems using a 0–1 integer nonlinear programming (INLP). The application of INLP to the optimal layout design of a sewer system includes two INLs, which represent ground surface elevations (i.e., from an upstream INL to the next downstream INL), a recursive procedure. Now, considering the flow of the system (i.e., from INL i to INL j), the computations are performed over the possible set of drops in crown elevations for each vector of possible connection of nodes on INLs i, j, and k. Flow directions for the set of nodes for all upstream and downstream node connections (outlets) denote which of these nodes' connections are possible for a sewer system layout. A vector of possible connections is needed for each connection.

Table 1
Wastewater system costs for different cost functions [4]

Number of WWTP	14	10	5	2	1
Low economies of scale $C = aQ^{0.95}$	100	101.4	104.4	112.1	121.1
Medium economies of scale $C = aQ^{0.86}$	101.1	100	100.8	106.8	112.9
High economies of scale $C = aQ^{0.75}$	106.9	103.1	100	104	107.3

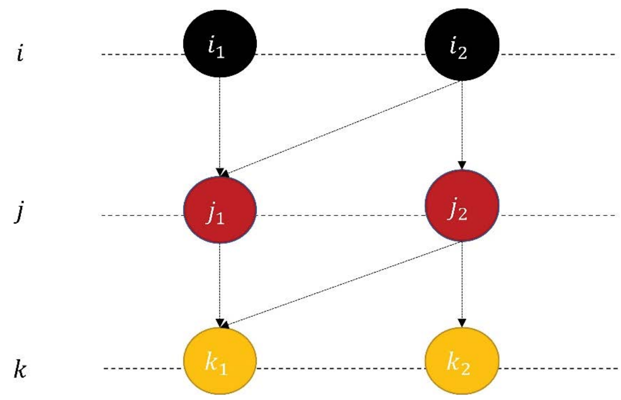

Fig. 1. Iso-nodal line layout and possible nodes connections.

Table 2
Overall cost functions associated with wastewater reuse systems

Reference	Overall cost
Brand and Ostfeld [2]	= 0.33 \$/m^3 (capital costs of WWTP) = 2.88 $Q^{0.99}$ (capital cost of WWTP). = 0.0825 $Q^{0.96}$ (operation and maintenance costs of WWTP).
Mays et al. [3]	= 80 $Q^{0.461}$ (capital cost of pipeline). = 4.56 × 10^{-3} L (distance in mile) $Q^{0.495}$ (operation and maintenance costs of pipeline). All flow rates Q are in gallons per day. = 1.85 \$/m^3.
Al-A'ama and Nakhla [8]	capital cost (= 1.33 US\$/m^3), tertiary treatment (= 0.16 US\$/m^3), collection (= 0.3 US\$/m^3) and distribution (= 0.06 US\$/m^3).
Kajenthira et al. [9]	Secondary WWTP in the range of 0.13–0.63 US\$/m^3. Tertiary WWTP in the range of 1.19–2.03 US\$/m^3.
Al-Zahrani et al. [10]	WWTP reuse ranges from 0.82 to 2.03 US\$/m^3 with an average cost of 1.43 US\$/m^3.

This vector has a dimension equal to the number of possible flow directions from each upstream node on INL i to downstream nodes on INL j and the same from the upstream node on INL j to downstream node on INL k. Each position in the vector of possible connections: which either has a value of 1, implying possible connection of the nodes, or a 0, implying no possible connection.

Fig. 2a shows the drainage directions for a stage n between INLs i and j. For each of the upstream nodes (i_n = 1, 2, 3) on INL i, there are four flow directions: one to each downstream node (j_n = 1,2,3,4). As an example, if the only possible connection of node i_n = 3 is to node j_n = 3, then $T_{3,3}$ = 1, $T_{3,1}$ = 0, $T_{3,2}$ = 0, and $T_{3,4}$ = 0. The concept of the vector of possible connectivity is shown in Fig. 2b. Indeed, more than one node on INL i may have a possible connection to a node on INL j, allowing for branches, so that the tree type network of a storm sewer system can be defined. Each node on INL i must have a possible connection to a node on INL j. The total vector of possible connectivity T_n at any stage n includes all possible connections.

The optimization computations are performed for each possible connection in stage n of INLs (i, j, and k) by considering flows at nodes at the upstream and downstream of each possible open flow connection. Once the decisions for each possible connection at stage n of the system have been considered by the optimizer, the next step is to determine the minimum cost layout (connection of nodes) for that pipe connection. For connectivity optimization, it is difficult to incorporate the flow directions from upstream nodes as a second decision variable at the GAMS optimization. The main difficulty is the inability to compute the flow rates for the succeeding downstream pipes. To solve this difficulty, a special equation was built up in the optimization code. In order to compute these flow rates for the optimization in the next downstream node, connectivity must be defined for the previous upstream node. However, connectivity can be defined using MINLP in GAMS after the computational procedure over all pipes (minimum costs) is completed. The total vector of possible connectivity T in Fig. 2b shows as following:

A connectivity model at nodes (i_n = 1 or 2) on INL i can be formulated using the costs required to continue draining each node (j_n = 1 or 2) on INL j through the next downstream node on INL j for each of the possible connections in node (i_n = 1 or 2). The possible connections for a simple network are shown in Figs. 3 and 4. The total minimum cost for each connection to drain the nodes on INLs i and j to INL k through the nodes on INL j can be computed. From the optimization computations, the minimum cost design for each possible connection and each downstream node up to INL j are known. The minimum total cost up to INL k can be computed by performing the computations for each possible connection between INLs j and k, taking into account the minimum costs up to INL j.

This gives a minimum total cost for portions of the system up to INL k, considering each of the possible connections between INLs i and j in addition to the costs to continue draining the flow through the next downstream INL. Essentially, this amounts to performing designs for each possible drop within the corridors defined by the possible connections from INLs j to k. The cost of placing nodes on INL k is included. Once the minimum cost required to continue draining each node on INL j through the next downstream pipe for each possible connection in connection pipe is known, a model can be formulated to select the connectivity or layout for each pipe connection.

3.1. Mathematical formulation

The connectivity approach is used to define the minimum cost connections of nodes once the computations have been completed for each possible connection between

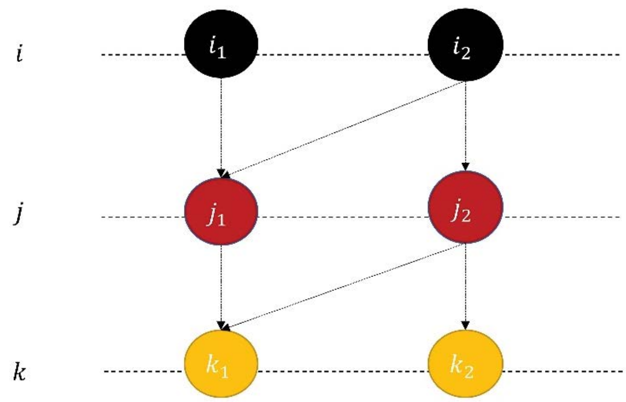

Fig. 3. Possible layouts to drainage line k.

$$T_{1,1} = 1 \qquad T_{2,1} = 0$$
$$T_{1,2} = 1 \qquad T_{2,2} = 1$$
$$T_{1,3} = 0 \qquad T_{2,3} = 1$$

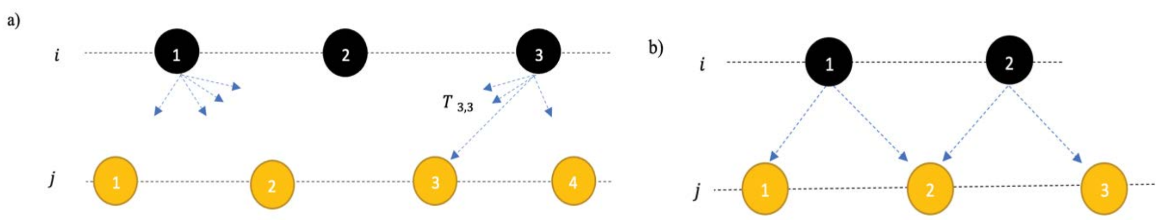

Fig. 2. (a) Drainage directions and (b) possible connections (adapted from Mays [1]).

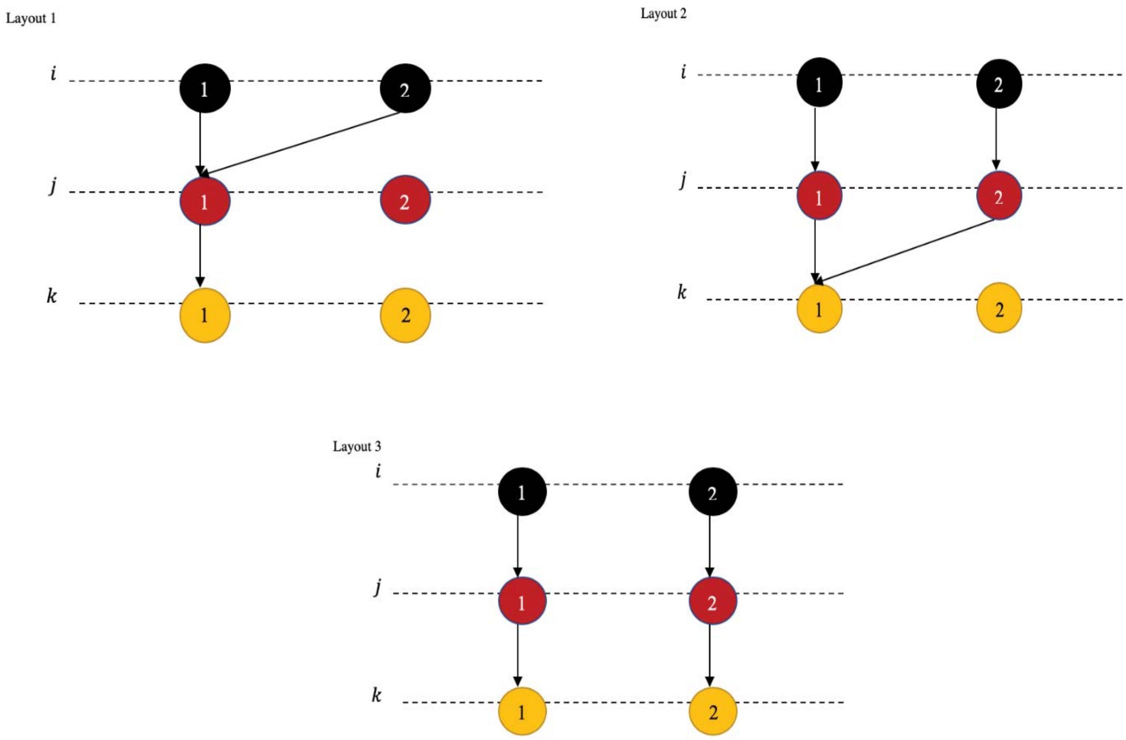

Fig. 4. Possible layouts.

nodes n on INL i and nodes n on INL j before proceeding to the downstream nodes n on INL j to nodes n on INL k. The minimum cost layout must be chosen so that each upstream node on INL i is connected to downstream nodes on INL j by only one pipe, which must be one of the possible connections.

The model constraints allow for only one pipe to drain node n (i.e., the summation of the 0–1 variables, each representing a layout that allows node n to be drained, is equal to 1). Because each upstream node n must be drained, a constraint exists for each of these nodes n = 1, 2, 3, … N. Similarly, constraints can be developed to satisfy the restriction that each node on INL j is drained by only one pipe connecting to node n on INL k.

- The flow in pipes from each source node i must flow through one collection node j, which can be satisfied as follows:

$$\sum_j a_{i,j} x_{i,j} = 1 \quad \forall\ i \tag{1}$$

where $a_{i,j}$ is equal to 1 if there is a possible pipe connection from node i to node j and is equal to 0 if there is not a possible pipe connection from node j to node k; the binary decision variable $x_{i,j}$ is defined as either 0 or 1 where $x_{i,j}$ = 1 indicates a pipe connection between nodes i and j and $x_{i,j}$ = 0 indicates no connection.

- The flow from each collection node j must flow through one WWTP node on INL k, which can be satisfied as follows:

$$\sum_k b_{j,k} y_{j,k} = \begin{cases} 1 & \text{if } \sum_k QI_{j,k} > 0 \\ 0 & \text{if } \sum_k QI_{j,k} = 0 \end{cases} \tag{2}$$

where $b_{j,k}$ is equal to 1 if there is a pipe connection from node j to node k and $b_{j,k}$ is equal to 0 if there is not a possible pipe connection from node j to node k; the binary decision variable $y_{j,k}$ is defined as either 0 or 1 where $y_{j,k}$ = 1 indicates a pipe connection between nodes j and k and $y_{j,k}$ = 0 indicates no connection.

- Continuity constraints for flows in the system states that all the system must be in equilibrium, so that the flow produced at source nodes on INL i must be sent to WWTP nodes on INL k. The continuity equation at source nodes on INL i to collection nodes on INL j is

$$QR_i = \sum_j QS_{i,j} a_{i,j} x_{i,j} \quad \forall\ i \tag{3}$$

The continuity equation at collection nodes on INL j to WWTP nodes on INL k is

$$\sum_i QS_{ij} a_{i,j} x_{i,j} - \sum_k QI_{j,k} b_{j,k} y_{j,k} = 0 \quad \forall\ j \tag{4}$$

- Lower and upper bound constraints. However, bounds play a significant role in nonlinear (NLP) models. To avoid an undefined operation, such as division by zero, it may be essential to provide bounds. In NLP, a definition of a reasonable solution space will assist in efficiently finding a solution.

$$Q_{\min_{i,j}} a_{i,j} x_{i,j} \leq QS_{i,j} \leq Q_{\max_{i,j}} a_{i,j} x_{i,j} \quad \forall (i,j) \tag{5}$$

and

$$Q_{\min_{j,k}} b_{j,k} y_{j,k} \leq QI_{j,k} \leq Q_{\max_{j,k}} b_{j,k} y_{j,k} \quad \forall (i,j) \tag{6}$$

where $Q_{\min}$ and $Q_{\max}$ are respectively the minimum and the maximum amount of wastewater through the system.

The objective is to select a set of possible layouts to satisfy the above constraints such that the minimum cost of the complete layout is selected for two stages of the system. The cost of each possible layout is determined by selecting the cheapest layout of the possible connections associated with flows. The cost of all upstream pipes, the WWTP, and the cost of the possible layout represent the cost coefficients, CPIP, CPIP1, and CWWTP, for the objective function. The objective function can be expressed as:

$$\begin{aligned} \text{Min cost} & \sum_i \sum_j \text{CPIP } QS_{i,j} a_{i,j} x_{i,j} \\ & + \sum_j \sum_k \text{CPIP2 } QI_{j,k} b_{j,k} y_{j,k} \\ & + \sum_j \sum_k \text{CWWTP } QI_{j,k} b_{j,k} y_{j,k} \end{aligned} \tag{7}$$

The optimization model, defined by the above equations, represents a 0–1 integer nonlinear programming optimization problem.

3.2. Test Example 1

To build the model in GAMS and ensure that the model formulation is correct, two examples were considered: the vector of possible connections with different costs associated with the flow and the vector of possible connections with same costs associated with flow, Fig. 5. Overall, continuity and connectivity constraints were used to ensure that the model would run perfectly through these two examples, changing the costs of pipelines and of WWTPs, and a possible path either way from source nodes on INL i to collection nodes on INL j or collection nodes on INL j to WWTP nodes on INL k). In test Example 1, the possible paths for $a_{i,j}$ and $b_{j,k}$, are considered, as shown in Tables 3 and 4:

It was assumed that all total costs of installation, operation, and maintenance from source nodes on INL i to collection nodes on INL j, CPIP, and total costs of installation, operation, and maintenance from collection nodes on INL j to WWTP nodes on INL k CPIP2 were included, as shown in Tables 5–7.

The optimum configuration of Example 1 shows that the flow tries to go through the possible paths allowed in the system and, at the same time, takes a minimum cost path, so

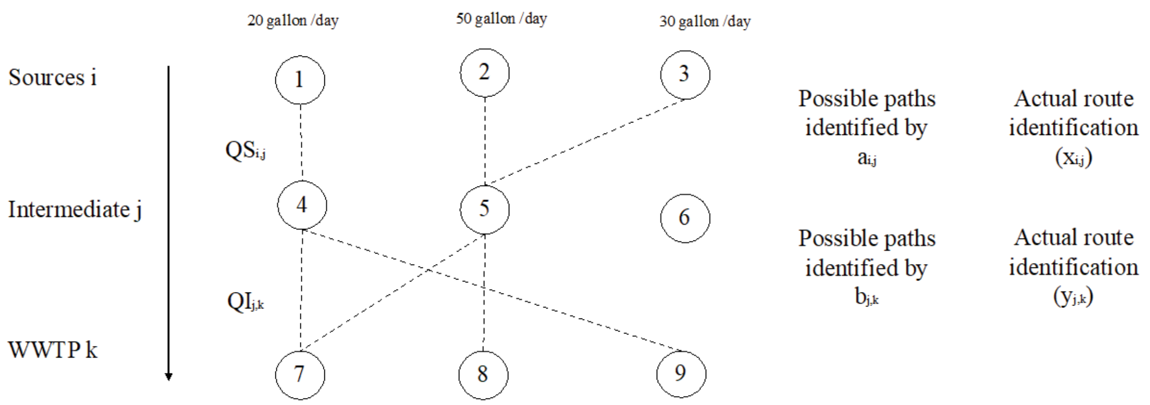

Fig. 5. Input values for the model in GAMS for Example 1.

Table 3
Possible paths of draining wastewater from sources nodes i to intermediate nodes j, $a_{i,j}$, for Example 1

Possible paths		Candidate intermediate nodes		
		n4	n5	n6
Sources node	n1	1	No possible path	No possible path
	n2	No possible path	1	No possible path
	n3	No possible path	1	No possible path

Table 4
Possible paths of draining wastewater from intermediate nodes j to WWTP nodes k, $b_{j,k}$, for Example 1

Possible paths		Candidate WWTP nodes		
		n7	n8	n9
Candidate intermediate nodes	n4	1	No possible path	1
	n5	1	1	No possible path
	n6	No possible path	No possible path	No possible path

Table 5
Assumption of the total costs of installing, operating and maintenance from sources nodes i to intermediate nodes j, CPIP, \$/gallon for Example 1

Total costs		Candidate intermediate nodes		
		n4	n5	n6
Sources node	n1	\$2	No possible path	No possible path
	n2	No possible path	\$5	No possible path
	n3	No possible path	\$1	No possible path

Table 6
Assumption of the total costs of installing, operating and maintenance from intermediate nodes j to WWTP nodes k, CPIP2, \$/gallon for Example 1

Total costs		Candidate WWTP nodes		
		n7	n8	n9
Candidate intermediate nodes	n4	\$3	No possible path	\$3
	n5	\$3	\$5	No possible path
	n6	No possible path	No possible path	No possible path

Table 7
Assumption of the total costs of new plant construction and operating and maintenance of wastewater treatment plants, CWWTP, \$/gallon for Example 1.

Candidate WWTP	\$/gallon
n7	\$2
n8	\$2
n9	\$3

all discharges are ended by n7, which has the lowest WWTP cost (\$2/gallon/d). Fig. 6 with total costs of \$820/d.

3.3. Test Example 2

Example 2 performed the model for a different model parameters inputs that considered more possible paths and the same cost values for objective functions. The flowing Tables 8 and 9 and Fig. 7 are the input data for the example system. In test Example 2, the possible paths for $a_{i,j}$ and, $b_{j,k}$, are as follows:

The total costs of installation, operation, and maintenance from source node i to intermediate node j, CPIP, and total costs of installation, operation, and maintenance from intermediate node j to WWTP node k, CPIP2 are assumed to be equal to \$1 /gallon as shown in Tables 10 and 11.

Table 8
Possible paths of draining wastewater from sources node i to intermediate nodes j, $a_{i,j}$, for Example 2

Possible paths		Candidate intermediate nodes		
		n4	n5	n6
Sources node	n1	1	1	No possible path
	n2	No possible path	1	No possible path
	n3	No possible path	1	1

Table 9
Possible paths of draining wastewater from intermediate node j to WWTP nodes k, $b_{j,k}$, for Example 2

Possible paths		Candidate WWTP nodes		
		n7	n8	n9
Candidate intermediate nodes	n4	1	1	No possible path
	n5	No possible path	1	No possible path
	n6	No possible path	1	1

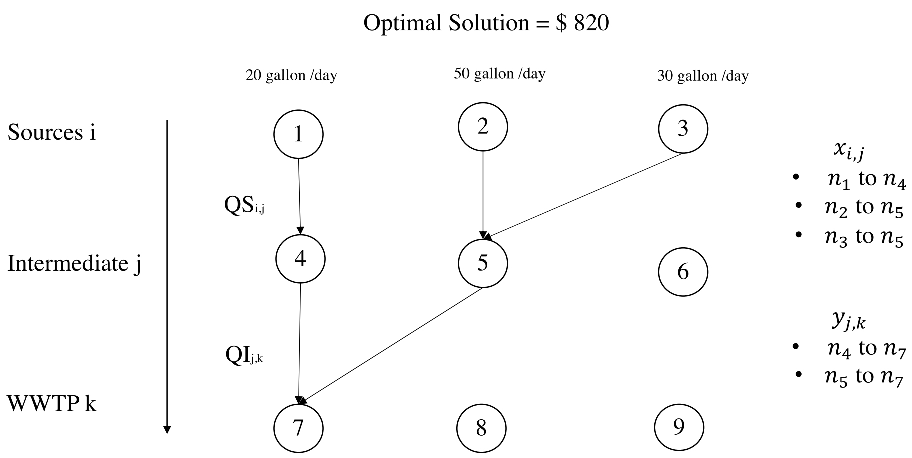

Fig. 6. Optimum configuration for Example 1.

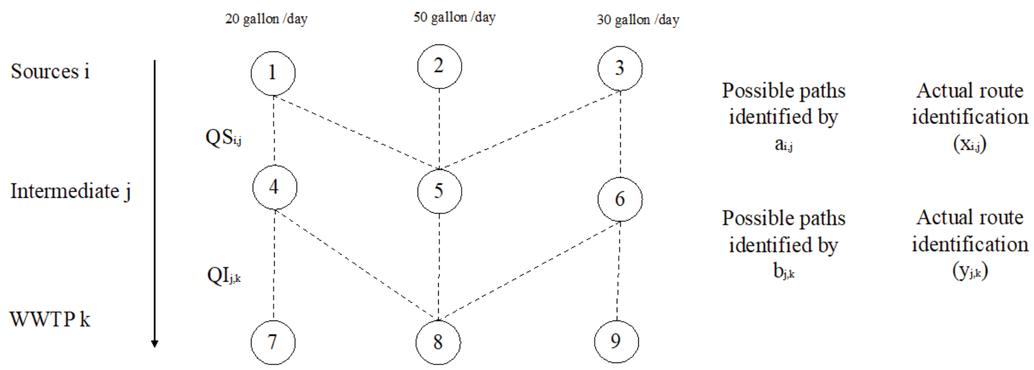

Fig. 7. Input values for the model in GAMS for Example 2.

Table 10
Assumption of total costs of installation, operation, and maintenance from source node *i* to intermediate node *j*, CPIP, $/gallon for Example 2

Total costs		Candidate intermediate nodes		
		n4	n5	n6
Sources node	n1	$1	$1	No possible path
	n2	No possible path	$1	No possible path
	n3	No possible path	$1	$1

Table 11
Assumption of total costs of installation, operation, and maintenance from intermediate node *j* to WWTP node *k*, CPIP2, $/gallon for Example 2

Total costs		Candidate WWTP nodes		
		n7	n8	n9
Candidate intermediate nodes	n4	$1	$1	No possible path
	n5	No possible path	$1	No possible path
	n6	No possible path	$1	$1

Assumed costs of new WWTP construction, operation, and maintenance would be equal to $1/gallon for all nodes. The optimum configuration for test Example 2 is shown in Fig. 8, with total costs of $300/d.

4. Conclusion

The application of the model is illustrated through two example systems and the results are discussed. The simple examples demonstrate that using the method allows for significant cost saving for large systems while further testing and developments may be needed. In the model formulation, it minimizes costs without considering the capacity limitation of a WWTP. The objective function minimizes total costs subject to continuity constraints and the connectivity model. At the starting point, the costs for each path are defined (paths from source nodes to collection nodes and paths from collection nodes to WWTP nodes), as are costs of new plant construction, operation, and maintenance. The reason for using this procedure is to check the quality of the model from a coding perspective. From a coding perspective, the mathematical formulation should be applied to the objective function, continuity constraints, and connectivity constraints only, so that later it can add more constraints for different purposes. This was applied to make sure that the model would work perfectly without any issues and it examined the mathematical formulation for continuity and conductivity constraints.

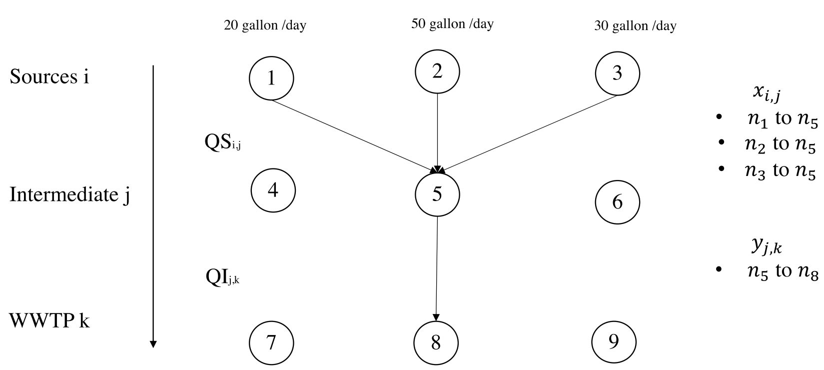

Fig. 8. Optimum configuration for Example 2.

The primary purpose of developing such models is to encourage decision-makers to plan regional wastewater systems with minimum costs. In the future, one thing can be added to the design purpose model is hydraulic constraints [16,17]. Hydraulic constraints include: (1) only commercial pipe diameters are considered, (2) minimum and maximum pipe slopes allowed, (3) continuous slope, (4) minimum pipe cover depths allowed, and (5) WWTP facilities elevations.

Symbols

Sets

i — Set of wastewater sources nodes on INL i

j — Set of the possible location of intermediate (collection) nodes on INL j

k — Set of possible location WWTP nodes on INL k

Parameters

QR_i — Amount of wastewater produced at sources for a node on INL i

$QWWTP_{max}$ — Maximum amount of wastewater that may be treated at a node on INL k

Q_{min} — Minimum flow allowed in the pipe system

Q_{max} — Maximum flow allowed in the pipe system

CPIP — Discount cost of installation, operation, and maintenance from source node i to intermediate node j

CWWTP — Discount cost of new WWTP construction, operation, and maintenance.

CPIP2 — Discount cost of the installation, operation, and maintenance from collection nodes on INL j to WWTP nodes on INL k.

$a_{i,j}$ — Possible paths of draining wastewater from source node to intermediate node j.

$$a_{i,j} = \begin{cases} 1 & \text{possible path from node } i \text{ to node } j \\ 0 & \text{no possible path from node } i \text{ to node } j \end{cases}$$

$b_{j,k}$ — Possible paths of draining wastewater from collection nodes on INL j to WWTP nodes on INL k.

$$b_{j,k} = \begin{cases} 1 & \text{possible path from node } j \text{ to node } k \\ 0 & \text{no possible path from node } j \text{ to node } k \end{cases}$$

Variables

$QS_{i,j}$ — Flow carried from source node i to intermediate node j

$QI_{j,k}$ — Flow carried from intermediate node j to WWTP node k

$x_{i,j}$ — Binary variable defined as either 0 or 1 where $x_{i,j} = 1$ indicates a pipe connection between nodes i and j and $x_{i,j} = 0$ indicates no connection

$y_{j,k}$ — Binary variable defined as either 0 or 1 where $y_{j,k} = 1$ indicates a pipe connection between nodes j and k and $y_{j,k} = 0$ indicates no connection

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