

## Estimation of solids distribution and settling velocity of solid particles in secondary clarifiers: large-scale measurements and numerical modeling

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### ABSTRACT

Within the framework of environmental preservation, a self-monitoring must be imposed on the wastewater treatment plants (WWTPs) and industries to make the industrialists and the decision-makers responsible for the quality of their discharges. The main objective of this study is to develop a mathematical model that allows simulating the variations of the sludge blanket height (SBH) according to the concentrations, integrating the different operating parameters of the WWTPs. This model is validated by a wide range of full-scale experimental results to obtain an operational model, which can also be used in simulation tests. On the other hand, the calibration of the sludge volume index values allowed the model to reproduce the variations of the SBH and ensure a good liquid/solid separation.

*Keywords:* Sedimentation velocity; Secondary clarifier; Clarification and thickening zones; Mathematical modelling; Biological treatment

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### 1. Introduction

Several countries around the globe have begun to search for solutions to conserve drinking water. They have been reusing wastewater to supply the increasing need for water consumption.

At the present moment, with the industrial and economic population growth, large cities are inadequate to purify wastewater by conventional processes, so decision-makers have taken it upon themselves to build wastewater treatment plants that use several treatment processes. The most frequently used treatment process is the biological process by activated sludge.

Wastewater treatment plants require a step series involving physical, physicochemical, and biological treatments to obtain purified water according to specific standards. They are essential for the economic and social development

of any country. Also, it protects the health of the population against pathogenic bacteria and viruses. The activated sludge treatment process is suitable for different types of communities, except for extremely small communities (with fewer than 1,000 residents). This technique is very successful in the world, and it has a highly important purification efficiency for the reduction of the values of the different forms of pollution to ensure the protection of the receiving environments. This type of treatment plant requires self-monitoring with professional maintenance.

The activated sludge process is a biological process in which microorganisms oxidize and mineralize organic matter. The composition of microorganisms depends on the nature of the wastewater and also on the design and operating parameters of the wastewater treatment plant (WWTP) [1]. These microorganisms are maintained in suspension, either by blowing air into the plant or with using turbines.

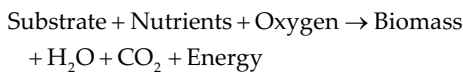
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Dissolved oxygen is necessary for the growth of microorganisms to ensure that the microbial population remains active. Organic compounds and even nitrogen and phosphorus substances constitute the principal nutrition for the development of these microorganisms.

Biological treatment by activated sludge is based on a microbial culture maintained in suspension. This aerobic culture is placed in the aeration tank [2,3]. The principal organization of a simple activated sludge treatment plant is shown in Fig. 1.

The final step in the treatment process is the separation of the sludge from the treated water by settling. The sedimentation is the last link in the treatment process before the treated water return to the natural environment. This settling is provided by the secondary clarifier placed after the aeration tank. The recirculation of the activated sludge process is so-called secondary wastewater treatment. Its operation requires two main tanks: the aeration tank (bioreactor) and the secondary clarifier (secondary settler). The aeration tank is where the sludge is degraded by microorganisms during oxidation operation when separated organic matter is retained by bio-flocs [1,4]. The biological process in WWTP is realized by different types of bacteria (heterotrophic and autotrophic). The principal microorganisms in this process are bacteria, whereas fungi, algae, and protozoa are of secondary importance [5,6]. The metabolization of organic matter can be explained in a simplified form:



The separation of the obtained sludge is realized in a secondary clarifier. The purified water is separated from the sludge by gravity sedimentation. The settled sludge is removed or returned to the aeration tank (Fig. 1) [7,8]. To provide a constant biomass concentration, the excess biomass produced is generally removed at the outlet of the clarifier. This biological sludge may require reduction and or stabilization treatments before disposal [9]. The study of settling velocity as a function of different sludge indices, which is also based on Vesilind parameters and sludge volume indexes (SVI), has allowed the development of more powerful correlations. The SVI values are influenced by the settling tank geometry (height, diameter, etc.), which consequently affects the design and simulation of the secondary settling process. These indices were readjusted during the disturbances by integrating all the operating data obtained at the treatment

plant. The secondary clarifier and the aeration tank are concerned with the admission of overloads. In the aerator tank, the growth of microorganisms is affected by the decrease in the sludge age. The consequences of hydraulic overloads on the secondary clarifier can increase the ascensional velocity and the mass flux.

The secondary clarifier of an activated sludge treatment plant is mainly used to separate the suspended solids contained in the treated water. The clarifier is the final stage of the process, where the treated water is discharged into the natural environment through the overflow. According to the water quality requirements, a tertiary treatment is necessary, such as chlorine dosage. A part of the sludge concentrated at the bottom of the settling tank is returned to the aeration tank to ensure a desired concentration of biomass in the aeration tank; the other part of the biomass is sent to the sludge treatment unit. In accord with several authors [10,11], it is possible to identify three factors that limit the performance of the secondary clarifier: (i) the conditions of the outlet zone (risk of sludge blanket rise), (ii) the ratio of recycled suspended solids, and (iii) fluctuations in the suspended solids content of the effluent.

Several definitions of sedimentation are given in the literature. These definitions vary widely from author to author. Tiller [12] defines settling as the separation of suspended particles from water under the influence of gravity. Lee et al. [13] define sedimentation (settling) as a mechanical separation operation by gravity difference. This author also notes that the role of decantation is to get a clarified liquid and, at the same moment, a concentrated sludge at the bottom of the clarifier.

The solids-flux theory is used to characterize the liquid/solid separation rate and to study the settling tanks operating performance. The authors [11,14–20] indicate that one-dimensional sedimentation models have a constant and homogeneous horizontal concentration distribution. These models are also called layer models. The most widely used models in the literature are based on the solid-flux theory developed by Kynch [21]. Kynch considered that the sedimentation velocity of solid particles ( $V_s$ ), at any point in the settling tank, depends only on the local concentration of the particles ( $C$ ) (total suspended solids (TSS)). The settling process is defined by the continuity equation, and it is independent of the real forces that influence on the solid particle.

For illustrating solids-flux theory, two important mechanisms are considered for the movement of solid particles to the bottom of the settling tank: hydraulic flow and sediment flow. The first phenomenon is due to the downward

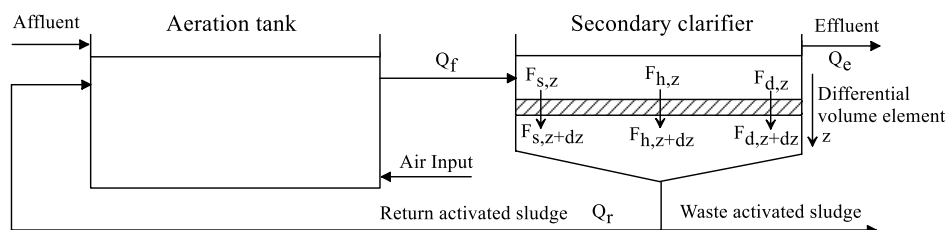


Fig. 1. Coupling of the aeration tank and secondary clarifier.

movement of TSS and is related to the output flow ( $Q$ ) (recycling or removal). The hydraulic velocity ( $V_h$ ) in a clarifier of constant section ( $S$ ), sometimes called transport velocity, is evaluated from the flow rate. The hydraulic mass flow ( $F_h$ ) is given by the following expression:

$$F_h = V_h C = \frac{Q}{S} C \tag{1}$$

The second phenomenon is due to the force of gravity. A solid suspension of concentration in a liquid phase tends to fall to the bottom of the clarifier with a settling velocity ( $V_s$ ) because of the action of gravity. The relation between this speed of fall of the particles and their concentration is defined by the mass flow expression ( $F_s$ ) following:

$$F_s = V_s C \tag{2}$$

The result of the two Eqs. (1) and (2) represents the solids-flux ( $F_{s,h}$ ) in the secondary clarifier:

$$F_{s,h} = F_s + F_h = (V_s + V_h) C \tag{3}$$

Based on the operating parameters of a wastewater treatment plant, such as residence time, sludge loading, aeration, and sludge age, Gernaey et al. [22] specify some objectives of wastewater treatment plant modeling: (i) simulate bacterial biomass growth and substrate removal (carbonaceous nitrogen and phosphorus materials), (ii) predict treatment quality with the concern is to preserve water environments, (iii) decrease treatment costs by optimizing energy consumption and the quantities of sludge produced, (iv) optimize the operation of the treatment plant by acting on the model parameters in case of disturbances, and (v) dimension the installations and structures of the wastewater treatment plants to ensure good efficiency and rehabilitation of these structures.

Wastewater treatment plants modeling (essentially activated sludge plants) is thus a widely used method to optimize plant operation and improve the efficiency and reliability of plants and facilities. According to Gernaey et al. [22], modeling can be employed to modify the operating parameters of wastewater treatment plants to ensure economic treatment. It is also used to better understand the operation of facilities such as the aeration tank and secondary clarifier.

Several mathematical models describing the separating process of biomass from purified water in secondary clarifiers are cited in the literature. As an example, the works of Gernaey et al. [22], Gillot et al. [23], Torfs et al. [24], and Bakiri and Nacef [15]. Modeling and control of plants were frequently used for evaluating the quality of wastewater treatment plants functioning by activated sludge. Some authors [4,11,18,19,24–26] believe that WWTP modeling aims to size the installations, in especially the one-dimensional (1-D) modeling, and control of secondary clarifiers.

The present study focuses mainly on the hydrodynamic model of the one-dimensional secondary clarifier, based on the solids-flux theory and the continuity equation, which may have been used to optimize the WWTP. The secondary clarifier models with integrating settling velocity models as a function of the sludge volume index are examined.

## 2. Materials and methods

### 2.1. Wastewater treatment plants studied

Three municipal wastewater treatment plants (WWTPs) were used for this study. These WWTPs operate with an activated sludge process. Sétif wastewater treatment plant (SWWTP) was put into service in March 1996. The domestic wastewater comes essentially from the city of Sétif. The biological treatment is realized by activated sludge in extended aeration at low load.

The wastewater treatment plant of Sidi Merouane (SMWWTP) was commissioned in July 2009. It ensures the treatment of all the effluents admitted at the entrance of the WWTP, which come from the wastewater of Mila, Garem Gouga, Sibari, Ras-El-Bir, Annouche Ali, and Sidi Merouane. After the treatment of these waters, they are drained into the dam of Beni Haroun.

Ibn Ziad wastewater treatment plant (IZWWTP) is the first plant of Constantine that was put into service in May 1997. It is located 12 km from the municipality of Hamma Bouziane. It is designed to accommodate 69,120 m<sup>3</sup>/d for 330,000 population equivalent (PE) and is immediately discharged into the river El-Rimal. The main operational conditions of each WWTP are presented in Table 1.

It is important to note that the volumes of water received by the treatment plants with the quality of this water are very strongly dependent on rainfall and the connected networks. The data presented in Table 1 can be used in the mathematical modelling and optimization of the functioning of the wastewater treatment plants.

### 2.2. Types of activated sludge

It should be noted that the sludge used in this study came from the aeration tank of three wastewater treatment plants. The decantation experiments were used immediately after sampling. The samples taken on the settling column tests are carried out weekly. The physicochemical parameters studied are temperature, pH, sludge settleability, TSS and stirred specific volume index (SSVI). The three types of activated sludge used in this study are compiled in Table 2.

Table 2 shows the physical and chemical characteristics of the sludge to be decanted (mixed liquor entering the

Table 1  
Main parameters characterizing affluents received by wastewater treatment plants

parameters	WWTPs		
	SWWTP	SMWWTP	IZWWTP
pH	7.58 ± 0.93	7.25 ± 1.25	7.14 ± 1.05
Maximum peak flow, m <sup>3</sup> /h	1,690	1,370	8,640
Average flow inlet, m <sup>3</sup> /h	802	856	2,880
BOD <sub>5</sub> load, kg/d	17,820	8,220	25,620
TSS load, kg/d	23,100	12,330	29,850

Table 2  
Different sedimentation tests performed in settling column tests

WWTPs	pH	T (°C)	Settleability (mL/L)	SSVI (mL/g)	TSS (g/L)
SWWTP	6.92 ± 0.80	18 ± 4	265 ± 80	[71–205]	[1.20–4.20]
SMWWTP	6.60 ± 1.05	19 ± 3	290 ± 95	[37–85]	[1.80–3.50]
IZWWTP	6.75 ± 0.94	21 ± 5	875 ± 145	[135–350]	[2.80–7.40]

secondary clarifier). They are microorganisms which are not the same from one station to another. It is very important to identify these characteristics to control the settling process. According to the results of Table 2, it can be observed that the sludge index of the IZWWTP is within the standards, which range from 135 to 350 mL/g, so the activated sludge of this plant is in good working conditions, the same remark concerning the quality of the sludge of the SWWTP. However, the sludge of the SMWWTP exceeded 85 mL/g, in this case, the sludge has poor sedimentation characteristics.

### 2.3. Measurement of activated sludge parameters

#### 2.3.1. Total suspended solids

The determination of TSS of the different activated sludge was performed by centrifugation (AFNOR NF T 90-105-2 standard) at 4,500 rpm for 20 min. The sediment was obtained and dried in an oven at 105°C for 24 h.

#### 2.3.2. Sludge volume index

The SVI represents the volume occupied by one gram of sludge after 30 min of settling in a one liter settling column test. This index represents the ratio between the sludge volume after 30 min in mL/L and the quantity of TSS in g/L introduced in the settling column tests. The SVI is expressed in mL/g. The SVI is the standard for the other indices. SSVI: stirred specific volume index. It corresponds to SVI but with an initial mass fixed at 3.5 g/L and a stirring speed of 1 rpm.

#### 2.3.3. Initial settling velocity

The measurements are carried out in settling column tests of about two liters (AFNOR T 97-001 standard). They consist in observing the height of the liquid–solid interface in the test tube as a function of time and generally for 2 h of settling. The  $V_s$  value is measured in m/h. It depends on the TSS and on the stirred specific volume index (SSVI). This velocity is used to determine the ability of the sludge to settle.

#### 2.3.4. Settling column tests

To examine activated sludge settling, we use a glass test tube with a diameter of 78 mm and a length of 460 mm.

### 2.4. Model for secondary clarifier

The objective of this section is to develop a mathematical model that can be used to simulate the variation of the sludge blanket height as a function of the concentrations. The model was developed by integrating the different

operating parameters of three wastewater treatment plants in Algeria. This model must be validated by a wide range of full-scale experimental results to obtain an operational model, which can also be used in simulation runs.

Considering a cylindrical volume element ( $dV$ ), a cross-section constant, in the thickening zone at a distance between  $z$  and  $z + dz$ . Any attempt to develop a mathematical model requires a thorough understanding of the process (Fig. 1). The mass balance can be expressed mathematically as:

$$F_t|_z = F_t|_{z+dz} \pm (r_c) dz + \frac{\partial C}{\partial t} dz \quad (4)$$

where  $F_t = F_s + F_h + F_d$ ; is the total solids-flux,  $g/m^2 \cdot s$ ;  $F_s$  is the gravity settling flux,  $g/m^2 \cdot s$ ;  $F_h$  is the hydraulic flux (divided into two fluxes: the upward bulk flux and the downward bulk flux),  $g/m^2 \cdot s$ ;  $F_d$  is the dispersion flux of suspended solids in the axial direction;  $C$  is the TSS,  $g/L$ ; The depth of the secondary clarifier ( $z$ ) is calculated from the top layer downwards,  $m$ ; ( $r_c$ ) is the rate of reaction and is a function of  $C$ ,  $g/L \cdot h$ .

The three fluxes are important for determining the profiles concentration in the secondary clarifier. Differentiating and after simplification, we obtain:

$$\frac{\partial F_t}{\partial z} \pm (r_c) + \frac{\partial C}{\partial t} = 0 \quad (5)$$

where

$$F_t = F_{s,h} + F_d = F_s + F_h - D \frac{\partial C}{\partial z} = \left( V_s(C) + \frac{Q}{S} \right) C - D \frac{\partial C}{\partial z} \quad (6)$$

and

$$F_t = \left( V_s(C) + \frac{Q}{S} \right) C - D \frac{\partial C}{\partial z} \quad (7)$$

where  $V_s$  settling velocity can be expressed as a function of total suspended solids  $C$ ,  $m/h$ ;  $D$  is the dispersion coefficient of suspended solids in the axial direction, it was assumed that the diffusion coefficient is constant,  $m^2/h$ .

Several simplifying assumptions were considered for this model: (i) there is continuous thickening, that is, no vertical dispersion, (ii) the TSS is entirely uniform in a horizontal plane in the clarifier, (iii) the gravitational velocity is null at the bottom of the clarifier, (iv) there is no significant biological reaction that influences the TSS in the clarifier, (v) the gravitational velocity is a function only of the TSS.

After developing Eq. (7), we obtain Eq. (8) which is a nonlinear partial differential equation and in steady state.

$$\frac{\partial C}{\partial t} = -\frac{\partial F_t}{\partial z} = 0 \tag{8}$$

The fluxes, as shown in Fig. 2 and Eq. (5), are defined as:

$$F_h = \begin{cases} V_{he} C = \frac{Q_e}{S} C, & \text{clarification zone} \\ V_{hr} C = \frac{Q_r}{S} C, & \text{thickening zone} \end{cases} \tag{9}$$

$$F_s = V_s(C)C \tag{10}$$

$$F_d = -D \frac{dC}{dx} \tag{11}$$

For the volumetric flows balance, it should be noted that:

$$Q_f = Q_e + Q_r \tag{12}$$

Here,  $Q_f$ ,  $Q_e$  and  $Q_r$  are the feed volumetric flow rate, the exit volumetric flow rate, and the recycled volumetric flow rate respectively, L/h;  $S$  is the secondary clarifier section, m<sup>2</sup>; The hydraulic velocities.  $V_{he}$  and  $V_{hr}$  are independent of concentration, m/h; consequently.

Simplifying gives

$$\frac{\partial F_t}{\partial z} = \frac{\partial \left( \left( V_s(C) + \frac{Q}{S} \right) C - D \frac{\partial C}{\partial z} \right)}{\partial z} \text{ with } V_h = \frac{Q}{S} \tag{13}$$

In the present study, we write the material balance Eq. (8) in a dimensionless equation. For this purpose, we introduce the following dimensionless variables:

$$\xi = \frac{z}{L}; \quad \psi = \frac{C}{C_r} \text{ with: } u = \frac{D}{L} \text{ and } \vartheta_i = \frac{F_i}{C_r} \tag{14}$$

where  $L$ : is the length of the secondary clarifier, m;  $u$ : is the flow velocity, m/s;  $C_r$ : is the return activated sludge concentration.

$$\frac{\partial \vartheta_i}{\partial \xi} = \frac{\partial \left( (V_s + V_h) \psi - u \frac{\partial \psi}{\partial \xi} \right)}{\partial \xi} \tag{15}$$

According to Fig. 2 for clarification zone and by derivatives with finite differences, Eq. (15) takes the following form:

$$\frac{\Delta \vartheta_{i,i}}{\Delta \xi} = \frac{V_{s,i-1} \psi_{i-1} - V_{s,i} \psi_i + V_{hr} (\psi_i - \psi_{i-1}) + u_e (\psi_i - 2\psi_{i-1} + \psi_{i-2})}{\Delta \xi} \tag{16}$$

Using the same reasoning to obtain Eq. (16) for the thickening zone:

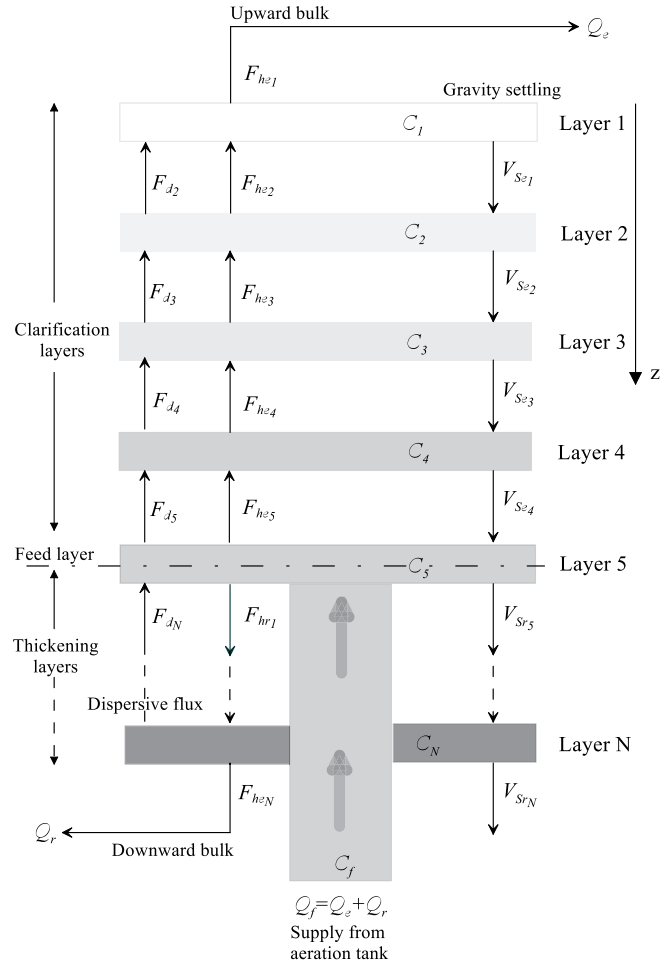


Fig. 2. Solids and velocities distribution in a secondary clarifier.

$$\frac{\Delta \vartheta_{t,i}}{\Delta \xi} = \frac{V_{s,i-1} \psi_{i-1} - V_{s,i} \psi_i + V_{hr} (\psi_{i-1} - \psi_i) + u_r (\psi_i - 2\psi_{i-1} + \psi_{i-2})}{\Delta \xi} \tag{17}$$

For solving the nonlinear continuity Eq. (15), many researchers in this subject area use numerical methods such as Runge–Kutta, finite difference, and iterative. Iterative methods are among the most used.

### 3. Results and discussion

The nature of the organic matter, the growth and structure of flocs, the viscosity of the affluent, the geometrical of the secondary clarifier, and the density of the particles all have a role in the settling process in activated sludge treatment systems.

The concentration of TSS in the clarification zone does not exceed 30 mg/L. It is very low, particularly the upper part of the secondary clarifier (top layers of the clarifier).

The secondary clarifier receives treated wastewater from an activated-sludge reactor by an output hole, which often has a high settling capacity. When a batch-settling column test is performed within the same supply, the formation of a solid-liquid interface is shown soon after settling begins [4,13].

The sludge is collected in the thickening zone and is located at the bottom of the secondary clarifier. The recycle and purge flows of the sludge circulate by gravity. The floc structure is adequately compacted. It allowed an increased concentration in this zone.

### 3.1. Evaluation of settling velocity and measurements

This section illustrates the semi-empirical models for describing the evolution of the liquid/solid interface over time. As a general practice, exponential functions are used to model the populations. According to the experimental results obtained, the variation of  $V_s$  as a function of SVI is an increasing exponential function (same for population growth). In the literature, we find three types of models describing the variation in sedimentation velocity. The most widely used current models are usually basing on Vesilind exponential model [27] ( $V_{s1} = k_1 e^{-n_1 C}$ ) and Yoshioka et al. power model [28] ( $V_{s2} = k_2 C^{(1-n_2)/n_2}$ ). Other authors use the Cho et al. function [29]. This function is simply the expression of Vesilind divided by the biomass concentration ( $V_{s3} = k_3 e^{-n_3 C} / C$ ), where  $k_i$  and  $n_i$  are the characteristic biomass parameters,  $C$  is the sludge concentration (total suspended solids), and  $V_{si}$  is the settling velocities.

There are many different types of expressions for settling velocity, as a function of sludge concentration that can be found in literature. As a result, the main objective of this work is to use the Vesilind (1968) [27] sedimentation model ( $V_s = ke^{-nC}$ ) to give a model for settling velocity as a function of sludge settling properties. The law velocity used in the proposed model is based on the Vesilind function.

The settling ability of activated sludge is mainly conditioned by the concentration of bio-flocs, the size and size distribution of bio-flocs, the nature of microorganisms, the geometry of the clarifier, and other parameters such as physicochemical parameters. For example, the presence of filamentous bacteria in excess causes a poor settleability of the sludge. The main difficulties in operating a wastewater treatment plant are related to the maintenance of the equipment and its biological nature. These problems are mainly due to filamentous bacteria. Two forms of filamentous bacteria are present: foaming and filamentous: foaming and filamentous bulking [6,20,30–34].

The most important problem is the difficulty in developing the sedimentation velocity law ( $V_s$ ) as a function of the TSS [2,24,35,36]. This sedimentation rate is also dependent on other factors such as viscosity and density of flocs. Several settling tests carried out in settling column tests are often performed with different initial concentrations (TSS) (Fig. 3). From this Fig. 3, it is easier to develop the settling velocity curves as a function of sludge concentrations (Fig. 4a). The solids flux due to gravity forces ( $F_g$ ) (Fig. 4b) can be calculated by performing settling velocity tests at different initial concentrations. Eq. (1), the hydraulic flux ( $F_h$ ), is a linear function of the solids concentration ( $C$ ), of the right slope ( $V_h = Q/S$ ) (Fig. 5). Note that the variation of the sedimentation velocity is not linear or constant in both zones (clarification and thickening).

Fitch [1], Torfs et al. [24], and Ong [26] consider the settling process of four sedimentation regimes. The diagram

is schematized in Fig. 5, established initially by Fitch in a precise way the effect of cohesion between particles and the concentration of suspended solids, where the zones of the different regimes are determined. Bürger and Wendland [37] confirm the existence of these four sedimentation regimes, and they only examined the two zones: clarification and thickening. Note that Cho et al. [29] also distinguished these regimes. The sedimentation regimes are determined by following the evolution of the appearance of a solid suspension with tests performed in a settling column tests.

It is imperative to understand the activity of biological flocs and their properties by studying their behavior and settleability. Most of these studies have been focused on empirically relating the settling velocity process to the different operational variables involved. The most used current models are based on Vesilind's exponential [18,20,24,36,38,39]:

Settling column tests are the most interesting source of information to determine sedimentation velocity and its characteristics. The characterization of sludge settling is important to determine the performance of secondary clarifiers as well as to evaluate the treatment efficiency of wastewater treatment plants.

The design and operation of secondary clarifiers are generally based on solid particles flow theory. Vanderhasselt and Vanrolleghem [40] reported that the baseline data required in this theory application can be obtained from multiple settling column tests. They also noted that initial settling velocities ( $V_s$ ) over a range of sludge concentrations ( $C$ ) are measured for different dilution experiments.

De Clercq et al. [41] consider that batch sedimentation tests are used to control the sedimentation process and to validate sedimentation models as well as to measure sludge quality. Note that settling column tests can be integrated into an automatic control system of a wastewater treatment plant (through the introduction of a PLC). The settling velocity formula used to describe the settling behavior of activated sludge in a secondary clarifier. Table 3 summarizes the settling velocity parameters obtained for three WWTPs included in this study.

The total solids-flux curve is obtained by addition of the two sedimentation flux curves ( $F_s$  and  $F_h$ ) (Fig. 5). A pin-point minimum can be observed when the two solids flux curves are added. It is called the limiting flux ( $F_L$ ). Where  $F_L$  is the maximum mass flux capacity in the thickening zone at equilibrium condition. The minimum point of intersection of the curve with a horizontal line provided the value of  $F_L$ . The corresponding limiting concentration ( $C_L$ ) represents the solids concentration between the sludge layer and the bottom of the secondary clarifier. At the concentration ( $C_L$ ), the transport flux is the equal to the total flux, which represents to the solid matter concentration in the recycling flow. This approach is based on the principle that sedimentation due to gravity is negligible in the lower part of the secondary clarifier [16,38,42,43].

### 3.2. Validation of the model

Generally, many modes of sedimentation are considered in the settling process: flocculation, clarification, thickening, and compression. In a settling column test, the classification of these regimes is immediately apparent. Bürger and

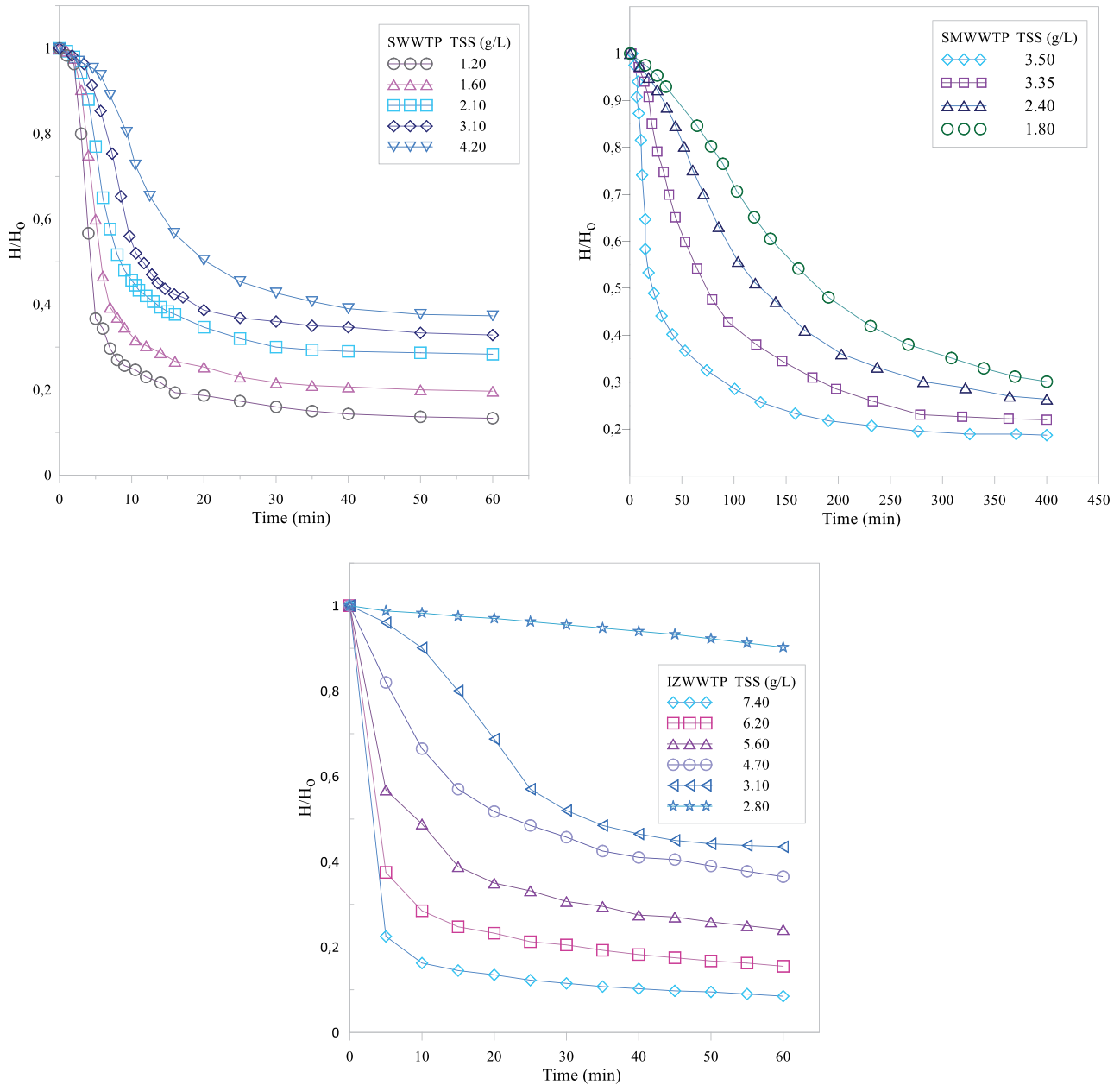


Fig. 3. Different types of activated sludge and settling column tests experiments.

Wendland [37] confirmed the existence of different sedimentation regimes but only investigated at two of them: clarification and thickening zones. Because the thickening and compression zones are of the same nature, we assume only as a single zone. For this reason, the term thickening has been reserved for this manuscript. The thickening zone is the lower part of the secondary clarifier where the sludge is stored. In this zone, the structure of the flocs is suitably consolidated and of greater density.

The one-dimensional (1D) model is considered with no biological reaction in the secondary clarifier to minimize discontinuity between layers. This assumption is justifiable because the aeration tank is where the bulk of the chemical and biochemical interactions occur. The model [Eq. (15)]

ignores particle aggregation and assumes that the system mass remains constant.

The settling process model for secondary clarifiers does consider back mixing (sludge dispersion) in the upward direction but no radial direction. The radial dispersion is neglected because of the effects of hydraulic and sedimentation flows (gravity).

A finite differences method is used to solve the differential Eqs. (16) and (17). This numerical method takes into account two zones (Fig. 2), each with five (5) layers: clarification zone  $I = 1, I = M-1$  and thickening zone  $I = M, I = N$ , that is,  $M = 5$  and  $N = 10$ . It is also important to note that the thickness of each layer is the same (=constant). The height of the sludge blanket, the concentration, and the settling velocity are all

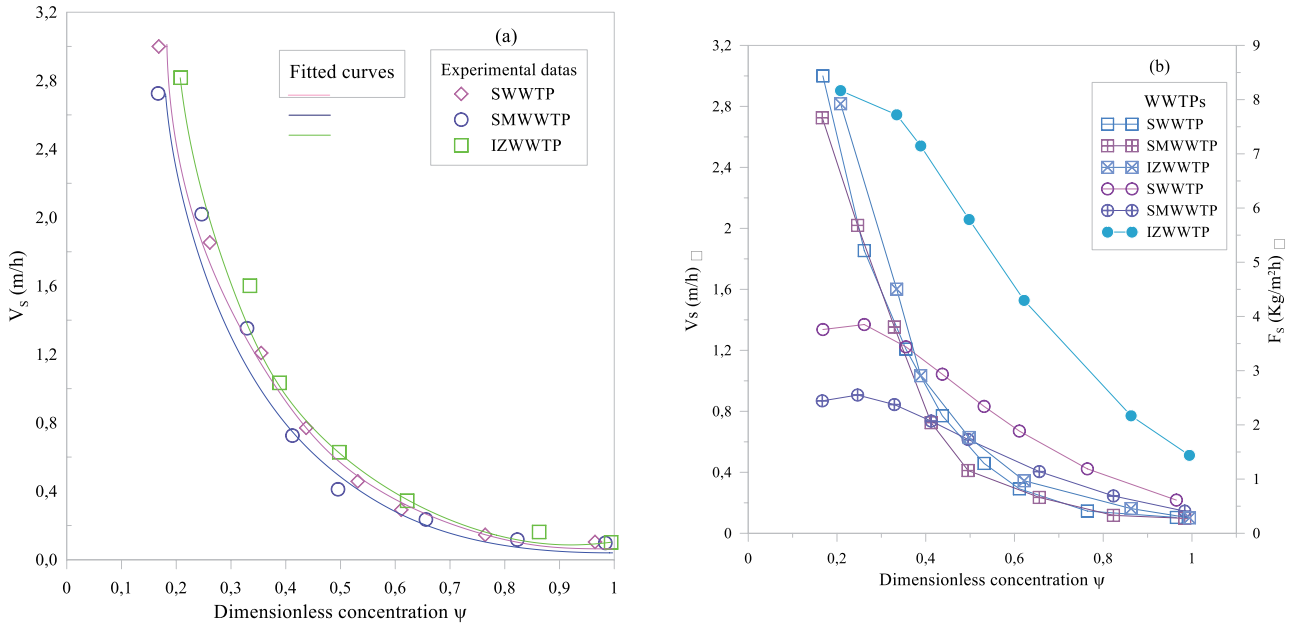


Fig. 4. Estimation of parameter values from three full-scale activated sludge: (a) sedimentation velocity and (b) flux of solid particles.

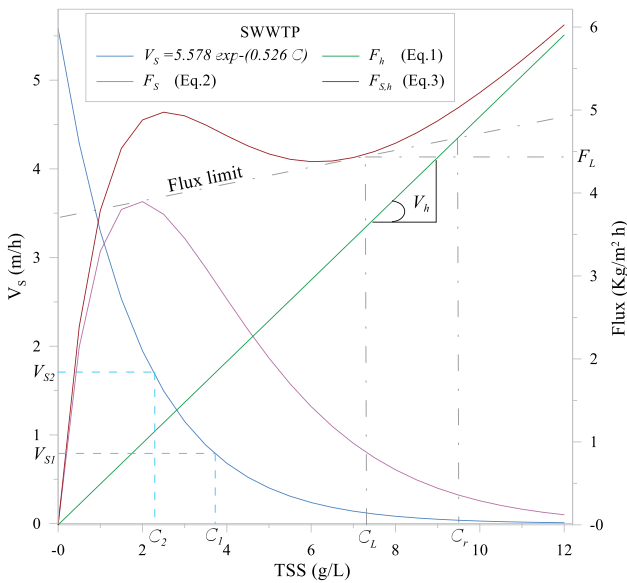


Fig. 5. Graphical analysis of the solid particles flow theory in SWWTP.

variable in this model. The mean squared error (MSE), measures the average of the squares of the errors, represented in Fig. 6, is defined as:

$$MES = \frac{1}{N} \sum_{i=1}^N (\psi_s - \psi_M)^2 \tag{18}$$

where  $N$  is the number of data points;  $\psi_s$  is the dimensionless simulated depth, g/L;  $\psi_M$  is the dimensionless measured depth.

The hydraulic feed flow ( $Q_f$ ) is divided into two flows: upward bulk flow (ascending clarification zone,  $Q_u$ ) and

downward bulk flux (descending thickening zone,  $Q_d$ ).  $C_f$  represents the initial concentration, that is, concentration of the mixed liquor in the aeration tank entering the secondary clarifier. It varies during the day and depending on the peak flow rate (low and medium load). This concentration is subjected to a settling in the secondary clarifier. It is concentrated at the bottom of this tank, and it is almost zero at the surface. For example, the concentration feeding the secondary clarifier is 5 g/L. This concentration is almost zero at the top layer, and it can attend at 12 g/L at the bottom of the clarifier.

The experiments realized on the three wastewater treatment plants study the settling properties of activated sludge on a large scale. It noted that these treatment plants mainly treat municipal wastewater. The study sites, especially the secondary clarifiers, which are of the same geometry, of a cylindrical cone bottom tanks with centre feed. The measurement of sludge blanket height as a function of sludge concentration was performed for the three plants included in this study and with different loads (802, 856 and 2,880 m<sup>3</sup>/h).

A series of samples were obtained at different depths and, at the same time, at four radial depth points. The average sludge concentration  $C_f$  at the inlet was 2.7, 2.65 and 5.1 g/L for SWWTP, SMWWTP, and IZWWTP, respectively. The influent well feed the secondary clarifiers of the plants (SWWTP, SMWWTP, and IZWWTP) at a depth of about 1.85, 1.55 and 2.10 m, respectively. Based on the results obtained from Fig. 6, the modelling of sludge blanket height (SBH) as a function of total suspended sludge fits the experimental results at the three treatment plants, with MSE values that are extremely very close to zero.

#### 4. Conclusion

In this study, the clarification and thickening are considered during the settling process in the secondary clarifier.



Table 3  
Vesilind parameters for settling velocity models

WWTP	SSVI (range) (mL/g)	TSS (range) (g/L)	$k$ (m/h)	$n$ (L/g)	R-square
SWWTP	71–205	1.20–4.20	$5.578 \pm 0.193$	$0.526 \pm 0.054$	0.975
SMWWTP	37–85	1.80–3.50	$4.970 \pm 0.158$	$0.715 \pm 0.055$	0.966
IZWWTP	135–350	2.80–7.40	$5.803 \pm 0.140$	$0.259 \pm 0.014$	0.986

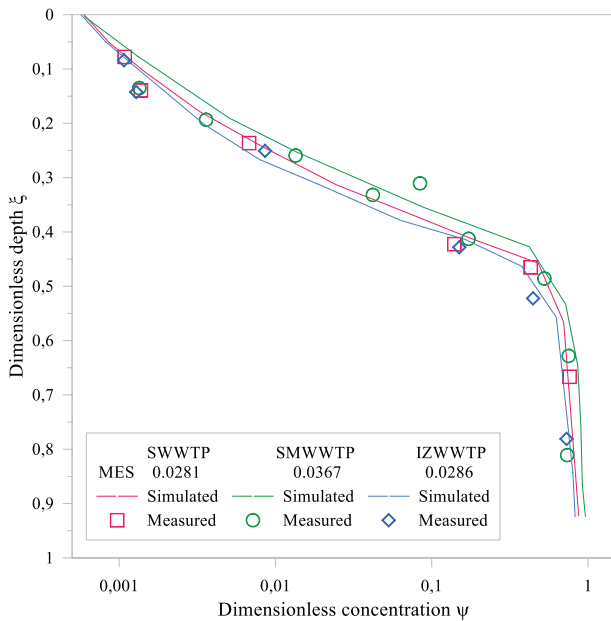


Fig. 6. Estimation and validation of secondary clarifiers model using three different types of activated sludge systems.

Based on the Vesilind function and data obtained from three full-scale activated sludge, the settling velocity expression was developed as a function of stirred specific volume index in this model (SSVI). It has been integrated into the secondary clarifier model, given the results from the batch settling tests. Using some experimental data from the study sites that were collected from three WWTPs, which were then employed to validate the model. The mathematical modelling of the overall solid particle concentrations and the height sludge blanket (SBH) also confirm actual experimental results. This dynamic analysis and simulation would be a great addition to the system in place for hydrodynamic excess monitoring and control.

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**Symbols**

- $C$  — Total suspended solids (TSS), g/L
- $C_L$  — Limiting concentration, g/L

- $C_r$  — Return activated sludge concentration, g/L
- $D$  — Dispersion coefficient of suspended solids in the axial direction,  $m^2/s$
- $F_h$  — Hydraulic flux,  $g/m^2 \cdot s$
- $F_{he}$  — Upward bulk flux,  $g/m^2 \cdot s$
- $F_{hr}$  — Downward bulk flux,  $g/m^2 \cdot s$
- $F_d$  — Dispersion flux of suspended solids in the axial direction,  $g/m^2 \cdot s$
- $F_L$  — Limiting flux,  $g/m^2 \cdot s$
- $F_s$  — Gravity settling flux,  $g/m^2 \cdot s$
- $F_t$  — Total solids-flux,  $g/m^2 \cdot s$
- $L$  — Length of the secondary clarifier, m
- $k_i$  and  $n_i$  — Settling parameters
- PE — Population equivalent
- $Q_f$  — Feed volumetric flow rate,  $m^3/s$
- $Q_e$  — Exit volumetric flow rate,  $m^3/s$
- $Q_r$  — Recycled volumetric flow rate,  $m^3/s$
- $(r_c)$  — Rate of reaction,  $g/m^3 \cdot s$
- $S$  — Surface area of the secondary,  $m^2$
- SSVI — Stirred specific volume index
- SVI — Sludge volume index
- SBH — Sludge blanket height, m
- $u$  — Flow velocity, m/s
- $V$  — Cylindrical volume,  $m^3$
- $V_{he}$  — Upward hydraulic velocity, m/s
- $V_{hr}$  — Downward hydraulic velocity, m/s
- $V_s$  — Settling velocity, m/s
- $Z$  — Depth of the secondary clarifier, m
- $\xi$  — Dimensionless length
- $\psi$  — Dimensionless concentration
- WWTP — Wastewater treatment plant

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